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Investigation of Problem-Solving and Problem-Posing Abilities of Seventh-Grade Students*

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Abstract
This study aims to examine the effect of multiple problem-solving skills on the problem-posing abilities of gifted and non-gifted students and to assess whether the possession of such skills can predict giftedness or affect problem-posing abilities. Participants' metaphorical images of problem posing were also explored. Participants were 20 gifted and 85 non-gifted seventh graders, and quantitative and qualitative research methods were used for data collection and analysis. The relationship between multiple problem-solving skills and giftedness was investigated, and a strong corre lation between problem solving in multiple ways and problem-posing abilities was observed in both the gifted and non-gifted students. Moreover, problem solving in multiple ways was observed in both the gifted and non-gifted students. Metaphorical images were based on the participants' experiences with problem posing, and they associated their positive or negative metaphors depending on their problem-posing performance.

Keywords: Problem solving in multiple ways • Problem posing • Seventh-grade students • Metaphor

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Problem solving and problem posing are accepted essential components of mathematics education worldwide. Many studies related to the two concepts have been conducted (Brown & Walter, 1999; Cankoy & Darbaz, 2010; Dede & Yaman, 2005a, 2005b; Leung, 1996; Silver & Cai, 1996; Schoenfeld, 1992), and research in this area continues to expand rapidly because of its importance to world governments. While problem solving is defined as the heart of mathematics education (Cockcraft, 1982; Dede & Yaman, 2005a, 2005b), problem posing can be identified as one of the coronary vessels. The National Council of Teachers of Mathematics (NCTM) (1980) emphasized that students should solve mathematics problems in different ways and generate their own problems in given situations.

If a problem is considered as difficult (Kilpatrick, 1987), problem solving refers to overcoming the difficulty. Problem solving is accepted as a central activity in education programs in many countries such as Australia, Japan, Korea, Singapore, and China, which were top performers in PISA 2012 (OECD, 2013). Most commonly known problem-solving steps were introduced by Polya (1957), who cited four steps to solve a mathematical problem: understanding the problem (can you state the problem in your own words?); devising a plan (look for a pattern or equation or examine related problems); executing the plan (implementing a strategy and checking operations and links); and looking back (checking the results). Many researchers (e.g., Abu-Elwan, 2002) include problem posing or creating a new problem after final steps. Furthermore, problem posing is incorporated as a feature of mathematics teaching in many countries (e.g., Japan) that employ it as a means of analyzing problems and enhancing students' problem-solving competence (Silver, 1994).

Problem posing in education was introduced by Freire (1970) for the first time as an alternative to banking education. Problem posing entails the generation of a new problem or reformulation of a problem from given situations or problems (English, 1997; Grundmeier, 2003; Silver, 1997; Stickles, 2006). Problem-posing strategies have been examined by some researchers (Brown & Walter, 1990; Stoyanova & Ellerton, 1996). The Turkish Primary Education Mathematics Curriculum states that "students are able to solve and pose problems which require calculating with fractions." Some students cannot comprehend fractions because of imperceptions. Therefore, students should acquire deep understanding of fractions and feel flexible in studying fractions (Milli Eğitim Bakanlığı [MEB], 2009).

In recent years, problem solving and problem posing have been used as tools for identifying students' thinking and understanding in mathematics learning. Because problem solving is a daily necessity, it is a talent that must continually improve to support our continued existence (Skemp, 1987). Because mathematics is not solely a batch of numbers, it has to be presented as an approach to memorizing concepts and thus allowing students to be the ones who research solutions, discover connections and relationships, and realize required abstractions (Schoenfeld, 1992). Students who solve problems that they created gain required experience and achieve victory in their discoveries (Polya, 1957). Problem solving is a learning process that we go through both at school and throughout our daily lives (Jonassen, 1997). Students who follow previously memorized paths in a traditional approach do not have the opportunity to create their own approaches (Hines, 2008). Individuals attempt to solve their problems based on their experience and knowledge even when they do not explicitly know the solution. The effort that individuals put into a task is called problem solving (Toluk & Olkun, 2001).

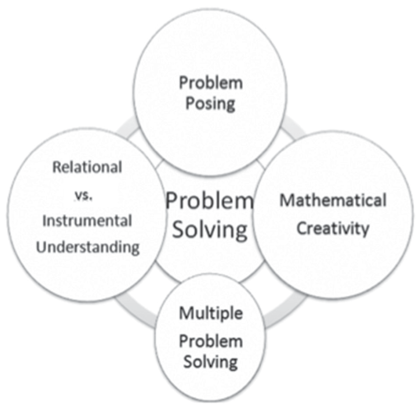

Figure 1: Interplay among Problem Solving and Mathematics Creativity, Multiple Problem Solving, Relational vs. Instrumental Understanding, and Problem Posing.

The literature reports that problem solving centers around four main constructs:

1- Mathematical creativity (Mayer, 1970)

2- Multiple problem solving (Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005)

3- Relational vs. instrumental understanding (Forrester & Chinnappan, 2010)

4- Problem posing (Cai, 1998)

The four main constructs and their relationship with problem solving are explained in the sections below.

Problem Posing and Problem Solving

Problem posing is a problem-solving activity. It can be defined as the creation of new problems from given events and situations. For primary school students, problem posing is the center of education in Singapore, which was the highest-performing country in PISA 2012, and has been adopted as an unchangeable element in mathematics education because of a reform that was enacted in Turkey in 2005 (Ministry of Education Singapore, 2006, p. 5). Related situations can be, but are not limited to, free, semi-structured, and structured problem-posing situations that are to create new problems (Abu-Elwan, 2002; Akay, Soybaş, & Argün, 2006; Cankoy & Darbaz, 2010; Lowrie, 2002; Schoenfeld, 1992; Silver & Cai, 1996; Yaman & Dede, 2005). Yaman and Dede (2005) discuss the importance of problem posing and problem solving in science and mathematics education. They surveyed the literature and concluded that problem posing and problem solving were strongly related. According to Cankoy and Darbaz (2010), people who cannot understand a problem will not be able to find and use suitable strategies; moreover, they will not be able to explain what they are doing and why (relational understanding) and will ultimately lose the motivation to solve the problem. The process of problem posing positively affects problem-solving capability (Grundmeier, 2003). Thus, similar to problem solving, problem posing has also been seen by researchers as the center of mathematics (Silver, 1997).

Conceptual knowledge and operational knowledge are involved in problem solving (Bernardo, 1999). It is reported that students who have difficulty solving arithmetic operations also have difficulty constructing and solving problems. Factors that affect problem-solving capability are attitude toward the problem, understanding, reasoning, and experience (Van de Walle, 1994). Problem solving is a sequential activity in traditional education; students follow what their teachers do (Evancho, 2000). Mathematics education curricula should be formed solely around problem-solving activities and determined that individuals must be open-minded, curious, and patient to improve their problem-solving capabilities (NCTM, 1980). From this viewpoint, the NCTM places more importance on problem solving and mathematical curiosity. Since 2005, Turkey's education program, including the structuring approach, has been in progress and practice. Unlike the approach used in traditional education, students will have the opportunity to conduct their learning activities in flexible, original, and interrogatory environments (Hinchliffe, 2001; MEB, 2005, Nelson, 1999). Students' critical capacities can be determined by considering their capabilities concerning assessing, analyzing, and creating relationships (Jonassen, 1997). Students with critical thinking skills can comment naturally on subjects from various aspects and make flexible assessments (Vander & Pintrich, 2003). In addition, they must organize their knowledge successfully and compare and abstract operations accordingly (Chance, 1986; Slattery, 1990). The primary challenge in improving mathematics education is to improve problem solving by constructing knowledge, and it is considered that the notion of mathematics as solely classroom based should be abandoned and that it should be emphasized that mathematics is a tool for real life (MEB, 2009). It can be taught that problem solving is the center of mathematics education programs. Problems are solved using questions that can be answered in routine, memorable, and formulated ways, i.e., students have been known to solve problems using unconventional methods and have also solved open-ended situations (NCTM, 1980).

Problem posing is a creative activity, and many instruments that measure creativity include the problem-finding dimension (Silver, 1994). Wertheimer (1945) emphasized Einstein's opinion that thinking during problem designing and problem solving and finding the right question are much more important than finding the right answer. Jay and Perkins (1997) stated that the key to creativity is to produce a new problem or make significant modifications to the current problem to create a new problem. Silver (1997) reported a similar detection. Although it was not empirically proven, Yuan (2009), in his doctoral thesis, described how problem posing was used in some works to measure creativity, i.e., he studied the relationship between problem posing and creativity. Originality requires superior talent, and fluency and flexibility are parts of a natural structure (Leikin, 2009). Solving problems using different approaches from those of others is a hallmark of creativity.

Students must be provided with the opportunity to create problems on subjects they are studying, and they should be prompted on those subjects. Thus, it will be much clearer how students attribute meaning to subjects they study (Hiebert & Wearne, 2003). Problem posing is maintained as an important approach for avoiding the creation of teacher-centered education programs (Shor, 1992). Freire (1970) emphasizes that problem posing is a communication method that easily results in social interactions. Using problem posing, teachers can obtain appropriate information concerning subjects that students are highly interested in (Freire, 1970).

Problem posing positively affects problem-solving capability (Grundmeier, 2003). Therefore, similar to problem solving, problem posing has also been accepted by researchers as the center of mathematics (Silver, 1997). Problem posing requires in-depth thinking because it is a different way to approach subjects. Classroom environments and teachers are the most significant elements of problem-posing activities. Students require classroom environments in which they are comfortable, flexible, and interrogative and in which they are not ashamed of what they produce during problem-posing activities; the person who is responsible for creating such environments is a teacher (Moses, Bjork, & Goldenberg, 1993). Students' critical thinking abilities are improved by problem posing. Students do their best to produce original ideas during problem-posing activities, thereby enhancing their creativity. Then, they begin to pay attention to logical relationships and question sentence formations as they start posing problems. Their problem-solving capacities grow more efficient as they question whether solutions exist for problems they create (Cai, 1998; Cankoy & Darbaz, 2010; English, 1997; Silver, 1997). Abu-Elwan (1999) conducted a problem-posing study with teacher candidates to establish that teachers' support of students during problem solving is insufficient; teachers must create problems that force students to struggle to find solutions, which will improve their mathematical thinking capabilities. He concluded that semi-structured problems were more efficient for improving problem-posing capabilities.

Lowrie (2002) attempted to compose a conceptual frame for five- and six-year-olds that would allow them to create their own problems. The study emphasized that open-ended questions are relatively much more difficult because they have more than one solution. Five-week training was given to 25 children from the 1st class in producing open-ended questions, and they were encouraged in the activity throughout the study. Although it is known that open-ended questions facilitate mathematics understanding more than standard questions, it has been observed that they are not widely used by teachers and are not incorporated into textbooks. One way to improve flexible thinking capacity is problem-posing activities. Thus, the study of Lowrie focused on producing open-ended problems. Just as teachers serve as experts when helping students gain mathematical understanding, they were also role models during the study's problem-structuring activities. Teacher–student interactions were also observed, and audio records of the interactions were captured. The author found that 13 students produced required open-ended questions at the end of the study, emphasizing that they were very excited about problem-posing activities, especially in subjects that they liked. It has been established that subjects that students like the most are the ones for which they prefer producing problems, and the consideration of this is very important for students during problem posing. In addition, it has been found that students' attitudes toward problem solving are highly correlated with their teachers' approaches.

Mathematical Creativity and Problem Solving and Problem Posing

The questioning of mathematics creativity and defining it by placing concepts in the proper frame began in 1960 and is still ongoing, although the research has not yet identified a universally accepted method for diagnosing mathematics creativity. Various assessment criteria have been taken into account concerning this subject. Mathematics creativity does not refer to forming discoveries from things that do not exist. Rather, it refers to discovering new connections as a result of formal changes to things that already exist (Ervynck, 1991). Sriraman (2005) defined mathematics creativity as two different products that emerge from both cognitive processes and result-oriented endeavors. This article focused on how mathematics creativity emerged from cognitive processes based on the sampling composition of its secondary school students and how the school approached creativity in mathematics. The most distinctive feature of mathematics creativity is that it entails using original solution methods that differ from conventional methods (Sternberg & Davidson, 2005). However, Romey (1970) states that making original connections among current mathematics knowledge, concepts, or approaches can be assumed to indicate mathematics creativity. Cornish and Wines (1980) maintain that adapting well-known mathematics knowledge into similar simple events can be put into practice because of mathematics creativity. That is, math creativity at school can entail both incorporating traditional mathematics into the standards of the developing world and looking for answers to open-ended math problems and situations (Haylock, 1987).

Balka (1974) suggested three criteria for math creativity: fluency, flexibility, and originality. In addition, asking the proper questions to find the missing information have been the major elements in improving math creativity. Semi-structured problem posing is one example of this situation. Carlton (1959) states that the prediction of changes based on other changes in the hypotheses of the problems may support mathematics creativity. Structured problem posing, including the What-If-Not strategy developed by Brown and Walter (1990), is an example of this situation. Flexible problem-solving ability has been associated with math creativity. Asking original questions to solve problems and presenting solutions from various viewpoints have also been referred to as mathematics creativity.

Sheffield (2008) also reached a similar conclusion that although students who possess mathematics creativity solve their problems and reach their conclusions using different and original methods, they also repeatedly study the problems and the solutions. Leung (1997) discussed the relationship between creativity and problem posing by comparing the characteristics of each concept and concluded that creativity is in the nature of problem posing; that is, creating a problem is a creative activity (Leung, 1996). However, Silver (1994) emphasized that it was not clear that there was a relationship between problem posing and creativity. Problem-solving and problem-posing activities on research-based math education contributes to student creativity. Understanding the problem, the first stage in problem solving, can be the beginning of the creative process (Getzels & Csikszentmihalyi, 1962). It can be stated that mathematics creativity has a close correlation with problem solving and problem posing. In addition, students' problem-solving methods that offer more than a single solution or their confirmations of their results are associated with the flexibility aspect of math creativity (Silver, 1997).

In problem posing, the students examine the various problems, as well as analyzing them and writing them up, using their own statements. Silver (1997) maintains that mathematics creativity has also been correlated with superior intelligence.

Creativity can be determined by an original solution to a problem that no one has solved before (Polya, 1945). Levav-Waynberg and Leikin (2009) stated that solving problems using a variety of ways can be a marker of creativity and teach-ability. They described that geometry has been a proper field for showing more than one way to solve a problem, and then they assessed their subjects' geometry knowledge and creativity using a geometry problem. Leikin (2009) stated that finding more than one solution method identifies and establishes creativity. That is, using multiple solution methods both develops creativity and assists in identifying it. For example, if a student reaches a solution using a different method from of others, he or she has a higher level of creativity than do others. In summary, problem posing, mathematics creativity, and problem solving have common characteristics.

Relational and Instrumental Understanding

The functionality of knowledge has been questioned since student-based education recently replaced traditional education. Education addresses two facets of knowledge, instrumental and relational (Baki, 1998). Instrumental knowledge is the operations that have already been used based on certain rules and formulas. The correct application of algorithms is the main topic rather than seeking to answer "why." In contrast, the meanings of concepts and the relationships among them are the main topics of conceptual knowledge. Relational knowledge has been related to symbolizing math concepts and making meaning out of the operations themselves (Soylu & Aydın, 2006). Therefore, it has been suggested that mathematics problems should be structured so that they require using both relational and operational knowledge (Baki, 1998). Unless the required importance has been provided for both relational and instrumental knowledge, there will likely be failures. When problems that require instrumental knowledge are solved in the classroom, students do not gain in-depth knowledge concerning abstract mathematics concepts (Bekdemir & Işık, 2007). Related research on fractions and both instrumental and relational knowledge have been carefully conducted (Toluk & Olkun, 2001). Studies find that the fact that students do not regularly face fractions and that they do not conduct relational learning activities that use fractions or make them concrete can cause fractions to be considered a difficult subject. It has been stated in the literature that students are much more successful with instrumental knowledge compared with relational knowledge. In this work, a question that requires both relational and instrumental knowledge at the same time was used for the study's problem-posing test. The seventh question on the test was prepared by balancing both operational and relational knowledge. That is, the correct things had to be carried out correctly and the required interpretations had to be realized after the operations to solve the actual problem.

Determination of Students' Thoughts about Problem Posing Using Metaphors

A metaphor is an essential mechanism of the mind that lets us know how we think and how we express our thoughts in language (Lakoff & Johnson, 1980). One of the more effective ways to identify students' thoughts on problem posing is metaphors. Research on metaphors that dates back to the work of Aristo (B.C. 386–322) looks at the use of language and eloquence, and Lakoff and Johnson (1980) determined that even our mentality is formed with metaphors.

Metaphors are used as pedagogical, assessment, and mental tools in education (Saban, Koçneker, & Saban, 2006). They make it easy to conceptualize and help to configure knowledge. Most research on using metaphors in mathematics education presents that metaphors highlight the importance of education. Metaphors produce a conceptual relationship between a source domain and a target domain because they link different senses (Lakoff & Johnson, 1980). Although conceptually, metaphors are related to the person who creates them, teachers use them to help increase students' understanding (Lakoff & Nunez, 1997). That is, in discussions of abstract concepts, the use of metaphors provides the coherence of meaning.

Metaphors are experiences that are acquired from our daily lives, and they are conceptual. They are indispensable for comprehending abstract notions. Because of their conceptuality, metaphors are shaped according to different cultures (Lakoff & Johnson, 2005). Picker and Berry (2000) asked seventh- and eighth-graders five different elementary schools in five different countries to draw their mathematics teachers. The analysis of the metaphors revealed that students had drawn threatening, violent, despotic, and rigorous figures. In the 1996 study by Inbar, 409 primary students and 254 educators participated offered metaphors for the concepts of student, teacher, and school principal.

Students were perceived as vegetation by many of the educators, and the educators were perceived as a super power by many of the students. The school principal was conceptualized as an authority figure by the students and educators, and they conceived of school as being framed by the world.

Ben-Peretz, Mendelson, and Kron (2003), studied 60 teachers of vocational and technical courses and found that they perceived themselves as a zookeeper, a maestro, a judge, and a puppeteer. Specifically, teachers of low-performing students defined themselves as a zookeeper, but teachers of high-performing students defined themselves as a maestro.

Frant, Acevedo, and Font (2005) proposed to investigate the dynamic process of teaching and learning graph fiction in high school in Spain. Researchers sought answers to the following questions: what kind of metaphors did teachers use to explain the graphic representation of functions, did the teachers realize the metaphors they used, the effect of the metaphors on the students, and the role played by metaphors in negotiating meaning.

Metaphors are widely used in understanding people's perceptions in different situations and different concepts. This study explores students' metaphorical images of problem posing after they performed the problem-posing activities.

The Importance of the Study

Researchers such as Cai (1998) and Crespo (2003) studied to find correlations between problem solving and problem posing, and Levav-Waynberg and Leikin (2009) found significant relationships between problem solving using different methods and creativity. Nonetheless, no researcher has investigated the link between problem solving by multiple methods and problem posing. Furthermore, students' views of problem posing were investigated through metaphor analysis. Establishing how students (gifted and non-gifted) use the particular method of metaphors to pose problems will contribute to the literature.

Problem posing has been studied by many researchers in mathematics education. However, this study was the first in the literature to use a problem-posing activity with multiple choice questions; when the test questions were being developed, misleading options were inserted, thus requiring students to confront both problem posing and problem solving. In addition, the students' teacher no longer has to lose time attempting to determine how to evaluate posed problems.

Research Questions:

1. Is there a significant relationship between problem solving ability using multiple methods and problem posing ability?

2. Is there a significant dependence between multiple problem solving and giftedness?
3. What are gifted and non-gifted students' metaphorical images of problem posing? What differences, if any, exist between gifted and non-gifted students' metaphorical images of problem posing?

Research Design

The main goal of this study was to investigate the effect of multiple problem-solving skills on the problem-posing abilities of gifted and non-gifted students. Another goal was to explore these students' metaphorical images of problem posing. The research model of this work was a survey, a descriptive model that aims at describing situations without interfering with or changing the situations. Non-experimental research is conducted in natural settings, with numerous variables that operate simultaneously. This study was designed to seek the answers to the research questions by employing both quantitative and qualitative techniques.

Participants

Eighty-five non-gifted public school students and 20 gifted private school students, all in the seventh grade, participated in the study. Seventh grade was chosen. Because these students could solve fraction problems not only arithmetically but also algebraically; the participants had learned algebraic solutions in the sixth grade following the secondary school mathematics curriculum.

The gifted students, who were enrolled in a full special class, were drawn from two private schools. Criterion sampling was used, and all of the gifted students who participated in this work had obtained scores of 135 or above on the Wecshler Intelligence Scale for Children (WISC-R). The WISC-R is one of the most common scales for assessing giftedness (Savaşır & Şahin, 1995). The non-gifted sample was drawn from one public school in Istanbul. Convenience sampling was used for selecting the non-gifted participants. The students were easy to recruit, and the researchers did not consider selecting participants who were representative of the entire population.

The full special class is very important gifted students' learning (Rogers, 2002). Considering the environmental factors (class selection, school selection, etc.), special classes have been dedicated to exceptional students to contribute to their academic gains without classifying them by the fields in which they are more talented. The classes are referred to in the literature as "complete special classes." The "complete special class" environmental factor has been recognized as important for talented students to make academically defined gains (Rogers, 2002). Talented individuals show extraordinary performance in at least one field and put their signatures on creative ideas. They mature earlier than their peers and continue to mature and develop skills well past the time when others' skills have peaked. For example, whereas normal individuals might progress during a defined time period, more talented individuals continue to progress until much later ages (Winner, 1996). It has been defined in the literature that talented students can easily understand concepts, show flexible thinking, are open to exploring new things, examine the details, and possess high levels of ethical sense (Reynolds & Birch, 1988).

Data Collection Tools

The problem-solving task consisted of five fraction problems to be solved in multiple ways. The test was constructed as 10 questions for the pilot application, and it was presented to experts for review. The problems that were produced by the researcher were designed to be solved in three ways, arithmetic, visualization, and algebraic. The students were asked to choose five problems and solve them using more than one solution in the pilot. Because the students in the pilot study could solve the problems in items 1, 3, 5, 9, and 10 in more than one way, these problems were selected and used in the master work. The reliability of the test using these selected items was calculated as .857 in the pilot study. Divided test solutions were implemented for internal consistency, and the Cronbach's alpha, Spearman-Brown, and Guttman coefficients were calculated (.714, .833, and .809, respectively). The problem-solving test was determined to be reliable, and it showed internal consistency.

The problem-posing task consisted of twelve multiple-choice items. Below is the problem-posing question, which came from page 43 of the third-grade mathematics textbook (Erbaş, 2014) published by the Turkish Ministry of National Education for school year 2013–2014, which served as the infrastructure for this study.

From page 43 of the 3rd-grade mathematics textbook published by the Turkish Ministry of National Education, 2014

Table 1
The Study's Problem-Posing Question

In Turkish	In English
213 + 167 = 380 işlemi aşağıdaki problemlerden hangisinin çözümü olabilir? A) Osman 213 tane ceviz topladı. Recep ise Osman'dan 167 fazla ceviz topladı. İkisinin toplam kaç cevizi oldu? B) Bir markette cumartesi günü 213, pazar günü ise 167 şişe su satıldı. Bu markette iki günde toplam kaç şişe su satılmıştır? C) Erdem'in 213 TL parası vardır. Ağabeyinin parası ise Erdem'in parasından 167 TL azdır. Erdem'in ve ağabeyinin paraları toplamları kaç liradır?	Which one of the below problems can be matched with the operation of 213 + 167 = 380? A) Osman picked up 213 pieces of walnut. Recep picked up 167 more pieces of nuts more than Osman. What is the total amount of the nuts that both Osman and Recep picked up? B) On Saturday, 213 and on Sunday 167 bottles of water were sold in a market. What is the total number of bottles of water that were sold at this market on these two days? C) Erdem has 213 Turkish lira. His brother has 167 lira less than that. What is the total amount of money that both Erdem and his brother have?

The aim of this study was to investigate one factor that affects problem-posing capability, problem solving using multiple methods. The participants were 105 seventh-grade students, twenty of whom were gifted. Two achievement tests, one on problem posing and one on problem solving, were used as a data collection tool.

The experts were two Turkish teachers and four math teachers who were asked to validate the tests. The problem-posing test was multiple-choice. To provide internal consistency, split test analysis was used on both tests.

We prepared the problem-posing test based on a strategy that was developed by Stoyanova and Ellerton (1996). The test items were designed using semi-structured and structured problem-posing situations.

The problem-posing test was developed in two stages. First, we investigated whether the problem posing was realized with multiple questions and also whether there was a difference between the classical problem-posing operations and other methods. The situations were presented to the two teachers who were experts in their fields at the state university in addition to two other math teachers and one Turkish teacher at the school where the study was conducted. The experts presented their predictions regarding whether administering either of the tests would pose any problems for seventh-grade students. This study also used descriptive, non-empirical research methods to supplement the quantitative approaches. The semi-structured and structured problem-posing situations that were created by Stoyanova and Ellerton (1996) were used here, in the following formats:

a) Constructing fractions in the correct format for the given operation

b) Changing the data on a given problem

The 68 students from the state school first faced a classically structured problem-posing situation and then a situation with multiple possible solutions. The semi-structured and structured problem-posing situations were used in both applications and were exactly the same. However, the problems with only the multiple selections were prepared by the researchers and presented to the students as ready problems with related selection options.

Improper problems, impossible cases, unnecessary or excessive knowledge and other problems were the main misleading options on the problem-posing test. Furthermore, 5 of the problem-posing tasks were structured, and the remainders were semi-structured situations.

The achievement tests were reformulated during the design phase by two teachers who were experts in their fields. The opinions of the above-referenced experts were considered during the pilot study, and the group confirmed that the problems on both tests could be correctly understood by the students and that they met the students' cognitive levels. The pilot study was conducted 100 students, 10% of whom were gifted.

The reliability of the achievement tests was checked with the pilot study, and the value for the problem-posing test was .855 ($p < .05$). Divided test solutions were implemented for internal consistency, and the Cronbach's alpha, Spearman-Brown, and Guttman coefficients were calculated, at values of .706, .846, and .844, respectively. The problem-solving test was found to be reliable and showed internal consistency.

Direct observation was also used in this as one of the methods to support the quantitative findings. The metaphors created by the students were classified through content analysis. The most salient findings from the study were that: the gifted students could not produce entirely new ways of problem solving; the students who did finding multiple solutions had higher scores on the problem-posing test; and the metaphoric thoughts in the problem-posing activities have much more positive effects on the normal students.

Data Analysis

To date, many problem-posing studies have been conducted according to classical practices. In other words, researchers have evaluated problems that were posed by students according to certain evaluation criteria. Leikin and Lev (2013) evaluated posed problems in terms of correctness, creativity (fluency, flexibility, and originality), and connectedness. Silver and Cai (1996) evaluated posed problems according to correctness and semantic or linguistic difficulty. In our study, we aimed to identify the link between problem solving using multiple methods and problem posing. Therefore, we prepared the problem-posing task as a multiple-choice test. We preferred to use distracters in options, as did Singer and Voica (2012). Hence, we did not establish any evaluation criterion; we only assessed the correctness of problems posed by students. One point for the correct answer and two points for the alternative solution were given in problem-solving tests during the analysis of the metaphor data. One point was given for correct answers in the problem-posing test. A chi-square test with Yates's correction for continuity was performed to determine dependency between giftedness and problem solving using multiple ways.

After the achievement tests were administered, the results were analyzed using SPSS 18.0 software. The problem-solving test items were graded, presented in Table 2:

Table 2
Problem- Solving Test Grading

Points	Description
O	No answer or incorrect answer
1	One way (single) solution
2	Multiple (alternative) ways of solution

The frequency distribution for solving problems using more than one was assessed following the problem-solving test. The relationship between the students' problem-solving and problem-posing test responses were checked with the Spearman correlation coefficient. Because it is not a parametric measure, it was used as a special state of the Pearson correlation coefficient.

Yates's correction for continuity is mostly used when at least one cell in a table has an expected count smaller than 5 (Yates, 1934). This study used the chi-square test to identify the dependence between multiple problem solving and giftedness.

Content analysis was used on the metaphors that were generated by the students. Qualitative content analysis uses inductive reasoning, by which themes and categories emerge from the data through the researcher's careful examination and constant comparison. To organize the data in this study, the students' metaphors were listed and grouped into categories, and the data were analyzed qualitatively in three phases. The metaphors were independently a) coded by concept, b) classified by topic, source, and connection between topic and source, and c) examined for common characteristics. Subsequently, the researchers compared their lists of metaphoric images and found the least common denominator.

Results

The findings are discussed in the order of the research questions. The first question aimed to explore the relationship between multiple problem solving ability and problem posing ability.

One of the questions that remains in the literature is whether students who are successful in finding and producing alternative solutions are also successful at problem posing, that is, for this study, whether there was any correlation between problem-solving and problem-posing capabilities. The Pearson correlation coefficient was r = .760 when the students' responses were analyzed in SPSS 18.0. Thus, there was a strong correlation (p = .00 < .01).

Table 5
Correlations between Problem Solving and Problem Posing

	Correlations		
Problem posing	Pearson correlation Sig. (2-tailed)	1	.760***
Problem solving	Pearson correlation Sig. (2-tailed)	.760***	1

Note. N = 105, correlation is significant at the .001 level (2-tailed).

One hundred and five students, 20 of whom were gifted, participated in the work. Eighteen students presented at least one alternative way of solving the 5 fraction problems; surprisingly, not all 18 were gifted, only 13 of them. The distribution of the students who solved at least one question on the problem-solving test using multiple methods is shown in Table 3.

Table 3
Distribution of the Answers Given on the Problem-Solving Test

	Multiple Ways	Only One Way
Gifted Students	13	7
Non-Gifted Students	5	80

Table 4
Different Way of Asking the Same Type of Question

Problem posing with interpretation		Problem posing by modifying the problem	
"The sum of the ages of both Ali and his father is 54. If Ali's age is 4/5 of his father's age, how old is Ali?" Which one of the selections in the right column is correct?	a. The problem is correct. b. The problem has missing information. c. The problem has unnecessary information. d. The problem is impossible.	"The sum of the ages of both Ahmet and his mother is 45. If Ahmet's age is 2/7 of his mother's age, how old is Ahmet?" How could we re-pose this problem by changing the fraction and the total age?	a.55/ $\frac{3}{7}$ b.56/ $\frac{2}{5}$ c.52/ $\frac{1}{4}$ d.52/ $\frac{6}{7}$ e.50/ $\frac{2}{5}$

Whereas 65% of the gifted students gave the correct results on the problem-solving test, only 5.88% of the non-gifted students did so. Yates's correction for continuity value was .78. Because the degree of freedom in Table 3 is 1, $\chi^2_{0,01;1} = 6.63490 < 35.78$, and therefore, giftedness and problem solving by multiple ways were dependent variables. We also investigated the coefficient of contingency where n = sum of observed values. That coefficient was 00.707.

Five structured and 7 semi-structured situations were presented on the problem-posing test, and the students were asked to comment on the problems that comprised the four structured problem-posing questions. In addition, the students were asked one question in the manner presented below, which includes both interpretation and modification. In fact, this situation can be considered a way to increase the test's reliability.

Notes from Direct Observation: Non-gifted students were observed to greatly enjoy the problem-posing task. However, they reported that they would have liked it even more if the test had not included. In addition, some students did not solve the problems through visualization. When they were asked, they said that visualization was a fourth-year subject.

Three gifted students were not willing to solve problems and said that they hated mathematics. This researcher interviewed one of them:

Researcher: But you are a gifted student. How could you hate mathematics? You have a high WISC-R score.

Gifted Student: But we were not asked mathematics on the WISC-R test. It is related to reasoning.

Metaphors are clues that are related to a person's ideas about a concept (Levine, 2005). We wanted to know students' thoughts about problem posing, we were surprised to find that the gifted students generated negative metaphors related to problem posing. We asked them to provide their thoughts and experiences concerning the activities because their metaphors reflected their daily life experiences (Lakoff & Johnson, 2005).

Forty percent of the gifted students found the problem posing unnecessary when the produced metaphors were examined carefully, but only 3.8% of the non-gifted students did so. Moreover, the gifted students who could not produce any alternative solutions were among those who found the problem posing unnecessary.

It was observed that 30.76% of the gifted students who solved the problems in more than one way also considered the problem solving to be an unnecessary activity. The same percentage also used metaphoric statements that revealed that the focusing and the infrastructure which requires the capability and the experience have been significantly important. Another important point in the table is that no gifted students made mention of a lack of experience, which indicated that the students had previously posed problems.

Table 6
Categories of Generated Metaphors by Students

	Categories of Metaphors	**Gifted Students**		**Non-Gifted Students**	
		Solving a problem in multiple ways	Not solving a problem in multiple ways	Solving a problem in multiple ways	Not solving a problem in multiple ways
1	Inexperience			-	15
2	Needlessness	4	4	-	4
3	Complexness	2	-	3	24
4	Time-consuming	2	-	2	11
5	Demanding	4	-	-	18
6	Funny	1	-	-	18
7	Necessity	3	-	-	10

Non-gifted students who solved the math mathematical problems in more than one way found the problem posing to be difficult and time consuming and to require patience. It was noted in examining the table that most of the normal students who could not produce any alternative solutions found that using metaphors to pose problems was complicated and difficult. Table 6 summarizes the metaphors generated by both non gifted and gifted students whom were able to solve a problem in multiple ways and not able to solve the problems in multiple ways. Student metaphors were fell into seven categories. The purpose was to explore students' perceptions of problem posing through the lenses of metaphors. For instance, while one gifted students found the problem posing activity as enjoyable, 18 non gifted students found the problem posing as enjoyable as shown row 6 in Table under the Funny category. It was interesting that gifted students whom were able to solve the problem in multiple ways, their perception of problem posing varied from necessary activity to time consuming activity. Furthermore, non gifted student' perceptions whom were not able to solve the problem in multiple ways were mixed, but they found the problem posing activity useful and necessary.

That is, the normal students, whether they identified alternative solutions or not assess problem posing as complicated.

Discussion and Conclusion

Multiple problem solving and being gifted were observed as the dependent variables in this research, which supported the study by Levav-Waynberg and Leikin (2009). In addition, a strong correlation between multiple problem solving and problem posing capability was revealed. It can be concluded that it would be useful to encourage students to solve math problems in different ways. Problem-posing activities should be described to teachers during their in-service trainings and their importance should be emphasized accordingly. Separately, it could be useful to establish whether teachers in Turkey have used in-class problem-posing activities.

It is stated in the curriculum, which has been in development since 2006, that problem-posing activities are as important as problem-solving activities. This situation shows the importance of presenting problem posing to teachers during in-service trainings, although time could be an issue when the teachers have a syllabus they must follow. Most of the students, 87%, could not produce any alternative solution to any question. Meanwhile, students must be given the opportunity to solve problems in alternative ways and to produce problems in their own languages.

Arıkan and Ünal (2012) determined that eleventh-grade students were satisfied with only one solution rather than seeking alternatives; that is, both high school and secondary school students preferred to just solve the problems, not to pose new ones. It was observed here that the curiosity and the eagerness of the gifted students could be inferred from their scores on both the problem-solving and problem-posing tests. Their tendency to solve problems in multiple ways was more pronounced than the same tendency in the non-gifted students. Yates's chi square result determined giftedness and problem solving in multiple ways to be dependent variables. Hence, giftedness can be examined using problem solving by multiple ways.

The same type of problem-posing question was used in both interpreting and implementing the data modifications. Thus, it was possible to present the problem-posing situations with misleading choices. This study examined whether there was a correlation between the capabilities of both problem solving and problem posing and found a robust correlation. It can be inferred that the students who solved problems in multiple ways will also be more successful in posing problems, which supports the findings by Arıkan and Ünal (2014) and Cai (1998). In fact, there has been no significant evidence in the literature that there is always a correlation between problem solving and problem posing. On the contrary, another study by Crespo (2003) did not accept that there was such a correlation, and thus, whether this relationship exists is still under debate.

Most of the non-gifted students did not like having to work with fractions, and found it difficult to pose problems because of their lack of experience. It can be suggested that teachers should use the required materials on fractions as often as possible because so many students were not happy about working with them. It was found in the study that some of the students were confused by both compound and simple fractions and could not tell the exact difference between them; that is, they saw no difference between ¼ and 4/1. In addition, it was observed that the students assumed that the fraction solutions they had learned in the 4th grade did not relate to each other, and they perceived that the lessons were only for the 4th grade. For example, the students could not remember to use modeling, i.e., using boxes or drawing figures or shapes to arrive at a solution; even those who remembered to use the technique were prejudiced against it and felt that it mainly related to the 4th grade lessons. It can be concluded from this work that there were still significant defects in the students' practice even with the adoption of a structuring rather than memorizing approach even if that is required in the syllabus. It appears that the students in this study had merely memorized the subjects when they first learned them and no longer considered them after they completed that grade because they thought they would not need them anymore. Therefore, it appears that it would be very useful to teach fractions using real-life connections and building relationships in the materials because fractions comprise the infrastructure of numerous math subjects. Teachers can assess the students' common mistakes and conceptual misunderstandings, taking advantage of the current technology and creating environments in students have the opportunity to assess themselves based on their work. One of these assessment methods is problem posing, which gives teachers information on students' strong and weak points after they check the questions the students pose.

Levav-Waynberg and Leikin (2009) said that multiple problem solving is used to assess gifted students. Holton and Gaffney (1994) emphasized that mathematically gifted students take pleasure in numbers and mathematical subjects, and Villani (1998) depicted these students as solving math problems quickly and using different strategies to solve the same problem. Given that only 13 out of 20 students in this study could solve problems in multiple ways, the following question is raised: "Does giftedness necessarily mean giftedness at mathematics?" We might benefit from specific instruments to identify giftedness in mathematics.

Based on the results of the content analysis of the metaphors, it could be the case that the gifted students found it dull and uninspiring to choose from limited response options and instead preferred to construct and pose problems. The majority of the non-gifted students, however, found constructing problems difficult and complicated, and they used metaphors to reflect their inexperience.

Recommendations for Future Research

The complexity of the relationship between problem posing and problem solving was not completely addressed in this study; much more remains to be learned. For example, studies are needed to examine problem posing and problem solving in different branches of mathematics such as geometry and probability. From the teaching perspective, the role of classroom activities in building problem-posing skills and that of instructions in the process should be investigated.

References

Abu-Elwan, R. (2002). The development of mathematical problem posing skills for prospective middle school teachers. In A. Rogerson (Ed.), *Proceedings of the International Conference on Mathematical Education into the 21st Century: Social Challenges, Issues And Approaches* (Vol. 2, pp. 1–8). Cairo, Egypt.

Akay, H., Soybaş, D., & Argün, Z. (2006). Problem kurma deneyimleri ve matematik öğretiminde açık-uçlu soruların kullanımı. *Kastamonu Eğitim Dergisi*, *14*(1), 129–146.

Arıkan, E. E., & Ünal, H. (2012). Farklı profillere sahip öğrenciler ile çoklu yoldan problem çözme. *Bitlis Eren University The Journal of Science and Technology Journal*, *1*(2), 76–84.

Arıkan, E. E., & Ünal, H. (2014). Problem posing and problem solving ability of students with different socioeconomics levels. *International Journal of Social Science Research*, *2*(2), 16–25.

Baki, A. (1998, May). *Matematik öğretiminde işlemsel ve kavramsal bilginin dengelenmesi*. Paper presented at Atatürk Üniversitesi 40. Kuruluş Yıldönümü Matematik Sempozyumu, Atatürk Üniversitesi, Erzurum, Turkey.

Balka, D. S. (1974). Creative ability in mathematics. *Arithmetic Teacher*, *21*(7), 633–363.

Bekdemir, M., & Işık, A. (2007). Evaluation of conceptual knowledge and procedural knowledge on algebra area of elementary school students. *The Eurasian Journal of Educational Research*, *28*, 9–18.

Ben-Peretz, M., Mendelson, N., & Kron, F. W. (2003). How teachers in different educational context view their roles. *Teaching and Teacher Education*, *19*, 277–290.

Bernardo, A. B. (1999). Overcoming obstacles in understanding and solving word problems in mathematics. *Educational Psychology*, *19*(2), 149–163.

Brown, S. I., & Walter, M. I. (1983). *Problem posing: Reflections and applications*. Hillsdale, NJ: Lawrence Erlbaum.

Brown, S. I., & Walter, M. I. (1990). *The art of problem posing* (2nd ed.). Hillsdale, NJ: Lawrence Erlbaum.

Cai, J. (1998). An investigation of U.S. and Chinese students' mathematical problem posing and problem solving. *Mathematics Education Research Journal, 10*, 37–50.

Cankoy, O., & Darbaz, S. (2010). Problem kurma temelli problem çözme öğretiminin problemi anlama başarısına etkisi. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, *38*, 11–24.

Carlton, V. L. (1959). An analysis of the educational concepts of 14 outstanding mathematicians, 1790–1940, in the areas of mental growth and development, creative thinking and symbolism and meaning. *Dissertation Abstracts International, 20*(06), 2131.

Chance, P. (1986). *Thinking in the classroom: A survey of programs*. New York, NY: Colombia University Teachers College.

Cockcroft, W. H. (1982). *Mathematics counts.* London, UK: HMSO.

Cornish, G., & Wines, R. (1980). *ACER mathematics profile series: Number* test. Hawthorn, Victoria: , Australian Council for Educational Research.

Crespo, S. (2003). Learning to pose mathematical problems: Exploring changes in preservice teachers' practices. *Educational Studies in Mathematics*, *52*, 243–270.

Dede, Y., & Yaman, S. (2005a, September). İlköğretim 6., *7. ve 8. sınıf matematik ve fen bilgisi ders kitaplarının incelenmesi: Problem çözme ve problem kurma etkinlikleri bakımından*. Paper presented at XIV. Ulusal Eğitim Bilimleri Kongresi, Denizli, Turkey.

Dede, Y., & Yaman, S. (2005b). Identification of mathematical problem posing and problem solving skills for prospective mathematics teachers. *Eurasian Journal of Educational Research, 18*, 236–252.

English, L. D. (1997). The development of fifth-grade children's problem-posing abilities. *Educational Studies in Mathematics*, *34*(3), 183–217.

Erbaş, A. K. (Ed.). (2014). İlköğretim matematik 3: Öğrenci çalışma kitabı [*Elementary School Mathematics 3: Student Workbook]*. İstanbul, Türkiye: MEB Devlet Kitapları.

Ervynck, G. (1991). *Advanced mathematical thinking*. Hingham, MA: Kluwer, Academic Publishers.

Evancho, R. S. (2000). *Critical thinking skills and dispositions of the undergraduate baccalaureate nursing student*. Connecticut, CT: Southern Connecticut State University.

Forrester, T., & Chinnappan, M. (2010). The predominance of procedural knowledge in fractions. In L. Sparrow, B. Kissane & C. Hurst (Eds.), *Shaping the future of mathematics education MERGA33* (pp. 185–192). Fremantle, WA: MERGA.

Frant, J. B., Acevedo, J. I., & Font, V. (2005). Cogniçao corporificada e linguagem na sala de aula de matematica: Analisando metafores na dinamica do processo de ensino de graficos de funçoes. *BOLETIM GEPEM*, *46*, 41–54.

Freire, P. (1970). *Pedagogy of the oppressed*. New York, NY: The Continuum Publishing Corporation.

Getzels, J. W., & Csikszentmihalyi, M. (1962). *Creative and intelligence: Exploration with gifted students.* New York, NY: Wiley.

Grundmeier, T. A. (2003). *The effects of providing mathematical problem posing experiences for K-8 pre-service teachers: Investigating teachers beliefs and characteristics of posed problems* (Doctoral Dissertation). Available from ProOuest Dissertations and Theses database. (UMI No. 3083732)

Haylock, D. W. (1987). A framework for assessing mathematical creativity in school children. *Educational Studies in Mathematics, 18*, 59–74.

Hiebert, J., & Wearne, D. (2003). Developing understanding through problem solving. In H. L. Schoen (Eds.), *Teaching mathematics through problem solving: Grades 6 – 12* (pp. 3–13). Reston, VA: National Council of Teachers of Mathematics.

Hinchliffe, G. (2001). Education or pedagogy? *Journal of Philosophy of Education*, *35*(1), 133–148.

Hines, A. (2008). Global trends in culture, infrastructure and values. *Futurist*, *42*(5), 18–23.

Holton, D., & Gaffney, M. (1994). *Teaching talented students*. In J. Neyland (Ed.), *Mathematics education: A handbook for teachers* (Vol. 1, pp. 397–409). Wellington, New Zealand: Wellington College of Education.

Inbar, D. (1996). The free educational prison: Metaphors and images. *Educational Research*, *38*(1), 77–92.

Jay, E. S., & Perkins, D. N. (1997). *Problem finding: The search for mechanism*. In M. A. Runco (Ed), *The creativity research handbook* (pp. 257–293). Cresskil, NJ: Hampton.

Jonassen, D. H. (1997). Instructional design models for well-structured and ill structured problem-solving learning outcomes. *Educational Technology Research and Development*, *45*(1), 65–94.

Kilpatrick, J. (1987). *Where do good problems come from?* In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 123–148). Hillsdale, NJ: Lawrence Erlbaum.

Lakoff, G., & Johnson, M. (1980). *Metaphors we live by*. Chicago University Press.

Lakoff, G., & Johnson, M. (2005). *Metaforlar, Hayat anlam ve dil* [Metaphors we lived by] (trans. G. Y. Demir). İstanbul, Turkey: Paradigma Yayınları.

Lakoff, G., & Núñez, R. (1997). The metaphorical structure of mathematics: Sketching out a cognitive foundation for a mind-based mathematics. In L. English (Ed.), *Mathematical reasoning: Analogies, metaphors, and images* (pp. 21–89). Mahwah, NJ: L. Erlbaum Associates.

Leikin, R. (2009). Exploring mathematical creativity using multiple solution tasks. In R. Leikin, A. Berman, & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 129–145). Rotterdam, Netherlands: Sense Publishers.

Leikin, R., & Lev, M. (2013). Multiple solution tasks as a magnifying glass for observation of mathematical creativity. In J.-H. Woo, H.-C. Lew, K.-S. Park, & D.-Y. Seo (Eds.), *Proceedings of the 31st International Conference for the Psychology of Mathematics Education*, *3*, 161–168.

Leung, S. S. (1996). Problem posing as assessment: Reflections and re-constructions. *The Mathematics Educators*, *1*(2), 159–171.

Leung, S. S. (1997). On the role of creative thinking in problem posing. *ZDM, 3*, 81–85.

Levav-Waynberg, A., & Leikin, R. (2009). Multiple solutions for a problem: A tool for evaluation of mathematical thinking in geometry. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of Sixth Conference of European Research in Mathematics Education* (pp. 776–785). Lyon, France: Institut National de Recherche Pédagogique. Retrieved from http://www.inrp.fr/editions/editions-electroniques/cerme6/

Levine, P. M. (2005). Metaphors and images of classrooms. *Kappa Delta Pi Record*, *41*(4), 172–175.

Lowrie, T. (2002). Young children posing problems: The influence of teacher intervention on the type of problems children pose. *Mathematics Education Research Journal*, *14*(2), 87–98.

Mayer, P. (1970). Socialization and the social anthropologist. In P. Mayer (Ed.), *Socialization: The approach from social anthropology* (pp. 13–30). London, UK: Tavistock.

Milli Eğitim Bakanlığı. (2005). *İlköğretim matematik dersi 1-5.sınıflar öğretim programı.* Ankara, Turkey: Devlet Kitapları Basımevi.

Milli Eğitim Bakanlığı. (2009). *İlköğretim matematik dersi 1-5.sınıflar öğretim programı*. Ankara, Turkey: Devlet Kitapları Basımevi.

Ministry of Education Singapore. (2006). *Mathematics syllabus primary*. Retrieved from http://www.moe.gov.sg/education/syllabuses/sciences/files/maths-primary-2007.pdf

Moses, B., Bjork, E., & Goldenberg, E. P. (1993). Beyond problem solving: Problem posing. In S. I. Brown & M. I. Walter (Eds) *Problem posing: Reflections and applications* (pp. 178–188). Hillsdale, NJ: LEA.

National Council of Teachers of Mathematics. (1980). *An agenda for action: Recommendations for school mathematics of the 1980s*. Resron, VA: Author.

Nelson, L. M. (1999). *Collaborative problem solving*. In C. M. Reigeluth (Ed.), *Instructional design theories and models: A new paradigm of instructional theory* (vol. 2, pp. 241–267). Mahwah, NJ: Lawrence Erlbaum Associates.

Organisation for Economic Co-operation and Development. (2013). *Education at a glance 2013*. Retrieved from http://www.oecd.org/edu/eag2013%20(eng)--FINAL%2020%20June%202013.pdf

Picker, S. H., & Berry, J. S. (2000). Investigating pupils' images of mathematicians. *Educational Studies in Mathematics*, *43*(1), 65–94.

Polya, G. (1945). *How to solve it: A new aspect of mathematical method*. New Jersey: Princeton University Press.

Polya, G. (1957). *How to solve it: A new aspect of mathematical method*. New Jersey: Princeton University Press.

Reynolds, M. C., & Birch, J. W. (1988). *Adaptive mainstreaming (A primer for teachers and principals).* London, UK: Longman.

Rogers, K. B. (2002). *Re-forming gifted education: Matching the program to the child*. Scottsdale, AZ: Great Potential.

Romey, W. D. (1970). What is your creativity quotient? *School Science and Mathematics, 70*, 3–8.

Saban, A., Koçbeker, B. N., & Saban, A. (2006). Öğretmen adaylarının öğretmen kavramına ilişkin algılarının metafor analizi yoluyla incelenmesi. *Kuram ve Uygulamada Eğitim Bilimleri*, *6*, 461–522.

Savaşır I., & Şahin, N. (1995). *Wechsler Çocuklar için Zeka Ölçeği (WISC-R) el kitabı*. Ankara: Türk Psikologlar Derneği Yayınları.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition and sense – making in mathematics. In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 334–370). New York, NY: Macmillan.

Sheffield, L. J. (2008). Questioning mathematical creativity - questions may be the answers. In *Proceedings of the 5th International Conference on Creativity in Mathematics and the Education of Gifted Students* (pp. 29–34). Haifa, Israel.

Shor, I. (1992). *Empowering education: Critical teaching for social change*. University of Chicago Press.

Silver, E. A. (1994). On mathematical problem posing. *For the learning of mathematics*, *14*(1), 19–28.

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM*, *29*(3), 75–80.

Silver, E. A., Ghousseini, H., Gosen, D., Charalambous, C., & Strawhun, B. T. F. (2005). Moving from rhetoric to praxis: Issues faced by teachers in having students consider multiple solutions for problems in the mathematics classroom. *The Journal of Mathematical Behavior*, *24*(3), 287–301.

Silver, E., A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school. *Journal for Research in Mathematics Education*, *27*, 521–539.

Singer, F. M., & Voica, C. (2012). A problem-solving conceptual framework and its implications in designing problem-posing tasks. *Educational Studies in Mathematics, 83*(1), 9–26. doi:10.1007/s10649-012-9422-x

Skemp, R. (1987). *The psychology of learning mathematics.* New York, NY: Penguin.

Slattery, P. (1990). Encouraging critical thinking: A strategy of commenting on college papers. *College Composition and Communication*, *41*, 332–335.

Soylu, Y., & Aydın, S. (2006). Matematik derslerinde kavramsal ve işlemsel öğrenmenin dengelenmesinin önemi üzerine bir çalışma. *Erzincan Eğitim Fakültesi Dergisi, 8*(2), 83–95.

Sriraman, B. (2005). Are giftedness and creativity synonymous in mathematics? *Journal of Secondary Gifted Education*, *17*, 20–36.

Sternberg, R. J., & Davidson, J. E. (Eds.). (2005). *Conceptions of giftedness* (2nd ed.). New York, NY: Cambridge University Press.

Stickles, P. R. (2006). *An analysis of secondary and middle school teacher's mathematical problem posing* (Doctoral Dissertation). Available from ProOuest Dissertations and Theses database. (UMI No. 3219902)

Stoyanova, E., & Ellerton, N. F. (1996). *A framework for research into student's problem posing in school mathematics*. Mel Bourne, Australia: Mathematics Education Research Group of Australia.

Toluk, Z., & Olkun, S. (2001, June). İlköğretim ders kitaplarının problem çözme becerilerinin geliştirilmesi açısından incelenmesi. Paper presented at the X. Eğitim Bilimleri Kongresi, Abant İzzet Baysal Üniversitesi, Bolu, Turkey.

Van De Walle, J. A. (1994). *Elementary school mathematics*. New York, NY: Virginia Commonwealth University, Longman.

Vander, S., & Pintrich, P. (2003). *Learning to learn. The skill and will of college success*. Upper Saddle River, NJ: Prentice Hall.

Villani, C. J. (1998, April). *Meeting the needs of the gifted student in language arts and mathematics: An evaluative exploration.* Paper presented at the American Educational Research Association, San Diego, CA.

Wertheimer, M. (1945). *Productive thinking*. Westport, CT: Greenwood.

Winner, E. (1996). *Gifted children: Myths and realities*. New York, NY: Basic Books.

Yaman, S., & Dede, Y. (2005). Matematik ve fen eğitiminde problem kurma uygulamaları. *OMÜ Eğitim Fakültesi Dergisi*, *20*, 1–11.

Yates, F. (1934). Contingency table involving small numbers and the $\chi 2$ test. *The Journal of the Royal Statistical Society*, *1*(2), 217–235.

Yuan, X. (2009). *An exploratory study of high school students' creativity and mathematical problem posing in China and The United States* (Doctoral Dissertation, Illinois State University). Retrieved from http://gradworks.umi.com/35/28/3528254.html