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The Examination of Representations used by Classroom Teacher Candidates in Solving Mathematical Problems

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Abstract

This study was designed according to the mixed research method in which quantitative and qualitative research methods were used in order to identify the challenges confronted by classroom teacher candidates in solving mathematical problems and the factors affecting how they choose these representations. The population of this study consisted of 100 3rd year students attending the primary teaching department of Çukurova University during the 2012-2013 academic year. In identifying the samples, the criterion sampling method, a purposeful sampling method, was used. The data collection tools consisted of The Use of Multiple-Representations in Problem Solving Test and a semi-structured interview form prepared by the researcher. At the end of the research it was discovered that classroom teacher candidates could use different types of multiple representations in solving mathematical problems. However, they used spoken language and algebraic representations the most. During the interviews, teacher candidates emphasized that experience and the content of the problem are the most important factors in identifying which multiple representations to use. Another important result obtained from the research is that teacher candidates can have problems forming a suitable theme for problems and shifting between representations. For this situation, it can be suggested that giving importance to the concept of multiple representations in the teaching of mathematics as well as doing in-class application studies intended to solve and associate problems using the concept of multiple representations would be beneficial. In that way, teacher candidates can guide their students towards different points of view while solving problems.

Keywords

Multiple Representations, Classroom Teacher Candidates, Problem Solving, Mathematics, Polya.

The concept of using multiple representations has an important place in the teaching of mathematics (Cai, 2005; Cobb, Yackel, & Wood, 1992; Goldin, 1998; Janvier, 1987), especially in the understanding of mathematical concepts and interpreting them from different points of view through the use of multiple representations (Cathcart, Pothier, Vance, & Bezuk, 2006; Hjalmarson, 2007; Pape & Tchoshanov, 2001). According to Tripathi (2008), the use of multiple representations in teaching mathematics is a strong instrument that eases the understanding of mathematical subjects for students. Also, the use of multiple representations strengthens the understanding of students for learning how to form and solve problems in a mathematics course.

The new standards of the American National Council of Teachers of Mathematics (NCTM) published in 2000 especially emphasize the importance of the concept of "representation." Within this context, according to the NCTM, the use of diagrams, graphics, tables and symbols, as

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well as transitioning between them, is of capital importance in expressing mathematical thoughts and relations. Stemming from the NCTM (2000) standards, representations are also a part of the required abilities in the mathematics curriculum of Turkey for use in problem solving, communication (verbal lectures, written statements, images, graphics, concrete concepts), and associations (Milli Eğitim Bakanlığı [MEB], 2005, 2009). Stylianou (2010) also states that students should be effective in using representations and their transitions for solving mathematics and understanding mathematical concepts.

As a general term, a representation is a way to show an actual situation from a different point of view (Even, 1998; Goldin & Kaput, 1996). In mathematics, however, teaching representation is a part of forming or shaping a mathematical concept. The different representations that teachers use during in-class activities affect the knowledge, and accordingly, the success of students (Cai, 2005; Neria & Amit, 2004; Stylianou & Silver, 2004). That is why the understanding of representations and how to use different types of representations should be an active part of the teaching process (Hjalmarson, 2007; Pape & Tchoshanov, 2001). Researchers mostly focus on two different representation types regarding the classification of representations. These are internal and external representations (Cai, 2005; Goldin, 1998; Goldin & Shteingold, 2001). Within this scope, internal representations are defined as those that express a reality using mental models (Cai, 2005; Hiebert & Carpenter, 1992), cognitive diagrams developed via experiences, or abstractions of mathematical thoughts (Pape & Tchoshanov, 2001). External representations are the expression of a person's thoughts regarding a certain reality by use of visual objects (Cai, 2005) or the use of written or verbal words (Goldin & Shteingold, 2001) involving numbers, algebraic equations, graphics, tables, diagrams or charts (Pape & Tchoshanov, 2001). Also, concrete structures such as tables, graphics, images, diagrams that are used in problem solving, or the defining of mathematical concepts are considered to be external representations (Goldin & Janvier, 1998). In order to deepen the understanding of students, teachers must define a concept by using the multiple representation method, or more plainly, using different types of representations. Instead of using just one model, teachers must present a concept by using different representations and then make suitable transitions between them (Ball, 1990).

Representations, which are effective in forming or shaping a mathematical concept (Goldin & Shteingold, 2001), are part of the abilities required in mathematics curriculum, including problem solving, communication and association. Accordingly, in the problem solving process, the path of solving the problem must be given importance. At this level, how a student solves the problem, on which level he provides which representations, and how the representations help to solve problems must be emphasized.

The representation models in problem solving involve spoken symbols, written symbols, static figural models (or pictures), manipulative models and real world situations (Goldin, 1998; Lesh, Post, & Behr, 1987). In this regard, for example, spoken symbols about the concept of the number 65 can be "sixty five, seventy minus five," or "six tens plus five units." Written symbols can be "65 = 6x10+5x1," Static figure models or pictures can be "six tens and five units" models. Manipulative models can be something such as "sixty five dollars." Real world situations can include such scenarios as "How old is Ali?" or "What is the price of a school bag?" It can also be seen that these five representations are not independent from each other; each representation can transform into another model of representation (Cathcart et al., 2006; Van De Walle, 2007). One of the important properties of problem solving is the application and formation of suitable representations (Cai & Lester, 2005, p. 202). When the literature is examined, the studies, especially the one from Polya (1990), focus on four problem solving stages. Inextricably intertwined, these stages are: understanding the problem, planning, applying the plan, and evaluating the solution. At the stage of understanding the problem, the student is expected to figure out what the problem is, its conditions, what is required to answer it, and draw images related to the problem (Polya, 1990). The planning stage is the first step of solving the problem after understanding the problem. A plan is formed by using the experiences obtained from similar problems. At this stage, suitable strategies should be developed and a suitable mathematical model should be formed for the solution of the problem (Baykul, 2005). The third stage is the application of the prepared plan. The last stage, in which the solution is evaluated, the solution is checked logically. At this stage it should be questioned whether the solution makes sense or not.

In literature, it is seen that studies that address the representations used in solving mathematical problems generally focus on the primary school level (Ahmad, Tarmizi, & Nawawi, 2010; Cai, 2000, 2004; Cai & Hwang, 2002; Gagatsis & Elia, 2004; Hwang, Chen, Dung, & Yang, 2007; Kılıç, 2009; Monoyiou, Papageorgiou, & Gagatsis, 2007; Sert, 2007). However, although they are limited in number, there are also some studies which focus on the academic level (Delice & Sevimli, 2010a, 2010b; Gagatsis & Shiakalli, 2004; Herman, 2007; İpek & Okumuş, 2012; Villegas, Castro, & Gutierrez, 2009). In this context, Gagatsis and Shiakalli (2004) examined the verbal, graphical and algebraic types of representations that 195 teacher candidates used in solving problems of functions, how they transition from one to the other, and the effect of these transitions with solving the problem. In the research, how teachers made transitions between representations was examined by applying two tests consisting of six questions. In one of the tests the transition from verbal representation to graphical and algebraic forms was examined; and in the other, the transition of a graphical representation to algebraic and verbal ones were examined. At the end of study, it was found that the types of representations that teacher candidates preferred affected the ability to solve problems by 47%. In addition, it was also found that teacher candidates had low success in transitioning from graphical representations. Neria and Amit (2004) examined problem solving patterns, which representation models were preferred in the verification period, and the effect of these representational models on academic success. In total, 46 problems, short, open-ended and multiple choice, were posed to 164 candidates. The candidates talked told about problems in their explanations that they classified representation used in verification period as algebraic, verbal, numeric, diagram and graphical. At the end of research, it was found that most of the candidates preferred verbal representations. When the relation between the level of success and the representational models used were examined, it was observed that the success of candidates using algebraic representations was higher than the ones using other representational models.

Villegas et al. (2009), in their research with teacher candidates being educated in a mathematics department in Spain, examined their thinking ability during the process of solving problems and understanding mathematical concepts, as well as how they define them with representations and their ability to transition from one representation to another. In this context, with the observation data presented as the representations used by teacher candidates, the transitions between representations, their way of thinking, and how often they were used was examined. At the end of research it was found that there is a significant and positive relation between the success of the problem solving of a candidate and their ability to use representations. Similarly, İpek and Okumuş (2012) examined which representations primary school mathematics teacher candidates used in the problem solving stages and which problems they used with which representation. The data was collected from interviews done with 48 teacher candidates and also from multiple representations using a test. At the end of the research, it was found that teacher candidates, especially in the process of solving problems, use verbal representation more than algebraic, graphical and numerical representations. On the other hand, another important result was reached, especially in regard to the understanding of a problem: the candidates have difficulty forming suitable representations for a problem and doing transitions between representations.

Again, Monoyiou et al. (2007) in their research examined the representations used in solving non-routine problems by primary school teachers in Cyprus. 20 teachers attended the research. With the data obtained at the end of the interviews, it was found that the teachers mostly preferred algebraic representations.

On the other hand, Herman (2007) examined the strategies of teacher candidates giving algebra lessons and their beliefs about multiple representations. In the study that was formed using the experimental design, the teacher candidates had ten weeks of training, and the differences between the pretests and post-tests were examined. Also, semi-structured interviews were done with the candidates as well. The candidates were asked to transform questions from the post-test, as well as six questions from the pretest, into algebraic, graphical and table forms. At the end of the research, the results showed that teacher candidates preferred algebraic representations before training, whereas they also concentrated on graphical and table representations. Besides this, it was additionally found that the success of candidates in transitioning between representations increased after considering the results of the pretest. Delice and Sevimli (2010a) in their research examined the representations that teacher candidates used in the solution of definite integral problems, their ability to transition between representations, and the relationship between the success of problem solving and the current representations. 45 mathematics teacher candidates applied for the study, which was designed according to a case study having a qualitative interpretive paradigm. In the study data was collected through a classical written test, an interview form, a representation preference and transition test, a participant observation form, and document analysis techniques. At the end of research it was found that the ability of teacher candidates to use multiple representations in the process of solving problems was not at the expected level. It was also found that the candidates wanted to use algebraic representation for the solution of all problems and that they were accustomed to using this representation.

In short, in light of this data, it was seen that studies related to types of representations generally examine the mathematics teacher candidates teaching in the mathematics departments of primary or secondary schools. However, when the literature about the subject is examined, there has been no study which directly questioned how classroom teacher candidates apply multiple representations. Based upon this phenomenon, it was the aim of this study to identify the representational models that classroom teacher candidates use in the solving of routine and non-routine verbal problems, the factors affecting how they identify these representations, and the challenges they confronted in this process. In the direction of this main aim, the answers to the questions below were researched in this study.

1) What types of representations do classroom teacher candidates use for routine problems regarding Polya's problem solving stages?

2) According to Polya's problem solving stages, what types of representations do classroom teacher candidates use for non-routine problems?

3) What are the factors that affect how types of representations are chosen in Polya's problem solving stages?

4) According to Polya's problem solving stages, what are the opinions of teacher candidates about the problems they experienced while using representations?

Method

Research Model

This research was designed using a mixed research method in which quantitative and qualitative research methods were used for identifying the representations used by classroom teacher candidates during the problem solving period, the factors affecting their choice of representation, and the problems experienced in this process. The aim of using quantitative and qualitative research methods is to increase the advantages of combining the two methods and to decrease the disadvantage of research that uses quantitative or qualitative research by itself (Johnson & Onwuegbuzie, 2004). At the same time, examining the same fact from these different aspects can increase the scope, deepness and power (Creswell, 2003; Punch, 2005).

Studying Group

In the study, 100 students (73 females; 27 males) who are educating in 3rd year in a department of classroom teaching at Çukurova University during the 2012-2013 academic year, were studied. 73% of the students were female and 27% of them were male. The 3rd year students who formed the study group had completed the courses of Basic Mathematics I, II and Mathematics Teaching I. In the syllabus for Basic Mathematics I and II, mathematical concepts and calculations, numbers, ratios and proportions, equations and inequalities, algebraic expressions and verbal problems were studied. Within the scope of Mathematics Teaching I, subjects such as important abilities in mathematics education, associations, representations, communication, reasoning, estimations, and problem solving (i.e. strategies, Polya's problem solving stages, routine and non-routine problems, and their applications) were covered. The participating teacher candidates were given six hours of theoretical knowledge about Polya's problem solving steps (understanding the problem, planning, application of the plan, and evaluation of the solution), the multiple representations used in this context, as well as routine and non-routine problems within the scope of Mathematics Teaching I.

In the process of identifying the group to be interviewed, the criterion sampling method from the purposeful sampling methods group was used. According to this method, sampling was thought about and identified beforehand to be related to a specific aim or about a focused subject (Punch, 2005). In this study, teacher candidates who were willing to participate in the study, as well as having had completed the Mathematics Teaching I course successfully and having shown success in using the Multiple Representations. In Problem Solving Test, were chosen and semi-structured interviews were performed. Ten of the candidates were female and five of them were male. After the results of the

Multiple Representations in Problem Solving Test were evaluated, four of the teacher candidates were found to be at a low level, seven of them were at a medium level, and the remaining four were at a high level of success.

Data Collection Tools

As a data collection tool, the Use of Multiple Representations in Problem Solving Test and a semi-structured interview form were used. Five of the questions in the Use of Multiple Representations in Problem Solving Test were routine, the other five were non-routine verbal problems. During the preparation period for the non-routine problems, related literature was used (Inoue, 2005; Verschaffel, De Corte, & Lasure, 1994; Xin, Lin, Zhang, & Yan, 2007). The problems in this test could be solved by using different representations. In this context, the first problem, a routine one, and the sixth problem, a non-routine one, are defined below as examples.

1st problem, routine: AB road is 200 km. Two cars are moving from point A with a speed of 50 km and 30 km. When one of the cars reaches B and immediately turns back without stopping and reaches C, the other car has already reached C. According to this, how many kilometers is the road CB?

6th problem, non-routine: 1128 students are going to a picnic by bus. Each bus can take 36 students. How many buses are needed to carry those students?

In order to check the validity of the Use of Multiple Representations in Problem Solving Test, it was stated by two specialists that the test had scope and aspect validity as well as with its target behaviors. Also, the views of the specialists were compared using the Kappa test, and at the end of this comparison, the Kappa parameter was calculated at .83. Later, the pilot scheme was applied to three students who were studying in their 4th year and had completed their mathematics teaching courses successfully. During the application period, the students were informed about the aim of the research and were checked to see if there was any expression they did not understand in the problems. During this application no complication arose in terms of understanding the expressions and the duration. Semi-structured interviews with 15 teacher candidates were performed for the qualitative part of the research. When the interview form was being prepared, data obtained from the related literature and the opinions of specialists were taken into consideration. After it was prepared, the semi-structured interview form was examined by four academic members who are specialists on qualitative research and mathematics teaching, and it took its final form from the suggestions they gave. The semi-structured interview form was also sent to two specialists in the fields of both qualitative research and mathematics. Again the data received was compared by applying the Kappa test, and at the end of this comparison, the Kappa parameter was calculated at .77 and .82 respectively.

In the semi-structured interview form, the students were asked how they would generally identify their representations in Polya's problem solving stages, if they would use any additional representations, how to effectively use multiple representations correctly in problem solving, and if there was any problem with this stage. The negotiations that were recorded using a voice recorder were transferred to a computer, and 42 pages of pure data were obtained. At this stage, the teacher candidates were given the codes $GÖ_1$ for the first interviewee, $GÖ_2$ for the second interviewee, etc., during the interview.

Collecting Data

The teacher candidates who participated in the study were given the Use of Multiple Representations in Problem Solving Test by the researcher. This test was applied over approximately 60 minutes. After this stage, semi-structured interviews were done with the teacher candidates who showed different levels of success. These interviews lasted 15-20 minutes.

Data Analysis

Data obtained from the Use of Multiple Representations in Problem Solving Test was evaluated by taking Polya's four stages in problem solving into consideration. According to this process, for each stage the representation that teacher candidates used was determined by examining the stages of problem solving such as understanding the problem, making a plan, applying the chosen plan, and evaluating the result. In the coding of these representations related literature was used (İpek & Okumuş, 2012; Lesh et al., 1987). According to this, the following representations were coded:

1. Verbal Representation: Students are expected to understand a problem related to mathematics by expressing the solution or result of the problem verbally.

Table 1
According to Polya's Problem Solving Stages the Distribution of Representation Types Used in Routine Problems

P	Understanding a Problem					Choosing the Plan					Applying a Chosen Plan					Evaluation				
	KD	GT	CT	ST	B	KD	GT	CT	ST	B	KD	GT	CT	ST	B	KD	GT	CT	ST	B
	F	f	f	f	f	f	f	f	f	f	f	f	F	f	f	f	f	f	f	f
1	89	79	5	3	1	78	13	13	-	14	14	5	91	1	5	52	6	27	-	30
3	89	28	11	7	7	63	2	13	2	26	9	2	92	4	8	49	1	21	1	38
4	76	39	1	28	4	75	2	7	5	17	19	9	43	41	6	58	3	8	5	29
5	93	5	3	2	5	75	2	11	-	22	6	2	92	1	8	55	-	26	-	27
9	96	42	3	1	3	80	7	4	-	15	25	3	90	2	3	69	2	28	-	19

(P: Problem number, KD: Verbal Representation, GT: Graphical Representation, CT: Algebraic Representation, ST: Numeric Representation, B: Empty)

2. Graphical Representation: Using a shape, diagram or a number line in explaining the problem.

3. Algebraic (Symbolic) Representation: Using mathematical symbols in problems or using arithmetic calculations involving symbols.

4. Numeric Representation: Using a table or matrix in explaining the problem.

In order to verify the reliability of The Use of Multiple Representations in Problem Solving Test, 10 of the students' previously answered answer sheets were randomly chosen, analyzed separately by a second encoder (an academic member from the educational field of mathematics) and at the end of the calculations which were performed by using the *Consensus/(Consensus + Dissensus) X 100* formula offered by Miles and Huberman (1994), the accordance between the researcher and the second encoder was calculated at .97.

In the section of the study involving quantitative data, descriptive statistics were used, whereas with qualitative data, descriptive and content analysis were used. Descriptive analysis is the interpretation and summarization of data according to identified themes. However, in content analysis data is firstly conceptualized then according to the identified concepts suitable codes, themes, and when necessary, sub-codes are formed. While the codes were being formed, the pure data obtained from the interviews with the teacher candidates were read line by line, and some definite codes were formed according to the direct data and sometimes to the meanings which came out and were marked on the article. The coded data was grouped according to similarities and differences. Next, the related codes were brought together to make thematic coding (Yıldırım & Şimşek, 1999). In the analysis period of qualitative data, a specialist in teaching mathematics analyzed randomly chosen interviews from five teacher candidates as a second encoder and the accordance ratio between the two coders was calculated at .95. Also, the consistency of codes done by the researcher at two different times was also examined. For this, the researcher recoded the interview forms of five candidates three weeks later thereby testing his own consistency. At the end of this, the researcher calculated the reliability parameter at .98.

Findings

The findings obtained from the analysis of the Use of Multiple Representations in Problem Solving Test and the semi-structured interview forms from the classroom teacher candidates who participated in the study were presented in parallel with the secondary aims of the research.

Findings Related to 1st Secondary Aim

The first aim of research is to find representation types that the teacher candidates use in Polya's problem solving stages and at the end of this analysis the data which was obtained is shown in Table 1.

When Table 1 is examined, it can be generally said that the teacher candidates used almost all the different representation types in all stages of the problem solving period. In this context, it was seen that in all stages, verbal representation was used the most. On the other hand, in applying the chosen plan stage, it was seen that algebraic representation was mostly used. In the first stage of understanding a problem, it can be said that verbal and graphical representations are frequently used. For example in the "understanding a problem" stage of the 4th problem, the verbal representation of the teacher candidate, $Ö_{23,}$ takes place in Figure 1.

It is seen that teacher candidate $Ö_{23}$ used both verbal and graphical representations in the "understanding a problem" stage. When Figure 1 is examined, it is also seen that the teacher candidate

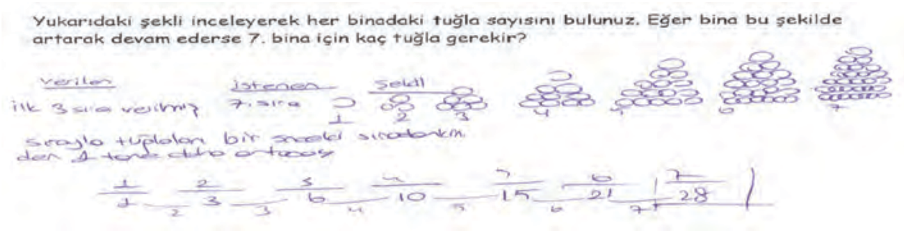

Figure 1: The verbal representation of teacher candidate $Ö_{23}$ during the "understanding a problem" stage of the 4th problem and their graphical representation.

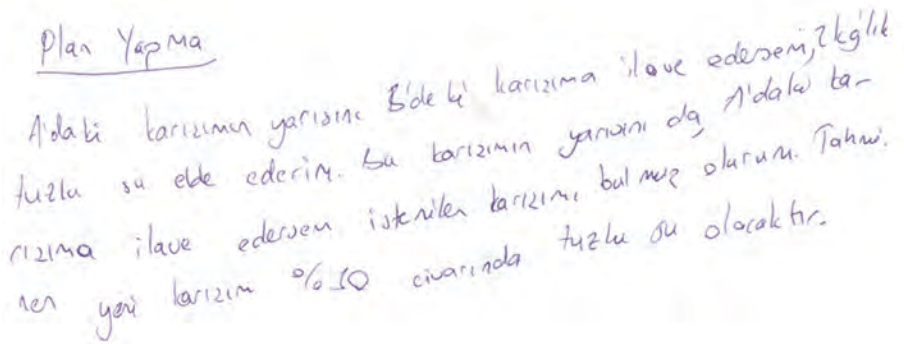

Figure 2: The verbal representation of teacher candidate $Ö_{28}$ used in the planning stage of problem #9.

verbally stated what is given in the problem: For the first three rows, each row involves one more brick than the previous, and after verbally stating that the number of bricks required for 7 rows is the question, he drew an image to understand this situation.

In the second stage of choosing a plan, when Figure 1 is examined, it is seen that the teacher candidate used verbal representation more than they used graphical and algebraic representations. However, only seven teacher candidates used numerical representations. For example, in the 9th problem, for the stage of choosing a plan, the verbal representation of teacher candidate $Ö_{28}$ is shown in Figure 2.

On the other hand, in the stage of applying the chosen plan, it is understood from Table 1 that algebraic representations were used by most teachers (approximately 90%). Verbal and graphical representations were used by only a few (approximately 10%) of the teacher candidates. In this context the example related to algebraic representation which teacher candidate $Ö_2$ used in the stage of applying the chosen plan is shown in Figure 3. When Figure 3 is examined, it can be seen that teacher candidate $Ö_2$,while solving the first problem, wrote a general formula in place of the given wording and reached the required answer by using a first order equation.

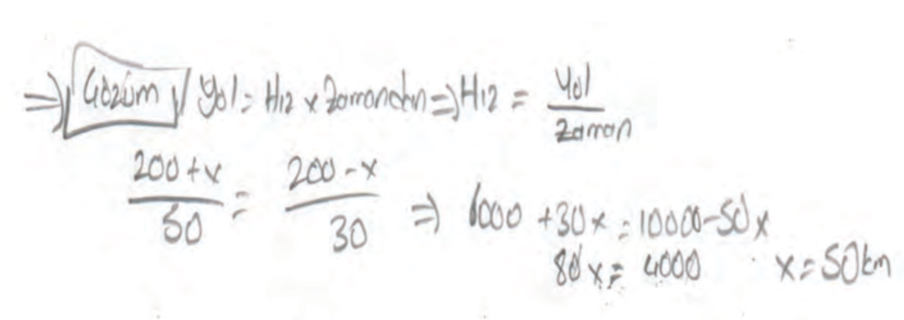

Figure 3: The algebraic representation teacher candidate $Ö_2$ used in the stage of solving problem 1.

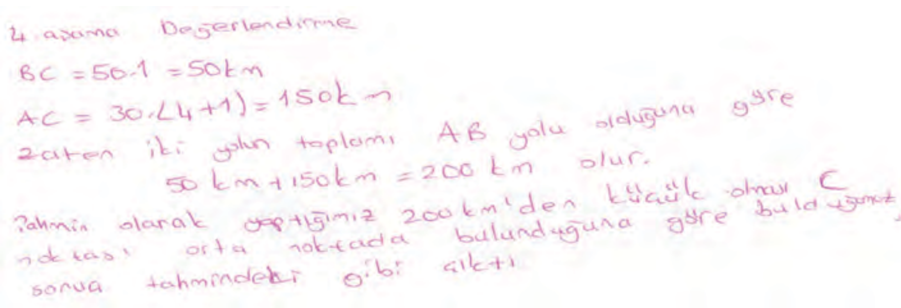

Figure 4: The verbal and algebraic representations that teacher candidate $Ö_{13}$ used during the evaluation stage of problem 1.

It can be seen from Table 1 that 25% of the teacher candidates left the last stage empty, the evaluation stage. It was found that teacher candidates mostly used verbal and algebraic representations during this stage. In this context the representation that teacher candidate $Ö_{13}$ used in the evaluation stage of the 1st problem takes place in Figure 4. When Figure 3 is examined, the stage of applying the chosen strategy, it can be seen that he verified the result by working backwards. The candidate concluded that the total length of the road is 200 km.

Findings Related to 2nd Secondary Aim

The 2nd secondary aim of the research was to find the types of representations that teacher candidates use during the stage of solving non-routine problems, and at the end of this analysis the data obtained is shown in Table 2.

When Table 2 was examined, it was seen that teacher candidates chose to use verbal representations in the solving, understanding, applying and evaluating stages of non-routine problems, whereas in the stage of applying the chosen plan, they mostly used algebraic representation. Also, in the stage of solving non-routine problems, it was noted that numeric representations were used little if at all.

It can be said that during the stage of understanding non-routine mathematical problems, verbal and graphical representations were frequently used. In problem #8, for example, in the stage of understanding, the verbal and graphical representations that teacher candidate $Ö_{19}$ used are shown in Figure 5.

When Figure 5 is examined, teacher candidate $Ö_{19}$ stated the plan verbally and then showed that the school can be in the middle or in the corner by drawing a picture. In this stage, it is seen that teacher candidate $Ö_{19}$ used both verbal and graphical representations.

It can be said that during the stage of understanding the plan, teacher candidates mostly used verbal representations rather than graphical or algebraic representations, however numerical representations were only used by two teacher candidates. For example, in the 6th problem during the stage of choosing a plan, the verbal representation of teacher candidate $Ö_{59}$ is shown in Figure 6. $Ö_{59}$ stated verbally how many buses were required by dividing the total number of students, 1128, by the number of students that each bus can carry. He then stated that the estimated result was less than 36.

In the stage of applying the chosen plan, it is shown in Table 2 that most of the candidates used algebraic representations followed by verbal and graphical

Table 2
According to Polya's Problem Solving Stages the Distribution of Representation Types Used in Non-routine Problems

P	Understanding Problem					Choosing the Plan					Applying of Chosen Plan					Evaluation				
	KD	GT	CT	ST	B	KD	GT	CT	ST	B	KD	GT	CT	ST	B	KD	GT	CT	ST	B
	F	f	f	f	f	f	f	f	f	f	f	f	F	f	f	f	f	f	f	f
2	91	65	3	1	4	66	9	5	-	-	17	34	73	1	14	47	4	13	-	41
6	92	27	2	2	6	75	5	7	-	21	21	2	96	3	1	63	3	28	-	18
7	89	46	1	-	3	76	4	7	1	18	21	16	84	2	3	62	7	29	1	19
8	91	55	1	2	2	77	8	7	1	14	13	26	85	1	4	70	4	16	1	18
10	93	53	5	2	2	78	3	11	-	15	20	8	92	1	4	65	3	25	-	21

(P: Problem number, KD: Verbal Representation, GT: Graphical Representation, CT: Algebraic Representation, ST: Numeric Representation, B: Empty)

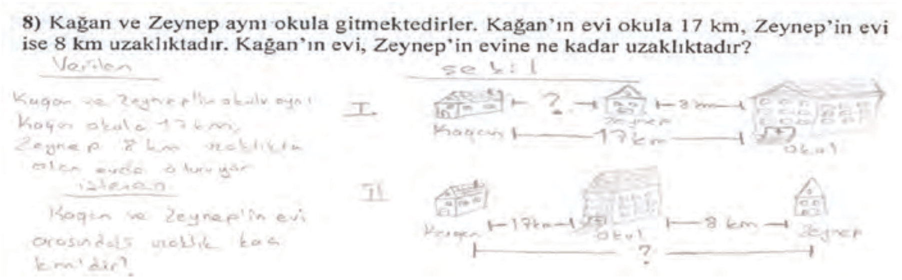

Figure 5: The verbal and graphical representations of teacher candidate $Ö_{19}$ in the understanding stage of Problem 8.

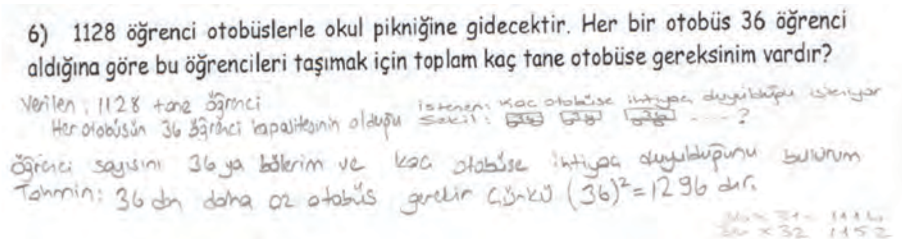

Figure 6: The verbal and graphical representations that teacher candidate $Ö_{59}$ used in problem 6 during the stage of choosing a plan.

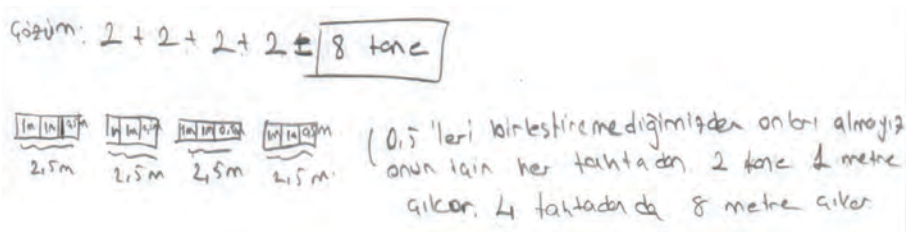

Figure 7: The algebraic, verbal and graphical representations that teacher candidate $Ö_{82}$ used in the problem solving stage of Problem 7.

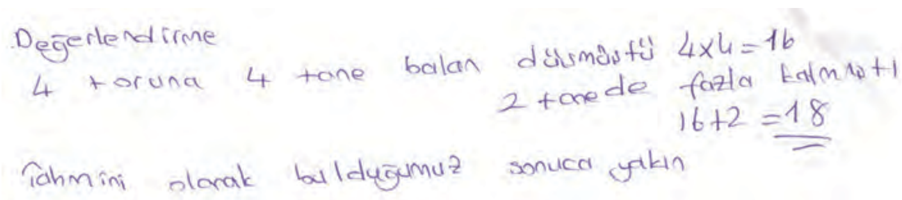

Figure 8: The verbal and algebraic representations that teacher candidate Ö13 used during the evaluation stage of the 10th Problem.

representations. According to this, the algebraic, verbal and graphical representations that teacher candidate $Ö_{82}$ used in the problem solving stage of the 7th problem is shown in Figure 7. Teacher candidate $Ö_{82}$, during the problem solving stage of the 7th problem, stated the solution as (2+2+2+2 = 8 pieces) algebraically, later he showed graphically there would be two full meters of wood in each 2.5 m piece of wood, so he stated verbally that there would be 8 full meter pieces of wood in the four 2.5 meter pieces.

In the last stage, evaluation, it can be seen in Table 2 that nearly 20% of the teacher candidates did not use any representations related to this stage. It was found that the rest of the candidates used mostly verbal and then algebraic representations. In this context, the verbal and algebraic representations that teacher candidate $Ö_{13}$ used in the evaluation stage of the 10th problem are presented in Figure 8. During the evaluation stage, he reached the result of 4 grandchildren have 4 balloons and there are 2 balloons left so (4x4 = 16) and (16+2 = 18) in total, there are 18 balloons.

Findings Related to the 3rd Secondary Aim

The third secondary aim of the study was to find the factors affecting how the participating teacher candidates understand problems and apart from this, they were asked the conditions for using different representations. The data obtained in this scope were applied to content analysis and the themes, codes and the frequency of each code that were formed are presented in Table 3.

Table 3
In The Stage of Understanding the Problem, the Themes, Codes and Frequency Distribution Related to Factors Affecting Teacher Candidates Choice of Representation

Theme	Codes	*f*
Verbal	Problems can verbally be defined well	9
	Habits (Cliched expressions)	5
	Verbal ability	4
	Easy	2
	Giving importance to communication	1
Graphical Representations	Understanding concepts better	9
	Objectifying questions	5
Situations for Using Different Representations	Yes	10
	Differentiation according to question	7
	Conditions depending on understanding	5

As can be seen in Table 3, the teacher candidates stated that they choose verbal and visual representations during the scope of understanding a problem. Also, most candidates (10) expressed that they can use different representations apart from these.

The first theme in the stage of understanding a problem is verbal representation. As a reason for choosing this representation, nine teacher candidates expressed that problems can be verbally defined well, five of them said cliched expressions, four of them said they were better at verbal representation, two of them thought it was easier, and one of them said he gives importance to communication. One comment from the opinions of the teacher candidates about this subject went like this: *"… when we were in math class, my teacher always told us 'If you tell it verbally, I will give you a point.' I think my habit comes from this. Also, my math was terrible at school but I always thought my verbal ability was better. That is why it is easier for me to understand and express it verbally…"* ($GÖ_4$)

The second theme in the stage of understanding a problem is graphical representation. When the teacher candidates were asked the reason why they used this representation, nine of them stated that they understood better using shapes and five of them said questions can be objectified with this representation. In this scope, teacher candidate $GÖ_2$ emphasized the importance of graphical representation with these words: *"In order to understand verbal data, I draw a picture or a graphic of it. I think it is necessary for understanding the problem because objectifying abstract data is more effective. Since objectified data is understood better, it is easier to reach the solution…"*

Lastly when the teacher candidates were asked the conditions for using different representations, ten of them stated that they prefer using different representations, seven of them said it depends on the question, and five of them said they decide which representation they use based on their understanding. The opinions of one of the teacher candidates on this subject went like this: *"…different representations can also be used according to the question. Graphics or different things can be used as well. I can use other representations according to the content of the question. It changes due to my understanding."* ($GÖ_{14}$).

The teacher candidates were asked which factors influenced them in identifying what representation to use in the stage of choosing a plan, and apart from this, the situations for using different representations. The answers given by the teachers were presented in Table 4.

Table 4
In the Stage of Choosing a Plan, the Theme, Code and Frequency Distribution Related to Factors Affecting Teacher Candidates for Identifying Representations

Theme	Codes	*f*
Verbal	Verbal ability	7
	The opinion of plan is suitable to verbal representation	6
	Problems can verbally be defined better	5
	Habits (Cliched expressions)	4
	Situations due to previous education	2
Graphical Representations	Understanding shapes better	3
	Objectifying questions	2
Algebraic Representations	Using formulas in problems	1
Using Different Representations	Yes	3
	Depending on understanding	2
	Deciding according to question	1

As can be seen in Table 4, teacher candidates stated that they choose verbal, graphical and algebraic representations during the scope of planning stage. Apart from this, a few of the teacher candidates stated they use different representations during the planning stage.

The first most common theme in the stage of understanding a problem is verbal representation. As a reason for choosing this representation, half of the candidates said they are better at verbal representation, six of them said verbal representation is suitable for planning, five of them expressed problems can be verbally defined well, four of them said cliched expressions, and two of them said it depends on previous education. One of the opinions of the teacher candidates who commented on this subject is as follows: *"Since my verbal skills are better, I use verbal representation in planning. Verbal representation is very suitable planning. Since we had a traditional education, it also had an effect. When I think about my primary school days, the teacher told us problems orally and wrote them on the board then we tried to solve them. Because of the education we had, we tried to use problems using traditional methods; that is why I mostly used verbal representations…"* ($GÖ_{12}$).

Also, for the planning stage teacher candidates used visual representations as the second most common theme, three of them expressing that they understand shapes better, and two of them said the questions are objectified this way. One opinion of the teacher candidates who commented about this subject went this way: *"I think problems can be well defined by using shapes. We start orally and then support them with graphics. At the beginning stage of a lesson, verbal representations are used then followed with graphical or algebraic representations…"* ($GÖ_{13}$).

Table 5
In the Stage of Applying the Chosen Plan, the Theme, Code and Frequency Distribution Related to Factors Affecting Teacher Candidates to Identify Representations

Theme	Codes	f
Algebraic Representations	Habits (Cliched expressions)	7
	Reaching a solution with mathematical calculations	5
	More understandable	5
	Situations due to previous education	4
	Need to use numeric data	3
	Inefficiency of verbal expressions in the solution stage	2
	Using mathematical symbols is better	2
Verbal	Ability of verbal reasoning	2
	Expressing the reason for a solution	2
Graphical Representations	Objectifying questions	2
Using Different Representations	Yes	7
	Depending on question	6
	Solving with shapes	2

Finally, as a last theme in the planning stage related to using different representations, three teacher candidates stated that they can use different representations. One opinion of the teacher candidates who commented on this said, "*… In fact I can use other representations. I make the plan according to how I understand (it) verbally, algebraically or graphically.*" ($GÖ_{1}$).

The teacher candidates were asked what factors influenced them in identifying representations in the stage of applying the chosen plan, and apart from this, what situations call for the use of different representations. The answers given by the teachers are presented in Table 5.

As can be seen in Table 5, teacher candidates stated that they mostly chose algebraic representations, they prefer to use verbal and visual representations later in the problem solving stage. Apart from this, seven of the teacher candidates stated they use different representations in the stage of solving.

The most common representation in the stage of applying a chosen problem was algebraic representation. As reasons for choosing this representation, half of the candidates said it was habitual, five of them said the result was reached by mathematical calculations, five of them expressed it to be more understandable, four of them said it depended on their previous education, three of them said it was because of the requirement of using numeric data, two of them expressed the inefficiency of verbal expressions during the solution stage, and two of them stated they use mathematical symbols better. One opinion of the teacher candidates who commented on this subject went like this: "*The reason why I generally use algebraic representation is problems cannot be solved sometimes by verbal representations. I use algebraic representations mostly to define the situation better. Because of the influence from previous education, using algebraic expressions is easier for me.*" ($GÖ_{13}$).

Also, for the stage of applying a chosen plan the teacher candidates used verbal representations for the second most common theme, as two of them expressed they wanted to use reasoning ability and two of them said they want to explain the reason for the solution. One opinion of the teachers who commented about this subject said, "*I solved the problem by using mostly algebraic expressions… I also use verbal representation to explain some problems.*" ($GÖ_{5}$).

The third most common representation for the stage of applying a chosen strategy was graphical representation. Within the same scope, candidate teacher $GÖ_8$ expressed his opinion like this: *"In the solution stage I think algebraic representation comes first, then visual representation, because they are more advantageous. As a result it is mathematics and thus the learner can understand and see it better..."*

Finally, the last theme in the stage of applying a chosen plan was related to using different representations, seven (half) of the teacher candidates stated they chose a representation according to the question. One opinion of the teacher candidates who commented about this subject said, "*It changes according to the question. I choose what is appropriate for each question. So when I look at the question, I am thinking about which representation can solve it more easily.*" ($GÖ_7$).

The teacher candidates were asked about the factors influencing them in choosing representations during the evaluation stage, and apart from this, what situations call for the use of different representations. The answers given by the teachers were analyzed using content analysis and the theme, code and the frequency of the different codes are presented in Table 6.

Table 6
For the Evaluation Stage, the Theme, Code and Frequency Distribution Related to the Factors Affecting Teacher Candidates Choice of Representation

Theme	Codes	*f*
Verbal	Interpreting the result of problem	8
	Checking consistency of the estimation with the result	4
	Reach the result by using a different way	3
	Thinking it would be better	2
Algebraic Representations	Crosschecking calculations	5
	Using a different way to solve the problem	5
	Situations due to previous education	1
Using Different Representations	Yes	2
	Depending on the question	2

As can be seen in Table 5, teacher candidates stated that they mostly use verbal representations in the evaluation stage; secondly, they prefer to use algebraic representations. Apart from this, a few of the teacher candidates (2) stated they use different representations in the solving stage.

The first most common theme for the evaluation stage is verbal representation. As a reason for choosing this representation, half of the candidates said it was to interpret the result, four of them said to check the consistency between their estimation and the result, three of them expressed to reach the solution by using a different way, and two of them said it would be better. One opinion of the teachers who commented on this subject was this: "*I use mostly verbal representation for evaluation as there should be a result. Here is given data about solving a problem. In interpreting the result of problem we do not use algebraic expressions. For example, if the result came out as 4.5 people, we interpret it as 4.*" ($GÖ_8$).

In the same scope, teacher candidates used algebraic representations as the second most common theme. Five of them expressed they wanted to crosscheck their calculations, and one of them said it was from the influence of previous education. One opinion of the teacher candidates who commented on this subject said, *"When we talk about evaluation, I crosscheck the result. When I search for another way, I explain it verbally. When I check the accuracy of the calculations, I use algebraic expressions since I am solving the problem using a different way."* ($GÖ_5$).

The third most common theme for the evaluation stage is using different representations. Two of the teachers said they chose the representation according to the question. One opinion of the teacher candidates who commented about this subject went like this: *"... using all representations in the evaluation stage depends on the question. If the question suggests the use of algebra, words or graphics, then I use that representation accordingly."* ($GÖ_{10}$).

Findings Related to the 4th Secondary Aim

The 4th secondary aim of the study is to find the factors which affect problem solving correctly during Polya's problem solving stage. The themes formed, codes and the frequency of each code for this data are presented in Table 7.

Table 7
Distribution of Themes, Codes and Frequencies Related to the Effect of Multiple Representations on Problem Solving

Theme	Codes	*f*
Understanding Of Problem Stage	Easier to understand	11
	Appeals to all students	9
	Different narrations of the same question	8
	Raising of awareness	2
	Objectifying of abstract data	2
Solving Stage	Focusing on solution of the same question with different point of views	9
Evaluation Stage	Checking consistency	6
	Discovering possible errors	2
	Providing an opportunity to reevaluate the result	1

Table 8
The Themes, Codes and Frequency Distribution Related to the Problems of Candidates in Using Multiple Representations According to Polya's Problem Solving Stages

Theme	**Codes**	**Sub Codes**	***f***
Forming Representation Suitable To The Problem	Graphical representation	Difficulty in expressing with images	8
	Algebraic representation	Finding the unknowns in algebraic expressions	5
		Duration	3
	Verbal	Influence of previous education	2
		Solving problems verbally	1
		Not making a plan in verbal representations	1
Transition Among Representations	Stage of transition from graphical representation to algebraic representation		3
	Stage of transition from algebraic representation to verbal		2
	Stage of transition from verbal to algebraic representation		1

As can be seen in Table 7, the teacher candidates were asked their opinions about the effectiveness of multiple representations in solving problems correctly during Polya's problem solving stages. The answers of the teacher candidates were collected under the themes of understanding the problem, solving, and evaluation. According to this, in the scope of understanding the problem in the first theme, most of the candidates (11) stated that multiple representations are helpful for understanding problems easily and some of them (9) said they appeal to all students. One opinion of the teacher candidates who commented about this subject went like this: *"In this age we all know that when we compare one student's visual intelligence to that of others they are all different. Each student understands differently; one of them understands visually while another understands verbally or numerically. That is why teaching mathematics using different representations appeals to all students and this helps them to understand better."* ($GÖ_9$).

In the same theme of understanding the problem, eight of the candidates expressed that teaching the same question in different ways helps. Two of them said it raises awareness in students and another two said that it helps objectify abstract data. According to this, teacher candidate $GÖ_{10}$ expressed his opinion like this: "*If I can use all representations within a single question, some students can understand verbally, some visually, some graphically, others algebraically. Their understanding capacities are different. Since children are not the same it is better to use all of them. In this respect I believe it is more beneficial.*" ($GÖ_{10}$).

As the second most common theme, most of the teacher candidates stated that they can focus on the solution of the same question by using different point of views during the solving stage. One opinion from the teachers' comments states: *"Providing different solution paths for the problem is formed via multiple representations. As a matter of fact, each representation type should be used, since there are individual differences among students. Students will choose what's best for themselves whether it is verbal, graphical or…"*

As the last theme in the scope of evaluation, most of the candidates (9) expressed that multiple representations are effective in problem solving. In accordance with this, six of the teacher candidates said it is effective for checking consistency, two of them said it's a way to discover errors, and one of them said it provides a recheck of the solution. One of the opinions from the teacher comments about this subject goes like this: "*Using different representations can show different ways of thinking. It can help one understand if their way is correct. I think doing something planned and regular is very important. Progressing in a planned manner in mathematical calculations makes finding the solution easy so we can check the accuracy of a solution. That is why it is good to use all representations in all stages.*" ($GÖ_6$).

As the last aim of the research, the teacher candidates were asked if they had any difficulties while using multiple representations according to Polya's problem solving stages. Five of the teacher candidates stated that they had no problem during that period. The themes, codes and frequency distributions related to the problems of the other teacher candidates are presented in Table 8.

As can be seen in Table 8, most of the teacher candidates expressed having problems in forming a suitable representation to a problem. Nearly half of them (6) said they had problems transitioning among representations according to Polya's problem solving stages. In this context, the codes in order of frequency are graphical, algebraic and verbal representations in the scope of forming a suitable representation to a problem. One opinion from the teacher candidate comments about this subject went: *"I can easily define* *problems verbally. However, since I need to think more when using images to show them, it is more complex for me. I cannot form the connection well. I think it comes from our parrot fashion of being taught. It is not easy for me to pass through images directly, I need to think very clearly… If we had been taught this way from, the beginning, it would be easy for me to transfer to images."* ($GÖ_4$).

On the same thematic aspect, five of the teacher candidates stated that they had problems during the algebraic representation period; three of them said they had problems in finding the unknowns. One of the opinions of the teacher candidates who commented about this subject stated: "*I have problems in formulating and writing problems algebraically. For example I know what data is given in an algebraic expression, but I cannot place them in the problem.*" ($GÖ_{15}$).

Similarly, in the scope of verbal representations, two of the teacher candidates stated that they had problems due to the influence from previous education. One of the candidates said he cannot solve problems with verbal expressions. One opinion from the teachers on this subject went like this: "*Yes, in the planning stage I had difficulty with verbal transitions…*" ($GÖ_8$).

As a second theme, the teacher candidates had problems in transitioning among representations. Three of them said they had difficulty in transitioning from graphical representations to algebraic representations, two of them with transitioning from algebraic representations to verbal, and one of them said he had difficulty in transitioning from verbal to algebraic representations. One teacher candidate opinion about this subject is like this: "*In some problems I can easily transfer algebraic representations to verbal however in some of them I cannot totally transfer it into algebra. I especially have difficulty in transferring verbal expressions into algebraic expressions. This situation changes according to the question or to my mathematical knowledge.*" ($GÖ_5$).

Discussion and Result

In this study which aims to identify the routine or non-routine mathematical problem solving abilities of classroom teacher candidates and the types of representations that they use during this period according to Polya's problem solving stages, the result that teacher candidates use different representation types in the stage of problem solving was reached. This result shows consistency with the results of other studies in the literature (Akkuş & Çakıroğlu, 2006; Akyüz, Coşkun, & Hacıömerliğlu, 2009; Bosse, Adu-gyamfi, & Chetham, 2011; Herman, 2007; İpek & Okumuş, 2012, Keller & Hirsch; 1998; Outhred, & Saradelich, 1997; Panasuk & Beyranevand, 2010).

Firstly, in the research it was found that the teacher candidates use "verbal expression" for the stages of understanding problems, planning solutions, and evaluation. Also, this finding which is supported by data from interviews and the analysis of quantitative data shows similarity with the results of other studies (Akkoç, 2005; Boulton-Lewis, 1998; Gagatsis & Shiakalli, 2004; İpek & Okumuş, 2012; Keller & Hirsch, 1998; Kılıç & Özdaş, 2010; Neria & Amit, 2004; Ural, 2012). It can be said that this situation originates from Polya's problem solving stages. In other words, at the stage of "understanding of a problem", students are particularly expected to define what the problem says or wants in their own words. In the second stage, which is "planning," it should be stated how the problem is going to be solved, and in the evaluation stage, the result is expected to be interpreted (Altun, 2009; Baki, 2006). In this scope, when the quantitative data is examined, it can be obviously seen that the reason "verbal expressions" are mostly preferred originates from the thought that they can be understood better, from the influence of previous education, and from the continuation of habits. This important finding which was obtained from the research shows parallels with the studies of Kılıç and Özdaş (2010). Kılıç and Özdaş reached the result that the participants' use of verbal statements, which have an important place in gaining mathematical abilities, stems from primary school years and solving problems about fractions in this way.

On the other hand, in the scope of the qualitative and quantitative data, it can obviously be seen that the teacher candidates mostly use "algebraic representation." The interviews done in this context show this situation originates from the teacher candidates because of being accustomed to this, the solution being able to be found with mathematical calculations, being easy to understand, and the influence from previous education. The results obtained from this research show consistency with the findings of Cai (2000; 2004), Cai and Hwang (2002), Çalık-Uzun and Arslan (2009), Delice and Sevimli (2010a), Gagatsis and Shiakalli (2004), Herman (2007), Kılıç (2009), Monoyiou et al. (2007) and findings of You (2006). In this scope, for example, Delice and Sevimli (2010a) at the end of their research also presented that the teacher candidates use algebraic representations for solutions of all mathematical problems. Similarly, Cai (2005) in his study found that teachers use algebraic representations mostly in the problem solving stage.

Another finding that was obtained from this study is that classroom teacher candidates who participated in the research use "numeric representation" the least. In this scope, Delice and Sevimli (2010a; 2010b), Kendal and Stacey (2003) and Herman (2007) also reached similar results. Delice and Sevimli (2010a) determined in their research that the reason teacher candidates do not use numeric representations is because they are not accustomed to questions related to numeric representations. Herman found that teacher candidates use "numeric representations" in the problem solving stage to check the accuracy of their result rather than to reach the result.

When the data relating the effect of the use of multiple representation in problem solving was examined, it was seen that multiple representations ease the understanding of problems, appeals to students having different intellectual talents, and helps to express the same question in different ways. This finding shows consistency with the results of Çalık-Uzun and Arslan (2009), Even (1998), and Herman (2007). In this context, Herman discovered that the use of multiple representation causes one to understand mathematics deeply.

On the other hand, when the teacher candidates that participated in the study were asked the conditions for using different representations, it was found that most of them obtained the representations according to the structure of the question and their perception of the question. This obtained data also shows similarity with the literature (Akkuş & Çakıroğlu, 2006; Gulkilik & Arikan, 2012; Herman, 2007; Keller & Hirsch, 1998; Özgün-Koca, 1998). In this context, Akkuş and Çakıroğlu (2006) found that the type of question, the structure of the problem and one's perceptions are important factors in obtaining representation types. Again, Özgün-Koca (1998), in their research, reached the result that viewing beliefs, thoughts, previous data and experiences as inner factors and problems themselves as outer factors are effective in solving problems. On the other hand, Herman (2007) found that representation types are chosen according to a given problem. Keller and Hirsch, (1998) reached the result that one's conceptions and experiences related to representations are influential in the choosing of multiple representations.

In the research, as the last secondary aim, the actual problems with types of representations that classroom teacher candidates use in the stage of problem solving, forming a theme suitable to the problem, and transitioning among representations were examined. In this scope it was clearly seen that the teacher candidates had difficulties in forming representations suitable to the problem as well as transitioning among representations, especially in the use of graphical, algebraic and verbal representations. This finding shows consistency with the results of Billings and Klanderman (2000), Çalık-Uzun and Arslan (2009), Delice and Sevimli (2010b), Elia, Panaoura, Eracleous, and Gagatsis (2007), Gagatsis and Elia (2004), İpek and Okumuş (2012), Patterson and Norwood (2004), and You (2006). For example, Çalık-Uzun and Arslan (2009) in their research showed that classroom teacher candidates mostly have problems with transitioning from graphical, algebraic, and verbal representations. Again, Delice and Sevimli (2010b) found that secondary school mathematics teacher candidates generally have problems with questions that require transitioning among representations. Even (1998), in his study in which he examined the transitioning abilities of mathematics teacher candidates, reached the result that the candidates are not very successful with transitioning between multiple representations in the context of flexible associations. Similarly, Patterson and Norwood (2004) found that in the transitions from graphical and algebraic representation types, teacher candidates experience problems due to inefficient time and information.

In short, in light of the results from above, despite teacher candidates being able to use multiple representation types in the mathematical problem solving stage, it can obviously be seen that they prefer mostly verbal and algebraic representations. This result shows similarity with the studies in literature done with mathematics teacher candidates (Delice & Sevimli, 2010b; Gagatsis & Shiakalli, 2004; Herman, 2007; İpek & Okumuş, 2012). However, the study of Pitts (2003), which shows differences with the findings of other studies in the literature attracts attention. Regarding this scope, his study which was done using mathematics teacher candidates, found that teacher candidates mostly use graphical and algebraic representations.

Also, teacher candidates mostly having problems in forming a theme suitable to the problem, and doing transitions among representations is another important piece of data obtained from the research. Accordingly, it can be suggested that giving importance to the concept of multiple representations and making class exercises which relate problem solving and the concept of multiple representations together could be beneficial. This way, teacher candidates can help students gain different point of views in the problem solving process. This study, starting from an actual evaluation of a condition, only examined the opinions of teacher candidates. However, a study relating to the applications and opinions of teachers on their duty can be suggested as well.

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