Prescriptions Guiding Prospective Teachers in Teaching Mathematics[*]

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Abstract

This study aims to investigate the nature of different mathematics teaching modes (prescriptions) that guide prospective teachers during their instruction. The participants were 24 junior prospective middle school mathematics teachers (19 females and 5 males) who were attending a mathematics methods course at a private university in central Turkey. Each participant was tasked with applying an innovative prescribed curriculum about equivalent fractions to a single elementary or middle school student; he/she is required to videotape the teaching session. We collected these teaching videos and analyzed them through a phenomenographic approach. Results suggest that the participants select and operate with certain teaching prescriptions while teaching. The mathematical knowledge of the teacher is not the only factor that affects his/her teaching; ingrained prescriptions influence and guide their teaching as well. All these teaching prescriptions are selected according to the teachers' strongly held beliefs about teaching and learning based on their prior schooling experiences and not on their experiences in their coursework. We also found that having prospective teachers operate with an innovative curriculum does not improve their teaching in the presence of these prescriptions.

Keywords

Mathematics teacher knowledge • Teaching prescriptions • Teacher's use of curriculum • Developing teacher practice • Prospective middle school teachers

Relevant Literature

Prospective teachers go through teacher training programs and take many courses that partially or mostly address teacher knowledge categories (as detailed later in the article) at different universities in the world. Thus, they gain experience in content-specific and generic knowledge types and in pedagogical reasoning that will shape their future teaching. What prospective teachers bring to those courses or to their programs is as important as what they are taught in those courses (Ball, 1988; Goulding, Rowland, & Barber, 2002) and the experiences they gain from their programs. What they bring to courses is crucial because of the two following reasons: (1) They are likely to operate from that knowledge in their future classrooms, which may not be parallel to the type of teaching that is expected of them when they graduate from those programs and enter the teaching practice. (2) This factor is also important to inform the field about the needs of teachers and teacher education programs. Such information may help us to revise and enhance teacher education programs and the teaching of those future teachers.

As Ball (1988, p. 46) mentioned,

> How can teacher educators productively challenge, change, and extend what teacher education students bring? Knowing more about what teachers bring and what they learn from different components of and approaches to professional preparation is one more critical piece to the puzzle of improving the impact of mathematics teacher education on what goes on in elementary mathematics classrooms.

Related information on this matter can be obtained through analyses of teachers' knowledge levels as they go through the programs using certain measurement instruments (e.g., Hill, Schilling, & Ball, 2004; Toluk Uçar, 2011; Türnüklü, 2005) or through analyses of teachers' actual teaching approach (e.g., Rowland, Turner, Thwaites, & Huckstep, 2009; Hacıömeroğlu, 2012). Our study falls into the latter category because we also believe that an investigation of teacher knowledge as they teach is informative in identifying teachers' needs and the nature of teaching knowledge. Such information can be obtained through investigations of in-service or prospective teachers. The current study strictly focuses on actual teaching of prospective teachers and deliberates the methods/ways of teaching, which we call prescriptions that guide these teachers' mathematics teaching. We pursued the following research question: *What teaching methods (prescriptions) guide prospective middle school mathematics teachers in their teaching of mathematics*? The relevant literature that oriented us in pursuing this question and the rationale for the study is detailed below.

The relevant literature suggests that mathematics teachers draw on various resources in teaching mathematics. One of these resources is mathematics teacher knowledge (Ball & Forzani, 2009), which has a special feature specific to mathematics teaching (Ball, Thames, & Phelps, 2008). Identification of the core of (mathematics) teacher

knowledge is pursued extensively in mathematics education literature (e.g., Hill et al., 2004; Rowland, Huckstep, & Thwaites, 2005; Shulman, 1986). Revealing how and in what ways teacher knowledge guides, limits, or shapes teachers' teaching approach is as important as understanding the nature of teacher knowledge. An articulation of the factors that affect the teaching of mathematics teachers may inform decisions as to what should be focused on in teacher education programs and policy development. The current study aims to contribute to such knowledge base through investigating prospective teachers' teaching.

Our search in the relevant literature also reveals the following questions that many researchers have struggled with for the last few decades: "What kind of knowledge do teachers need to teach effectively? What is the nature of the knowledge that is specific to teaching?" Ma (1999) concludes that teachers should have deep knowledge at their teaching level instead of knowing advanced mathematics because no direct positive correlation exists between the formal education that teachers receive and their teaching (Askew, Brown, Rhodes, Johnson, & William, 1997; Zembat & Yasa, 2015). Teachers' knowledge should also be "connected" and "profound" (Ma, 1999). In a number of studies, Deborah Ball and her colleagues (e.g., Ball et al., 2008) designed scenarios to test the specialized content knowledge needed by teachers to handle everyday tasks of teaching mathematics. Even though such efforts are valuable in terms of identifying the depth of teacher knowledge of a large number of teachers and for policy development purposes, their results do not "necessarily reflect how one would teach in practice" (Rowland et al., 2009, p. 24). In this sense, direct observation of teaching is important and necessary to understand the nature of teacher knowledge (Hegarty, 2000). Studies in the relevant literature generally investigate mathematics teacher knowledge through pure quantitative measures or interviews regardless of the actual teaching methods applied by teachers (e.g., Hacıömeroğlu, 2013; Hill et al., 2004; Krauss, Baumert, & Blum, 2008). The relevant literature also includes some studies that aim to improve the teaching approach of teachers (e.g., Hacıömeroğlu, 2012; Wilson, 1994) through predesigned courses or programs. However, these studies investigate how specially designed courses or activities affect or orient a single teacher's thinking about and understanding of teaching and trace changes (or steadiness) in their thinking about teaching during their participation. The current study differs from both genres of research because it aims to articulate the factors that affect a number of teachers' teaching approach. Such factors are determined by analyzing the teaching approach of teachers who use a prescribed curriculum.

The current literature also suggests that teachers need deep and connected knowledge. Askew et al. (1997, p. 3) stated that children need to have a "rich network of connections between different mathematical ideas". Having such a network is possible only if teachers know more than their students. Therefore, how

can we prepare more knowledgeable teachers to teach effectively? We know that mathematical knowledge alone is not enough for effective teaching. Thus, we need to determine the other factors that facilitate the effective teaching of teachers. In investigating these questions, Shulman (1986, 1987) suggested that content-specific teacher knowledge consists of subject matter knowledge (SMK), pedagogical content knowledge (PCK), and curricular knowledge (CK). SMK was defined as the "amount and organization of the knowledge per se in the mind of the teacher" by Shulman (1986, p. 9). This definition was later expanded and described as the knowledge of key facts, concepts, principles, explanatory frameworks, and the rules of evidence and proof in the discipline (Shulman & Grossman, 1988). PCK is about the knowledge of

> The ways of representing and formulating the subject which makes it comprehensible to others […] includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons (Shulman, 1986, p. 9).

CK is about knowledge of curriculum, instructional programs, textbooks, materials, and resources in general. Shulman (1986) added four generic categories of knowledge to this list: general pedagogical knowledge, knowledge of learners, knowledge of context, and knowledge of purposes of teaching and learning. Our work focuses mostly on PCK to better understand prospective teachers' moves (Brown & Wragg, 1993) as they teach. We do not specifically focus on the nature of PCK. Instead, we pay close attention to the factors that guide the teaching and instructional decisions of teachers.

Teacher education literature also suggests that teachers' beliefs and what they know are factors that affect their practice (Thompson, 1992). Beliefs and practice develop together (Cobb, Wood, & Yackel, 1990). One way to help teachers develop their practice is to provide them with opportunities to apply an innovative curriculum through the use of new teaching approaches (Borasi, Fonzi, Smith, & Rose, 1999; Clarke & Hollingsworth, 2002; Kaasila, Hannula, Laine, & Pehkonen, 2008; Kieran & Guzmán, 2010; Lloyd, 2002). Such experiences give teachers the opportunity to draw wisely on their SMK, PCK, and CK. SMK, PCK, and CK interact in a teaching situation (Rowland et al., 2005). Thus, an amalgam of these knowledge types guides teachers in their teaching. However, professional use of that amalgam is difficult, especially for prospective teachers. The current study investigates the source of this difficulty in the context of actual teaching of prospective teachers.

Theoretical Underpinnings of the Study

Shulman (1987) identified a number of sources that feed the conception of teaching by proposing a model of pedagogical reasoning and action. He considered these

sources as a cycle of activities involved in pedagogical reasoning; these activities are comprehension, transformation, instruction, evaluation, and reflection. We focused mostly on one of these resources in analyzing our data; the rationale for this approach is provided later. Shulman (1987) uses *comprehension* to refer to teachers' comprehension of the critical ideas to be taught, their understanding in several ways of what they teach, and their comprehension of the purposes of teaching. *Transformation* is the capacity to transform the comprehended content knowledge "into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students" (Shulman, 1987, p. 15). Transformation consists of *preparation* (critical interpretation and analysis of texts), *representation* (using analogies, metaphors, examples, demonstrations, and explanations), *selection* (selecting teaching modes, organizing, managing, and arranging), and *adaptation and tailoring to student characteristics* (considering conceptions, difficulties, language, culture, age, gender, interests, and attention). Shulman uses *selection* to refer specifically to teachers' "draw[ing] upon an instructional repertoire of approaches or strategies of teaching. This repertoire can be quite rich, including not only the more conventional alternatives, […] but also a variety of forms […] and learning outside the classroom setting" (Shulman, 1987, p. 16–17). *Instruction* is the third source that drives teaching. It is about aspects of active teaching and observable forms of classroom teaching, such as management, interactions, and questioning. The *evaluation* component is about checking student comprehension during teaching and testing it at the end of lessons. *Reflection* is about critically analyzing one's own and class' performance, thereby leading to new *comprehension* from current experience in connection with the first component above.

As the teachers teach, they *transform* the *comprehended* mathematical ideas through *selection* of specific teaching modes. In interpreting and analyzing the data, we mainly focused on the *selection* of teaching modes to which the participants referred during their teaching and the effect of that selection on the *transformation* process. This approach gave us an opportunity to delve deeply into the factors that guide teachers' teaching. We referred to the aforesaid cycle as an orienting framework, whereas we specifically focused on the *selection* part of that cycle analytically. The reason for this approach is explained in the Method section.

Method

The current study was designed as a qualitative study, and its details are given below.

Participants

The participants were 24 prospective middle school teachers (PMSTs) (19 females and 5 males) who were in their third year of the mathematics education program at a private university in central Turkey. They took a total of 29 courses in their program before the

study; these courses consisted of 12 general interest cultural courses, 12 pure mathematics courses, and 5 educational science courses (Introduction to Educational Sciences, Educational Psychology, Principles and Methods of Teaching, Educational Technology, and Philosophy of Education). Considering their coursework before the study, the participants had sufficient experience with certain fundamental issues in teaching (e.g., comprehension of mathematical ideas that cut across school curriculum, lesson design, analysis of student thinking, educational theories and their applications, transformation, instruction, and evaluation). During their participation in the study, the participants were about to complete the mathematics methods course, as will be detailed below.

Data Gathering Procedure

The participants were taking a mathematics methods course that mainly focused on certain mathematical concepts, how students reasoned about those concepts, and on teaching and learning of those concepts before they participated in the study. Throughout the methods course, the PMSTs were given sample curricular pieces to be applied in actual classrooms (see Appendix 1 for a sample) and asked to go through those pieces under certain conditions (e.g., they were told to use only knowledge of fractions and counting in learning fraction division). Such experience gave them an opportunity to think about how a student needed to reason in going through such a curriculum. They were then asked to reflect on those experiences by focusing on the mathematical ideas targeted in the applied curriculum and the mental or physical activities students would go through, that is, the possible learning trajectories of students. This step gave them an opportunity to reflect on the subject matter and learning of students. For the next step, they were given several teaching videos (a teacher who is teaching a student or a class of students) to analyze how targeted ideas and issues, such as classroom norms (Cobb et al., 1990) and responding moves (Brown & Wragg, 1993), are taught in those teaching samples. This complete experience during the methods course gave the PMSTs opportunities to think about mathematics education as an amalgam of subject matters and to learn and teach it with the use of mostly constructivist lenses (Piaget, 2001). In this sense, they gained experience about several curricular pieces for certain mathematical concepts from the perspective of students and teachers, and from mathematical point of views. During the methods course, the PMSTs also gained experience in developing lessons aligned with constructivist principles (Gallagher & Reid, 1981; Piaget, 2001). In this regard, the methods course is designed to help PMSTs develop constructivist ideas (different knowledge types, assimilation, and different abstractions), as sampled in Appendix 2, and to gain teaching experience before going to schools for practicum.

Toward the end of the methods course, the prospective teachers were informed about this study; they then volunteered to participate. They were taught a lesson on equivalent

fractions (see Appendix 1 for details) by using the aforementioned sequence of activities. This equivalent fractions lesson, which was originally developed by Simon (2003), was first applied to the PMSTs themselves by the first author during the methods course. In applying the lesson, the PMSTs were supposed to go through the lesson by using only basic knowledge of what a fraction was, how it was represented, and basic operations on natural numbers. Once they used the elementary school student lens to go through the lesson, they then used the teacher lens and analyzed the learning mechanism of the lesson and the pedagogical principles for the design of the lesson under the guidance of the first author (see Appendix 2 for an analysis of the lesson). Then, they were given the opportunity to analyze video cases of teaching that lesson and discuss the important parts of the designed lesson. After acquiring experience on the equivalent fractions lesson (and a similar experience for different concepts, such as fraction division and multiplication, measurement concepts, and geometry concepts, throughout the semester), the PMSTs were asked to teach it to an elementary or a middle school student who did not know equivalent fractions but had the aforesaid minimum prior knowledge to go through the lesson as part of this study. For the PMSTs, the condition for this project was to teach the lesson and videotape it. The PMSTs were expected to work in pairs so that one could operate the camera as the other teaches. Forty-eight PMSTs joined the study. However, because they worked in pairs and only one person in the pair taught the lesson, we collected 24 teaching videos from 24 individual PMSTs; these videos were their first complete trials without any direct outside help during actual teaching. These teaching videos were created by the PMSTs who did not receive any external help. Thus, we believed that these videos gave us a better understanding of the models that the PMSTs used in teaching mathematics. The teaching time for each video ranged from 10 to 48 minutes. The collected videos provide the data for this study.

Data Analysis Procedure

We analyzed the teaching videos of PMST through the phenomenographic approach (Marton, 1986), which mainly aims to reveal an understanding of participants' experiences. Åkerlind (2005, p. 324) summarized this form of analysis as follows:

> The analysis usually starts with a search for meaning, or variation in meaning, […] and is then supplemented by a search for structural relationships between meanings. […] In the early stages, reading through transcripts is characterized by a high degree of openness to possible meanings, subsequent readings becoming more focused on particular aspects or criteria, but still within a framework of openness to new interpretations, and the ultimate aim of illuminating the whole by focusing on different perspectives at different times. The whole process is a strongly iterative and comparative one, involving the continual sorting and resorting of data, plus ongoing comparisons between the data and the developing categories of description, as well as between the categories themselves.

With the use of such a method, we performed the following steps during the analysis: As part of the methods course, the first author had already gone through the videos before the actual data analysis for measurement and evaluation purposes. Doing so gave him the opportunity to familiarize himself with the kind of teaching in which the PMSTs engaged. When we started the data analysis for this study, we did not start with the best or the worst video. Instead, we started with an average teaching video that seemed to exemplify constructivist teaching but is rich enough to allow us to see some complications of teaching. We checked that video in detail and generated some questions for a "high degree of openness to possible meanings," as highlighted by Åkerlind (2005, p. 324). Some of the questions were (1) Which questioning styles (leading or probing) as part of the *instruction* did the PMSTs use during their teaching? Do they use questions to reveal student thinking when needed during their instruction? In which cases can they do it and in which cases can they not? (2) Which teaching modes do the PMSTs *select* and adopt during their teaching? How do these modes guide their teaching? (3) Do the PMSTs focus their students on mechanical processes or conceptual underpinnings involved in the instruction? What understandings do their teaching lead to—empirical or reflective abstraction? (4) How do the PMSTs interpret the prescribed curriculum they are to pursue? Do they follow the script strictly, or do they make changes in the curriculum as needed? What affects the PMSTs' instructional decisions? How do they start the instruction? (5) How do the PMSTs analyze students' prior knowledge? Do the PMSTs analyze the teaching session well enough to inform their teaching? How do they handle unexpected student questions? (6) To what extent do the PMSTs trust the student as they teach, and do they provide enough waiting time? (7) To what extent are the PMSTs careful about the mathematical language they use in their teaching? (8) How does their understanding of the concepts affect their teaching? (9) How do the PMSTs handle student misconceptions to inform their teaching? How do they handle students' cognitive conflicts? Can they perform on-the-spot analyses of student thinking and integrate that into their teaching? These questions need to be answered to fully investigate participants' different ways of teaching. Such questions were also pursued in the relevant research for different purposes, as highlighted in the literature review of Moyer and Milewicz (2002). These questions helped us understand in finer detail the resources that participants draw on during their teaching. Therefore, we find this approach informative in analyzing participants' ways of teaching. We then applied this sequence of questions, which belong to either SMK, PCK, CK, or pedagogical thinking, to other videos. This was the first phase of the analysis where we increased our familiarity with the data and started thinking about some overarching themes with regard to participants' ways of teaching (Miles & Huberman, 1994). This phase also enabled us to generate dense descriptions of each PMST's teaching, especially of their teaching mode selection.

We then watched the videos again and checked the themes through comparison (Strauss & Corbin, 1994) in the second phase of our analysis. As we watched the videos, we checked for counter evidence; thus, the themes became clearer in this phase. We also attempted to discern the themes that model the PMSTs' selection of teaching modes during the transformation. We ran through all the data to identify the driving forces behind the participant teachers' acts in their teaching by looking at the students' progression and at the teachers' responding moves (Brown & Wragg, 1993) in response to that progression. Our purpose was to identify ways of thinking that guided teachers' instructional decisions. In other words, we attempted to model the *selected*/adopted ways of teaching and thinking behind that teaching. We called these ways of thinking as *teaching prescriptions* that guide and shape teachers' teaching. Identifying and modeling these prescriptions that focus on the teachers' *selection* of teaching modes and the effect of those selections on the *transformation* process guided us in modeling the teachers' different teaching modes. Note that throughout this article, we use "modes" and "prescriptions" interchangeably.

In the third phase of the analysis, we identified the specific prescriptions that teachers used during their instruction. This step gave us the opportunity to distinguish the kind of teaching and the thinking used behind that teaching, i.e., the prescriptions. As we went through this data analyses cycle, we refined the main characteristics of the prescriptions and finalized them.

Rationale for Focusing Only on the Selection Part of the Cycle

Prospective teachers who are taking teacher education programs are transitioning from being a student to being a teacher; in other words, they are shifting from being "doers" to "teachers" (Brown, McNamara, Hanley, & Jones, 1999, p. 302). What factors make such a transition difficult or easy for these teachers? Finding an answer to this question is not easy when prospective teachers have too much load in terms of different teaching tasks. We believe that having them pass through Shulman's entire cycle of activities alone for teaching a specific mathematical topic would not help us see the details in this transition. However, focusing on specific part(s) of that cycle in detail would better help us understand the nature of obstacles commonly seen in such a transition. Some research in the relevant literature (Wilkes, 1994) has focused on and evaluated parts of the *transformation* process. Given this reason, we focused particularly on one of the items in the cycle, namely, selection.

Another reason is given for focusing on a specific part of the cycle. As discussed previously, comprehension, transformation, instruction, evaluation, and reflection are the main tenets of the pedagogical reasoning involved in teaching. We believe that different courses in teacher education programs are designed to help prospective teachers go through this cycle, and prospective teachers acquire experience in those

different parts as they go through the program. However, bringing those parts together is not an easy task for someone who is transitioning from being a "doer" to being a "teacher." Therefore, in addition to all the coursework taken by the participants, the first author helped prospective teachers gain rich experiences in the parts of this cycle during the methods course, as explained in the description of the methods course. Even though the PMSTs have acquired such experience, we considered that they, as novice teachers, would not be able to undertake the full load of these parts of the entire cycle. Therefore, we asked the PMSTs to use a prescribed curriculum and apply it to a single student instead of a whole classroom of students. We believed that applying this approach would not overwhelm the PMSTs with the heavy load of their teaching requirements, such as identifying critical ideas to be taught (comprehension), analyzing texts (preparation), generating examples (representation), management and interactions (instruction), and evaluation or reflection. Instead, they were to deal with a single student with a pre-planned and pre-analyzed curriculum piece with the use of their background knowledge and beliefs. Therefore, we reduced the entire cycle to a single part of the cycle, and we focused primarily on that part. In this way, we were able to investigate in detail what hampered, fostered, or affected the transition from being a "doer" to being a "teacher."

We did not especially focus on evaluation, reflection, and new comprehension parts of the cycle in detail because we are not pursuing these prospective teachers' development of certain teaching qualities. Instead, we mainly focused on the sources (or obstacles) that guide these teachers' decisions and teaching modes during their transformation of the preplanned instructional unit.

Results

The PMSTs analyzed equivalent fractions in detail under the guidance of the first author before the lesson. However, what they did in the lesson was almost in contrast to what they learned about the methods course. This finding was interesting. When we analyzed how the PMSTs taught children, we found that they constantly *selected* and used certain teaching models that we call prescriptions, which guided and shaped their teaching. Prescription in this context refers specifically to the PMSTs' adopted ways of teaching based on their current beliefs, knowledge about teaching, and their learning from their prior experiences. We observed that these prescriptions directed the teachers and guided their teaching approach; in a way, these prescriptions were their harbors and were common for all participants.

All participants *selected* two or more of the prescriptions at certain times/benchmarks in their teaching. These benchmarks are in accordance with the curriculum. For example, three main thresholds exist in the curriculum: questions 1–3 are the first,

questions 4–5 are the second, and question 6 is the third. The first three questions, or the first section, were designed to provide students with experience in finding the equivalence of given fractions without thinking much about the activity sequence (Phase 1). The second section in the sequence, or questions 4 and 5, was designed to help students reflect on the activities they go through and solve the given problems as if they used diagrams (Phase 2). In the third section, or question 6, students were expected to understand the logico-mathematical knowledge of 'size of the fractional quantities is independent of number of partitions they consist of' because the missing factor in the given fraction equivalence is in a different place (Phase 3). In each section, the PMSTs seemed to select different teaching prescriptions. We describe these teaching prescriptions with sample dialogs from the succeeding data.

Results through a Qualitative Look at the Data: Teaching Prescriptions Used by PMSTs

The teaching prescriptions used by the PMSTs are detailed below. These prescriptions are similar to a doctor's prescription for his/her patient. In other words, these are different modes/ways of teaching that the PMSTs used during instruction.

A. Dragging teaching prescription. In this teaching mode, the PMSTs 'dragged' students either to the target in the PMSTs' minds or to the right answer with the use of leading questions without paying much attention to the student's thinking. When asking leading questions was ineffective, the PMSTs either directly told the students what to do or gave them hints as to the right answer when the student gets stuck at some point in the instruction. Even if the students did not get stuck, the PMSTs might hamper the student's thinking during the instruction and direct his/her attention to the teacher's own way to obtain the answer. In this mode, the PMSTs showed the students what they should learn. Students were directed to the correct answer through different curriculum sections without having knowledge of the meaning and purpose of those sections. In this teaching mode, the teacher's role is to drag students' thinking toward a predetermined goal, and the student's role is to follow the teacher's lead. Dialog 1 is a typical example of this teaching mode. Throughout this paper, T denotes the teacher, and S denotes the student in dialogs.

Dialog #1

1 T: Here [*pointing to question-4a* ($\frac{2}{9} = \frac{?}{90}$)], we will do the diagrams, which we did before, without drawing them. Here we did it through drawing, right? This question asks how we would do it without actually drawing. How would you do it now?

2 S: Here it says 2/9. That's why—

3 T: (*interrupting the student*) We would draw 2 over 9, right? Then we would shade in 2 of them.

4 S: Then we would draw 10 over 90, shade in 10 of those.

5 T: Now we have 2 over 9. We would shade in two of 2 over 9. Then 90, since it is 10 times of 9, we would partition each part into 10. Since there is 2 here, when we multiply each part by 2, we would get 20. How would we do the one below [*pointing to question-4b* ($\frac{7}{9} = \frac{?}{72}$)]?

6 S: It says 7 over 9. And here, it says 72. Here, out of 72—

7 T: (*interrupting the student*) First, we would draw 7 over 9.

8 S: We would draw 7 over 9 (*repeating what the teacher said*).

9 T: We would shade in 7 of them.

10 S: We would shade in 7 of them (*repeating what the teacher said*) but here, 72 of them—

11 T: (*interrupting the student*) We would divide 72 into 9.

12 S: We divide 72 into 9 (*repeating what the teacher said*).

13 T: We would partition each part; (*with*) the number we divide by, right? You don't need to make a calculation. You don't need to find the result. It is enough to say the procedures.

14 S: (*thinks silently for about 10 seconds*)

15 T: Now, it is 7 over 9, right? Now, what do we do on these diagrams? We divide it into 9 parts, right? (*the teacher draws a whole rectangle, partitions it into 9 parts, and shades in 7 of them*) This area (*pointing to the shaded 7 parts*) was our 7 over 9. The question asks us to find "?" over 72. What can we do here? We would divide 72 into 9, making 8. We would separate each part into 8 pieces. When we separate into 8, we would count the shaded area (*referring to the number of parts in the shaded area*), then we would find the "?," right?

The teacher in Dialog #1 tells or explains to the student almost every step to be taken, interrupts him, and does not allow the student to think (lines #3, #7, #11). When no clear response is given by the student (line #14), the teacher goes through the entire solution process herself and performs direct teaching (paragraph #15). The dialog suggests that this teacher operates in *dragging* mode because she does not give the student enough opportunity to think and explain his thinking. She interrupts him continuously and leads him to the answer by stating it directly or giving hints. In this teaching mode, the student almost blindly follows the teacher without thinking about the steps to be taken and the concept of equivalence (lines #8, #10, #12). The teacher

seems to be one step ahead of the student. Thus, she drags the student to the point where she wants the student to be. Note that the teacher drags the student toward the answer throughout the teaching session, and the student follows her lead. The teacher's mode of teaching suggests that students learn as their thinking is dragged to the right answer. In this manner, the teachers of this category *select* dragging as a teaching mode to *transform* what they know into a form that fits the student's needs. However, such *selection* was not very effective in fostering student development of the concept of fraction equivalence.

B. Molder teaching prescription. The important thing for the PMST who is operating with this prescription is to make students to go through mental activities (or activity sequence) by giving students certain directions. The PMST who uses this prescription either asks the right question at the right place to help student overcome a difficulty and to untangle the conflicts at hand (if this approach does not work, then the process just continues) or sacrifices the quality of the question for the sake of helping the student. Even if the student makes a reasonable move to think about the equivalent fraction concept, the teacher ignores this move and leads the student to a predetermined activity sequence. In doing so, the teacher helps the student make an empirical abstraction as opposed to a reflective abstraction. The thinking that guides the teacher who uses this prescription is "if the student learns the topic empirically, then he/she actually learns it." The teachers who operate in this mode help students to progress through a particular mold frame (leading to empirical abstraction). Thus, this teaching prescription is called *molder teaching prescription*. Dialog 2.1 is a typical example of this teaching mode.

Dialog #2.1

The dialog for Question-2 ($\frac{2}{3} = \frac{?}{12}$):

1 S: (*draws a whole rectangle and shades in 2/3 of it*)

2 T: So how many parts do I have to divide this whole into?

3 S: Twelve.

4 T: I need to get 1 over 12s. Then, how many pieces do we need to separate each partition into?

5 S: Four (*partitions each shaded part into 4 and does not change the unshaded parts*).

6 T: (*pointing to the unshaded parts*] What about the others, (*is it*) the same way?

7 S: (*partitions each unshaded part into 4 pieces too*).

8 T: (*asks as if she wants the student to find the number of shaded parts*) Now, the shaded parts?

9 S: (*counts the shaded parts*) Eight.

10 T: (*as if asking about the result*) Then?

11 S: (*silently writes 2/3=8/12 on the paper*)
   The dialog for Question-3a ($\frac{3}{4} = \frac{?}{8}$):

12 T: All right. What do you need to do here?

13 S: To find the place of "?."

14 T: Yes, go ahead.

15 S: (*draws a diagram to represent* 3/4)

16 T: Now, how many pieces do we need to partition each part into so we get one over eights?

17 S: Two [*partitions only the shaded parts into 2 pieces*).

18 T: If you like, you can count (*the total number of parts*) to see how many there are.

19 S: Nine.

20 T: How many should there be all together?

21 S: Four.

22 T: (*in a tone indicating that he answered wrongly*] Why don't you draw again?

23 S: (*draws another representation of 3/4 next to the first one*)

24 T: Now, how many partitions do I need to separate each part into to get one-eights?

25 S: Two. (*partitions each part vertically into 2 pieces*)

26 T: How many shaded parts are there now?

27 S: Six.

28 T: Then?

29 S: (*silently writes 3/4=6/8 on the paper*)

Dialog #2.1 suggests that the important thing for the teacher here is to help student find "?" by passing through the following activity sequence: (1) identify the first fraction using a diagram, (2) partition only the shaded area with respect to the denominator of the second fraction, (3) partition unshaded parts after the teacher's intervention, and (4) count the number of pieces in the shaded area. Before each activity in the above sequence, the teacher asks a leading question (lines #2, #4, #6, #8, #16, #20, #24, #26) to help the student progress in the predetermined activity sequence. The interaction between the student and the teacher in this mode is in the form of "the teacher asks a question, the student answers it to progress to the next activity in the sequence." An interesting detail is that the student initially partitions shaded parts and ignores the unshaded parts because he follows the aforesaid activity sequence (lines #5, #17). He then pays attention to the unshaded parts after the teacher's interruption. This finding also suggests that the student silently goes through the sequence suggested by the teacher. The student operates with an understanding that 'the shaded parts are important to find the result,' whereas the teacher considers it a mistake (line #22) and corrects it each time the student operates this way. The same interaction pattern continues in other questions. This pattern suggests that the student follows the aforesaid activity sequence with the teacher's lead without thinking about what he does, and the teacher does not attempt to have the student think about what he does. What is important for the teacher in this teaching mode is to have the student progress through a particular mold frame, in this case, a particular activity sequence. This method is the teacher's *selection* of a certain teaching mode to *transform* his understanding of the content.

We also mentioned that teachers who operate with this prescription lead students to empirical abstraction as opposed to reflective abstraction. This finding is illustrated in Dialog 2.2.

Dialog #2.2

30 T: Let's look at these (*pointing to question 6*).

31 S: (*pointing to* 13/36) Divided by 36 and taken 13, (*pointing to ?/324*) divided by 324 and taken a number. Here, it (*referring to* 36) is three times as much as this number (*points to* 13)! We set up a relationship among these (*referring to the numerators and denominators of given two fractions*).

32 T: That (*referring to* 13/36)? Three times as much?

33 S: Not three times as much. It does not give an integer. If we set up a relationship between 36 and 324, if this number (*points to* 324) is nine times as much as this (*points to* 36), we find nine times of this number (*points to* 13).

34 T: […] How do you know that it is nine times as much? Did you multiply 36 and 9 first?

35 S: First, I multiplied by 10, which makes 360. But this is 324, so I multiplied by 9. This makes (*referring to* 13×9) 117.

36 T: Look at this one. You will use the calculator again.

37 S: (*pointing to* 9/72) Divided by 72 and taken 9. Taken 81 and divided into something (*pointing to 81/?*). This (*pointing to 81*) is 9 times as much as this (*pointing to* 9). Then, we find 9 times this (*pointing to* 72) (*computing on a calculator that is unseen by the camera*).

38 T: Let's look at the other one.

39 S: Divided by 54 and took part of it, divided by 702 and took 78 of it. Between 54 and 702 (*divides* 702 *by* 54), this (*pointing to* 702) is 13 times of this (*pointing to* 54). Then, this number (*pointing to* 78) will be 13 times of this (*pointing to "?"*). Then, we can find it through dividing 78 by 13. (*performs the division on paper*) Six times. And this should be (*pointing to "?"*) 6.

Dialog #2.2 suggests that the student does not understand the core of the fraction equivalence but memorizes a method (paragraphs #31, #33) to solve fraction equivalence problems. This operating seems to be at the level of empirical abstraction on the student's part and fostered by the teacher. The student operates with the knowledge that whatever the multiplicative relation is between the two denominators should be same as the one between the two numerators, which is understood on a numerical basis only (paragraphs #37–#39). What is important here is that the teacher is satisfied with his *selection* of this method and fosters it instead of making moves to support reflective abstraction. The lack of support may be due to the teacher's *comprehension* of the purpose of teaching (going for empirical abstraction) for fraction equivalence, which in turn seems to affect the teacher's *selection* of this mode.

C. Language-ignorant teaching prescription. The PMSTs who operate with this prescription are ignorant of either the content/quality of the mathematical language they use or of the influence of that language on students at some stage(s) of their instruction. The PMSTs in this mode of teaching check the degree to which the language used by the student is aligned with the language they use. If a mismatch exists, then the teacher perceives that the student does not learn the targeted topic. In other words, these teachers may consider that the student has learned if a tight one-to-one correspondence exists between the teacher's language and the student's language. Therefore, these PMSTs look for the closest match between two languages to make a conclusion about student learning. Sometimes, such a match is made possible by reducing the quality of

the questions asked to the students. What is important for the teacher in this mode is to help students to progress without paying attention to the quality of given mathematical explanations or the preferred language of students. Hence, the teacher who operates in this mode attempts to have the student *adapt* to the teacher's language by imposing his use of language on the student instead of adjusting to student characteristics (Shulman, 1987). Dialog #3 is a typical example of this teaching mode.

Dialog #3

1 S: [*explaining the solution to* $(\frac{7}{9} = \frac{?}{72})$] We first divide the rectangle into 9 and shade in 7.

2 T: Okay.

3 S: Then we divide into 72.

4 T: Okay.

5 S: We will divide each part into 8.

6 T: Very nice!

7 S: Since each part is divided into 8, 8 times 7 makes 56.

8 T: What did we find by multiplying 8 and 7?

9 S: Fifty-six.

10 T: Where do we find? Out of the whole—

11 S: We found the numerator.

12 T: We partitioned, remember? What is that numerator about?

13 S: Would you say that again?

14 T: Remember we divide into 9, shade in 7 of it? What is that about?

15 S: It is 56 divided by 72.

16 T: Remember you divided the whole into 9 and shaded in 7 of it. Then what is the 56 that you found here?

17 S: Fifty-six.

18 T: What is it?

19 S: It makes 7/9.

20 T: In fractions, you partitioned this (*referring to* 9), you divide it, and you shaded in this (*referring to* 7) What would that be here (*referring to?*/72) then?

21 S: We will shade in 56 of it.

22 T: It is the shaded area, this one (*pointing to* 72) is the *partitioned piece*, this one (*referring to* 56) is a *shaded piece*.

In Dialog #3, the teacher evidently makes effort (lines #10-#19 and paragraph #20) to have the student adapt to a certain vocabulary, such as the terms "partitioned piece" and "shaded piece," without thinking about what such vocabulary would add to the student's thinking. This behavior suggests that the teacher considers that student is at the expected level as long as the teacher's language and the student's language match closely. Otherwise, the teacher would not encourage a certain language even though the initial responses of the student were on target (lines #1–#9). The teacher's *comprehension* of the purpose of teaching seems to be to establish a fit between the student's language and the teacher's own language. Expectation of such a fit seems to affect the teacher's *selection* of this operating mode. As a result, she is attempting to make the student adapt to her language instead of *adapting and tailoring* the student's characteristics.

D. Self-centered teaching prescription. The PMSTs who operate with this prescription consider themselves a locomotive and in the center of the teaching-learning process. The PMSTs in this mode think that students cannot progress unless the teacher helps. In this mode, the student blindly follows the teacher's lead and is not given any chance to develop any ideas. Even if the student wants to develop a different idea at some point during the instruction, the teacher encourages the student to use the method that he/she offers instead of the one offered by the student. The PMSTs who use this teaching prescription do not trust the student and do not give him or her opportunities to think and reason, and they perform self-centered teaching. Thus, this prescription is called *self-centered teaching prescription*. Dialog #4 is a typical example of this teaching mode.

Dialog #4

1 T: I will also ask you additional questions. What would be "?" here (*referring to* 81/?= 9/72)?

2 S: This can't be done through drawing. First, we will divide this (*pointing to* 81) into this (*pointing to* 9). We will shade in 9 after we divide this.

3 T: What is my 9 about?

4 S: The area we will shade in, the number of it.

5 T: Does the question already give me the shaded pieces? These—what is my numerator about?

6 S: The numerator is the shaded area.

7 T: Yes, the shaded area. Then the question gave me the shaded area in both fractions.

8 S: This (*pointing to* "?") will be 9.

9 T: What would be my 9 here about (*pointing to the* 9 *that the student found by dividing* 81 *by* 9)? If I were to draw a diagram, what would I do with this 9?

10 S: I would divide 72 into 9. I remember now. Here, the question mark (*referring to* "?" *in the question*) will be 8.

11 T: Let me say it this way. Now I divide every part of 72 into 9, right? Then, how many parts do I have in total?

12 S: Eight.

13 T: I have 72 parts. I divide each into 9.

14 S: When we divide 72 by 9–

15 T: (*interrupts the student*) I don't divide 72 into 9. I divide every part of my 72 into 9.

16 S: Then we would draw 9. We would draw that shaded area with 72. As we did it before, right?

17 T: Very nice. Then what would I do at the end?

18 S: This 8, I would shade in up to 9.

19 T: I need to do what with 72 and 9?

20 S: Divide.

21 T: Okay (*with a discouraged sigh*). How many parts do I have? I have 72 parts? Am I right? How many pieces do I have 72 partitioned into?

22 S: Nine.

23 T: I divide into 9. Then how many pieces do I have at the end? In total?

24 S: Eight.

25 T: Don't we multiply 72 by 9?

26 S: Correct.

27 T: Because I will have a very big part, right?

28 S: Right.

29 T: Okay, let's multiply then. What would come in place of the question mark then?

30 S: Six hundred forty-eight.

Dialog #4 involves many different teaching and learning issues. However, we will only focus on the teacher's teaching mode that reflects his beliefs about teaching and the student's role in this interaction. The dialog suggests that the student understood the equivalent fractions neither conceptually nor procedurally given that the student either divided the numerators to find the missing denominator (paragraph #2) or divided the denominator of the second fraction to its numerator to find the missing denominator (paragraph #10). Then, a teacher-led discussion (paragraphs #9–#23) proceeded in the form of 'the teacher asks a question, the student answers wrongly, and the teacher asks another question, the student answers wrong again' until the teacher had the student find the answer (lines #25–#30). The teacher *selects* this teaching mode in interacting with the student.

In this dialog, the teacher has the mentality of 'the student can only progress following my commands or leads.' This mentality means that every move of the student is defined by the teacher, and the student makes his moves according to questions and guidance by the teacher. The teachers who use this prescription differ from the others in that they continue the instruction even though the student cannot answer any question correctly and have the student mindlessly find the answer. In this mode, the teacher sees the problem and the steps to take to solve it, and he/she has the student take the necessary steps even though the student's answers are wrong and has the student find the solution. The student then blindly follows the teacher's lead as if he/she has no choice but to obey the teacher after some point during the instruction. Therefore, the student cannot be considered mentally active, whereas the teacher acts as the driver and the host in this teaching mode.

E. Repetitive teaching prescription. In some teaching sessions, the students could not decide on the activity sequence they were supposed to go through to solve the given problems and had difficulty with higher-level problems (questions 4, 5, and 6) for which they were not allowed to use diagrams. In such cases, the PMSTs select the repetitive teaching prescription. In this prescription, the teacher leads the student to a previous point in the instruction (or to the very beginning of the instruction) and asks same questions repeatedly with the hope that it may help the student make progress and learn the targeted idea. This approach is repeated as long as a student

gets stuck or makes mistakes during the instruction. Sometimes, the teacher repeats the questions slowly in the same manner that a resident of a city would use to describe an unknown address to a foreign visitor word by word. Dialog #5 is a typical example of this teaching mode.

Dialog #5

1 T: Now I don't want you to draw on paper. What would you do if you were to draw? I want you to explain that in order to find the question mark. You don't need to write anything, just explain it. What would you do first?

2 S: I would separate into nine parts and shade in two of them. To divide into 90, I would do 10–

3 T: Yes, we would divide all into 10. How many parts did we shade in in the first drawn diagram?

4 S: Two.

5 T: Then we divide them into 10 parts. Then what would be the question mark?

6 T: (*the student thinks for a while*) *Should we take it over?*

7 S: Twenty.

8 T: Yes. Okay. Now let's do this one (*pointing to question 4b*) the same way.

9 S: I would divide into 9, shade in 7 of them. I would then do each—

10 T: How many parts would you separate it into to get 1 over 72? Should we think about it again? *Let's think about it again calmly. What did we do first?*

11 S: We divide into 9 and took 7 of them.

12 T: Okay. We divide into 9 and took 7 of them. We have 7 shaded partitions, right? We have an imaginary diagram. Okay, how many more parts do I need to partition these 7 parts into so that there are 1/72s?

13 S: Ten.

14 T: No, it can't be ten.

15 S: Twelve.

16 T: No, it can't be twelve either.

17 S: (*after thinking for about 25 seconds*) Eight.

18 T: Yes, if we divide into 8, what would that all be?

19 S: Seventy-two.

20 T: Yes. Okay. Then […] what would be the question mark?

21 S: Eight.

22 T: Now we divided into 9, we shaded in 7 of them, we divided each of seven parts into 8 parts.

23 S: Fifty-seven.

24 T: No! *Should you think about it again?*

With the use of the repetitive teaching prescription, as illustrated in Dialog #5, the teacher made the student start over (lines #6, #24, paragraph #10) when she realized that the student got stuck at some point or cannot make any progress. The problem here is that the teacher could not diagnose the sources of the problems in student's progress during the instruction and believes that she could relieve the student from his difficulties by going back to a starting point. The student did not internalize the activity sequence to follow in solving the given question, and the teacher herself did not identify the goal from the very beginning; the goal that the teacher identified is to find the "?." The teacher was not able to identify these issues with instant judgment during the student-teacher interaction. Moreover, the teacher may believe that she could have the student handle the problem by making the student start over. The teacher who used this prescription believed that starting over is an efficient teaching approach even though it does not work. This interaction pattern continued for approximately an hour for the teacher illustrated in the above dialog.

F. Curriculum-ignorant teaching prescription. Teachers who select and operate with this prescription consider the prescribed curriculum piece to make students solve a sequence of problems instead of aiming to teach students the concept of equivalent fractions. Therefore, these PMSTs ignored the hierarchical structure of the prescribed curriculum and considered each section in the curriculum independently as a threshold to pass without thinking about the big picture targeted by the curriculum. When students got stuck at some point, the PMSTs who adopted this prescription replaced some questions in the task sequence with others aimlessly without thinking about how they fit the hierarchy in the overall prescribed curriculum. They also did not pay enough attention to the purpose of using the materials (e.g., scientific calculator) in the sequence and therefore did not allow students to use them. In sum, the PMSTs who followed this teaching mode ignored the main tenets of the prescribed curriculum. Dialog #6 is a typical example of this teaching mode.

Dialog #6

Student-teacher interaction in solving Question-1 ($\frac{1}{2}=\frac{?}{6}$)**:**

1 T: Can we solve these problems with you?

2 S: Yes (*reads question-*1*a silently, shows* 1/2, *and separates each part into three pieces*).

3 T: What did you do first?

4 S: I first did one-half and divided it into 6. One-sixths needs to be shown first.

5 T: Then what would "?" take in?

6 S: Three.

*Student-teacher interaction in solving Question-*2 ($\frac{2}{3}=\frac{?}{12}$)**:**

7 S: (*reads the question silently, shows* 2/3, *and separates each part into different numbers of pieces so that there are 12 pieces in total*) This (*pointing to part of the shaded section*) over 12 is 1. Because this part (*counts the shaded parts*) is 4, it should be 4 here (*pointing to* "?").

8 T: Let's move on to the next question.

In Dialog #6, the teacher paid attention only to whether the student solved the problem or not. As soon as the student solved the problem, the teacher led the student to the next question without questioning what the student did or how the student thought. Dialog #6 suggests that the student performed two main activities: separating the parts (un)equally with respect to the denominator of the second fraction and counting the shaded parts one by one to find the value of the numerator. The student's counting of the shaded parts one by one suggests that he operates without knowing the relation between partitioning (e.g., partitioning each 1/3 into 4 equal parts) and how 3 becomes 12 (for the question, $\frac{2}{3}=\frac{?}{12}$). The teacher seemed to ignore this detail and let the student find the wrong answer (paragraph #7; the answer should be 8 instead of 4). As long as the student finds an answer for "?," the teacher is satisfied regardless of whether it is right or wrong and lets the student move on to the next question. This approach suggests that the teacher considers the curriculum as a collection of questions to be solved without thinking about the role of each section of questions (each phase), how they fit together, and how they foster students' development.

The teacher who operates in this mode also ignores the hierarchical structure of the curriculum. Such lack of attention to the different parts of the curriculum and their effects on the student development cause teachers to deviate from the prescribed curriculum

and modify it aimlessly. For example, after working on question 4 unsuccessfully, the participant teacher (from Dialog #6) asked the student a question like 4/5=?/60 and ended the instruction. The question 4/5=?/60 is not different from question 4 (7/9=?/72). The student already had some difficulty in solving question 4 and could not answer this extra question without the teacher's help either. This observation suggests that the student does not understand the core of fraction equivalence. The teacher ignored questions 5 and 6, and instead asked an extra question that is very similar to question 4; this approach suggests that the teacher does not understand the purpose of those two last questions and the sequence. In addition, the questions before question 6 are all similar; the numerator of the second fraction is missing, whereas question 6 has the missing denominator in the first fraction. This pattern is especially integrated into the curriculum piece to help students reflect on the interrelationship between numerators and denominators of the fractions, and not make an overgeneralization for equivalent fractions. The fact that the teacher skips question 6 and asks a question similar to the ones before suggests that the teacher ignores (or does not understand the value of) the main tenets and the hierarchical structure of the prescribed curriculum.

Curricular ignorance is demonstrated in another example. During the instruction, students are to use a basic scientific calculator to find the missing numerator in the given fraction equivalence. As they use the calculator, they also need to talk about why they perform certain calculations (the activity sequence). The purpose of using the calculator is to help the student not deal with complex calculations and to allow him/her to reflect on and pay attention to the activity sequence he/she is supposed to go through, as well as the multiplicative relations between the numerators and denominators. However, the teacher who uses this prescription skips this step and does not allow the student to use the calculator. This approach suggests that the teacher does not understand the purpose of using the calculator, its effect on the learning process, and the kind of abstraction it fosters. Therefore, the teacher ignores this part of the curriculum and *selects* and operates with a curriculum-ignorant prescription during the *transformation* process.

G. Student-ignorant teaching prescription. The PMSTs who *select* this prescription operate as if the student were not an actor in the teaching-learning process. They do not allow students' thinking to inform their teaching, and they basically ignore the students. These PMST also do not pay enough attention to the prior knowledge of the students, and they do not make judgments about student knowledge unless the student gets stuck or makes a mistake.

For example, in one of the teaching sessions, a student solves all the problems except question 6b by benefitting from the pure multiplicative relation on a numerical basis only between numerators and denominators of the given fractions. However, the student was not aware as to why she needed to multiply both the numerator and the denominator with the

same number. In question 6b, ($\frac{?}{54} = \frac{78}{702}$), the student multiplies 54 by 12, deletes it, tries 54 × 13 and finds 702, then calculates 78 × 13 and finds 1014, and announces it as an answer to the given question. This student had no problem in finding the answers to the given problems until question 6b because she used the aforesaid method for this question. The student had no problem with the first 5 questions. Thus, the teacher believed that she understood the core of fraction equivalence. This approach suggests that the teacher did not analyze the student's thinking appropriately and ignored it until question 6b where she actually had trouble. The place of the question mark changed in question 6b, thereby revealing the real understanding of the student. This situation was the first cue for the teacher to realize that the student actually did not understand the core of fraction equivalence until the end of the lesson. Such ignorance on the teacher's part affected the lesson, and the teacher ended the lesson.

Another example of operating with this prescription is teacher's ignorance of the prior knowledge of the student. The teachers who use this prescription start their instruction by ignoring the student's prior knowledge, as illustrated in Dialog 7.

Dialog #7

1 S: (*reads question 1b,* $\frac{1}{2} = \frac{?}{6}$*, aloud*) The question mark should be 3.

2 T: Yes. I mean you would partition each part into 3, right?

3 S: Because here (*pointing to the denominator of* 1/2) it says 2 and here (*pointing to the denominator of ?/6)* it says 6, so it is 3 times as much, that's why.

4 T: Let's divide each part into 3 then.

5 S: (*partitions each part of the rectangle into three equal parts*)

6 T: How many parts are in the shaded area?

7 S: Six divided by 3.

8 T: The shaded part, how many pieces are there?

9 S: Three.

10 T: Three, right? Then for the question mark—

11 S: It is 3.

Dialog #7 suggests that the student knew how to find the missing factor in equivalent fractions at the beginning of the lesson (lines #1-#2 and paragraph #3). The teacher started the lesson in spite of this fact and continued it by ignoring the student's prior knowledge (or by not even realizing it).

Results through a Quantitative Look at the Data: Frequency of the Selected Prescriptions

A detailed list of different prescriptions selected and used by the PMSTs is illustrated in Table 1. When 3B is present in a column, it means that the participant teacher referred to prescription B three times during that phase.

Table 1
List of PMST using Different Prescriptions as related to the Prescribed Curriculum

Name	Phase 1 Questions 1, 2, and 3	Phase 2 Questions 4 and 5	Phase 3 Question 6	Total Number of Prescription Types Used
Melek	B D / B C F G A	B / B C	G	6
Okan	B / B C	A B / B	F G	5
Reyhan	C E F		F G B	5
Betül	B	B C / B C	F	3
Fatmagül	A B C D F	F	B	5
Serpil	B D / B	B / B F	F	3
Halil	A B F / F	B F	B	3
Rana	B / A B / B	B / B / B		2
Rüveyda	B / B C D	B F	A / A B D F	5
Havva	B / B C D / B	B D E	B / B F	5
Hasan	B G A	F		4
Mahmut	B C F A B	A		4
Ahmet	C E F B		F	4
Gamze	B / B C F A B	F		4
Semiha	B D F A B D			4
Ela	A G	E F		4
Feyza	A A	D F	F	3
Meltem	B F G A B / B	E		5
Mavera	C B	F	B	3
Müge	B A B	A		2
Öykü	A C D A / A	F A D		4
Sinem	B / B C A B	F		4
Esma	B C			2
Nida	F		A B	3
Total	6 27 13 7 2 8 3 12 24 3 3 3	13 3 6 8 0 2 0	6 0	-

We also checked the dominance of the use of different prescriptions in the PMSTs' teaching. Results showed that 40% of the teachers referred to Prescription B, 18% to Prescription F, 16% to Prescription A, 11% to Prescription C, 8% to Prescription D, 4% to Prescription G, and 3% to Prescription E. This finding suggests that

Prescription B is used the most, whereas Prescription E is used the least. In addition, most prescriptions *selected* by teachers (%74) are one of A, B, or F.

Table 2

Distribution of the Prescriptions with respect to the Phases in the Curriculum

Prescription/Phase	Phase 1	Phase 2	Phase 3	Total
A	6	12	6	24
B	27	24	8	59
C	13	3	0	16
D	7	3	2	12
E	2	3	0	5
F	8	13	6	27
G	3	3	0	6
Total	66	61	22	149

An interesting detail is the point at which PMSTs move from one prescription to another as illustrated in Table 2 during the instruction. The PMSTs referred to prescriptions 66 times during the first phase (problems 1–3 in the given sequence), 61 times during the second phase (problems 4 and 5), and 22 times during the third phase (problem 6) of the prescribed curriculum. This finding suggests that the PMSTs especially refer to prescriptions during the first two phases of the instruction more than the third. The PMSTs mostly referred to prescriptions B and C during the first phase, whereas they referred to prescriptions A, B, and F mostly during phases 2 and 3. They also referred to different types of prescriptions in Phases 1 and 2 more than in Phase 3. Those phases in the curriculum are included in sequence with increasing difficulty for students. Therefore, the correspondence between moving from one phase to another and the change of prescription is interesting. Such correspondence suggests that different curricular demands challenge participants' ways of teaching, or choices of prescriptions, and create a need for them to question their teaching style.

Major Results and Discussion

This study suggests that mathematics teaching is affected not only by the mathematical knowledge of the teacher but also by these prescriptions, which are hard to break and ingrained in teachers, thereby influencing and giving direction to their teaching. Our focus is not specifically on the knowledge of teachers but on their teaching mode, the prescription, which they select and operate with. During their teaching, once a prescription has been selected, it takes over the control and drives the teaching. These prescriptions are described briefly and compared. Note that we do not claim that this list of prescriptions is exhaustive.

In *dragging prescription*, students are dragged to the correct answer through different curriculum sections without having the knowledge of the meaning and purpose of those sections. The teacher's role here is to drag students' thinking toward a predetermined goal through leading questions, whereas the student's role is to

follow the lead. In *molder prescription*, the teacher helps the student pass through a predetermined activity sequence that leads to empirical abstraction, with the thinking that "if the student learns the topic empirically, then he/she actually learns it." The main difference between the dragging mode and the molder mode is that the student progresses through leading questions in the former mode, whereas students make an empirical abstraction through a certain activity sequence determined by the teacher in the latter mode. In *self-centered mode*, the teacher thinks that students cannot progress unless the teacher helps; no such thinking exists in the previous two modes. The teacher who uses this mode does not trust the student, does not give him or her opportunities to think and reason, and performs self-centered teaching; the student blindly follows the teacher's lead and is not given any chance to develop ideas. The teacher who uses this prescription plays both the planner and actor roles, whereas the student unpurposefully accompanies the teacher and hands over all the authority to him/her. In this sense, the purpose of self-centered mode is to have the student parrot what the teacher says without any conceptual or procedural progression. To sum up, progression means helping students find answers to given problems through leading questions for a teacher who uses dragging prescription, helping students to empirically abstract a teacher-defined activity sequence for a teacher who uses molder prescription, and having students solve given problems through parroting the teacher for a teacher who uses self-centered prescription. The use of *repetitive teaching prescription* leads students to a previous point in the instruction (or to the very beginning of the instruction) and to ask the same questions repeatedly with the hope that it may help the student make progress and learn the targeted idea. Other three prescriptions are based on the idea of ignorance. *Language-ignorant prescription* ignores either the content/quality of the mathematical language that teachers use or of the influence of that language on students at some stage(s) of their instruction. *Curriculum-ignorant prescription* ignores the hierarchical structure of the prescribed curriculum by considering each section in the curriculum independently as a threshold without thinking about the big picture targeted by the curriculum. *Student-ignorant prescription* ignores the student by either not paying enough attention to his/her prior knowledge or not making judgments about student knowledge unless the student gets stuck or makes a mistake. What makes this prescription different from self-centered prescription is that the teachers who use it ignore students' prior knowledge and do not make a move until students get stuck. By contrast, self-centered prescription completely ignores the student and makes him or her parrot the teacher from the beginning to the end of the instruction without any thinking or learning on the students' part.

Previous research (e.g., Ball et al., 2008) emphasized the effect of teacher knowledge on student learning. Other studies (e.g., Boulton-Lewis, Smith, McCrindle, Burnett, & Campbell, 2001) suggested that teachers' conceptions of teaching and their conceptions of learning do not always match. Our study suggests that these prescriptions hinder

effective teaching. Even though we strengthen teachers' knowledge levels, we may not be able to help them teach effectively even with a prescribed curriculum because these prescriptions are major obstacles in the transformation process. In other words, these prescriptions may prevent one from shifting from being a "doer" to being a "teacher" because these prescriptions get in the way, take over control, and drive the teaching of PMSTs. None of these prescriptions was highlighted during the methods course or in any of the courses taken by the PMSTs. Where do they come from then? As Ball (1988) pointed out,

> … prospective teachers do not arrive at formal teacher education "empty headed"; instead they bring with them a host of ideas and ways of thinking and feeling related to math and the teaching of math, drawn largely from their personal experiences of schooling (p. 40).

This situation can be likened to that of a smoker who is unable to clear his/her lungs even after several years without smoking. Prospective teachers have been exposed to these different prescriptions (similar to smoking) for so long during their schooling that they cannot eliminate the side effects right away. Even a well-designed curriculum aligned with constructivist principles, experience with a methods course, and all coursework in their programs are not enough to ensure effective teaching because of the corrosion (in this context, prescriptions) ingrained in these teachers.

Ball (1988, p. 40) suggests and we concur that

> Why teachers, in spite of courses and workshops, are most likely to teach math just as they were taught. Mathematics teacher educators must find ways to address this conservative cycle […]. Changes in requirements or improvements in curriculum alone are unlikely to alter this pattern alone.

In our study, we found that having PMSTs operate with a well-designed curriculum does not improve their teaching unless they resist selecting the aforesaid prescriptions and operate with them.

Interestingly, most of the PMSTs (74%) referred to dragging, molder, or curriculum-ignorant prescriptions. Their choices may suggest that their past experiences and the teaching they were exposed to throughout their schooling are so dominant with respect to one or more of dragging, molder, or curriculum-ignorant teaching modes. In addition, we found that the PMSTs selected one prescription over another as they moved from one phase of the curriculum to another. The correspondence between moving from one phase to another and the change of prescription suggests that different curricular demands challenged the participants' ways of teaching and created a need for them to question their teaching style. Therefore, they moved to a different prescription to handle a different phase.

In addition, the PMSTs did not select a single or consistent prescription systematically throughout their teaching but changed the prescription they use or make different *selections* as needed. This approach is similar to that of a patient who is seeing different doctors for the same illness, getting prescriptions from each doctor, and using all the prescribed medications in hopes of getting well. In the same manner, the participating PMSTs *select* and refer to different prescriptions and apply a combination of them.

Previous research efforts (Hacıömeroğlu, 2012; Wilson, 1994) emphasize the difficulty of changing prospective teachers' limited views of teaching and mathematics despite their exposure to contemporary teaching techniques. In the current study, we find that the reason for resistance to change on the part of prospective teachers is dependence on these prescriptions. All these teaching prescriptions are selected during teaching not because they fully match the PMSTs' experiences in the methods course or their coursework in the teacher education program. They seem to be selected because of the teachers' beliefs about teaching and learning in accordance with their prior schooling experiences. Unless such experiential world is strongly shaken, a noticeable improvement in their practice cannot be expected. These prescriptions are obstacles for prospective teachers during the transformation part of Shulman's cycle. Thus, a crucial task is to make prospective teachers aware of the negative effect of such teaching modes on students and their teaching by providing them with rich related experiences. The relevant literature also suggests that teachers' beliefs and what they know affect their practice (Thompson, 1992) and that beliefs and practice develop together (Cobb et al., 1990). Therefore, treatment of teaching beliefs that are established through accumulated past experiences should be part of the equation and taken as seriously as teacher knowledge during teacher education.

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Appendix 1 – Equivalent Fractions Lesson used in PMST's Teaching

1. a. Draw a rectangle and show $\frac{1}{2}$ of it.

b. Then partition each section of this rectangle in such a way that it consists of $\frac{1}{6}$ s and determine what would be (?) in $\frac{1}{2} = \frac{?}{6}$.

2. a. Draw a rectangle and show $\frac{2}{3}$ of it.

b. Then partition each section of this rectangle in such a way that it consists of $\frac{1}{12}$ s and determine what would be (?) in $\frac{2}{3} = \frac{?}{12}$.

3. Find the answers to the following questions using diagrams only.

a. $\frac{3}{4} = \frac{?}{8}$ **b.** $\frac{4}{5} = \frac{?}{15}$ **c.** $\frac{1}{4} = \frac{?}{20}$

4. You will see that drawing becomes harder as we increase the numbers. Do not draw diagrams for the following questions. Instead, find what (?) would be by thinking as if you solve the problems through use of diagrams.

	Steps – What would you do?	Why would you do it?	What do you get as a result?
a. $\frac{2}{9} = \frac{?}{90}$			

	Steps – What would you do?	Why would you do it?	What do you get as a result?
b. $\frac{7}{9} = \frac{?}{72}$			

5. Use a scientific calculator to find the answers to the following questions. Write down each step and its result. Explain how each step is related to the diagram drawing.

	What did you do in the calculator?	Why?	How is this step related to diagram drawing
a. $\frac{16}{49} = \frac{?}{147}$			

	What did you do in the calculator?	Why?	How is this step related to diagram drawing
b. $\frac{13}{36} = \frac{?}{324}$			

6. Find the number corresponding to "?" with the help of a calculator.

a. $\frac{9}{72} = \frac{81}{?}$ **b.** $\frac{?}{54} = \frac{78}{702}$
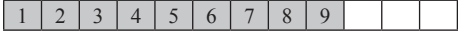
Appendix 2 – Learning Mechanism Driven by Equivalent Fractions Lesson and The Pedagogical Principles than went into the Lesson Design

In this lesson the PMST needed to teach students the logico-mathematical knowledge about fraction equivalence (*understanding that equivalent fractions represent same quantity that is independent of partitioning since they both refer to the same multiplicative relation between numerator and denominator*) instead of physical knowledge of it (*understanding that both the numerator and the denominator is multiplied by the same number*) by keeping in mind the assimilation principle (*one can only notice or learn what one already knows – new knowledge cannot be passively digested*). This lesson is designed using Hypothetical Learning Trajectory model of Simon (1995) and is based on the idea that students learn by reflecting on their own goal-directed activities (Simon, Tzur, Kinzel, & Heinz, 2004; Simon, 2014) through reflective abstraction (Piaget, 2001). The details of this design are given below.

All the questions in the given lesson ask for a common purpose: to get the unknown fraction (e.g., finding "?/4" as "2/4") by partitioning the fully given fraction (e.g., 1/2) with respect to the multiplicative relationship between the given two denominators (e.g., 4÷2=2). Shortly, the purpose is to get an equivalent fraction through further partitioning a given fraction. To reach this purpose in each problem, a student needs to go through the following physical or mental activity sequence (Simon et al., 2004) outlined in Table 3.

Table 3

A Sample Activity Sequence for Solving $\dfrac{3}{4} = \dfrac{?}{12}$.

Activity Sequence	Figures drawn as a result of the followed activity
Activity #1. Identifying the first/given fraction through drawing,	
Activity #2. Multiplicatively comparing the given two denominators/numerators,	e.g., (Denom. #2) ÷ (Denom. #1) = 12 ÷ 4 = 3
Activity #3. Repartitioning each part in the first/given fraction based on the result of the comparison in Activity #2,	
Activity #4. Counting the number of shaded parts,	1 2 3 4 5 6 7 8 9
Activity #5. Determining the missing-factor-fraction based on the result of Activity #4.	$\dfrac{9}{12} \rightarrow ? = 9$

Note that students need to go through this activity sequence (consciously or unconsciously) since they are to solve the problems based on diagram work. This is because the use of diagrams limits students' activities.

For a student to go through the above activity sequence and solve the given problems in the lesson, s/he needs to have the following prior knowledge: Prior Knowledge #1. Knowing the meaning and representation of fractions; Prior Knowledge #2. Being able to multiplicatively compare two natural numbers; Prior Knowledge #3. Basic knowledge of counting and arithmetic operations on natural numbers. Note that this list of prior knowledge is sufficient to handle the above activity sequence. Prior Knowledge #1 is sufficient to handle Activities #1, #3 and #5, Prior Knowledge #2 is sufficient to handle Activity #2 and Prior Knowledge #3 is sufficient to handle Activity #4.

During the lesson the student goes through these (physical or mental) activities using his or her prior knowledge to reach a common goal for each given problem. The first two problems in a sense help the student to go through these activities without any trouble. There is no direct help for the student about the third problem though he or she needs to solve the questions in Problem 3 under the monitoring of the teacher. Problem 4 is designed to help student go

through the same activity sequence mentally by thinking about what he or she would do if he or she is to draw diagrams. Problem 5 is designed to help student go through similar activities by focusing more on the multiplicative relationship involved in the given fraction equivalence questions with the help of the calculator. The calculator here is especially given to help focus on the multiplicative relations between numerator and denominator pairs. Problem 6 is designed in a way to help student reflect on these relations since the missing factor can be any one of the involved numerators or denominators.

In this sequence of questions the students can go through the first three problems without any real understanding of the fraction equivalence and focus on the diagram work and make necessary calculations to solve the problems collection of first three problems is called phase 1. If the instruction is ended at this point the student will probably make *empirical abstraction* of the followed activity sequence and be able to solve such problems within the presence of diagrams only. This is called empirical abstraction because, without any reflection over what is done so far, it is not reasonable to attribute any logico-mathematical knowledge to the student at this stage since one can only learn how to find the missing factor of the second given fraction based on fraction knowledge, counting, ratio by passing through these three problems without any probe. However, problems 4, 5 and 6 are designed to help the student reflect on the multiplicative relationship between numerators and denominators respectively and make a *reflective abstraction* through coordinating these relationships with the diagram work. Problems 4 and 5 together are considered to be phase 2 whereas problem 6 is considered to be phase 3.

All of the above analysis is shared with PMSTs during the methods course. What is also shared with PMSTs is the following Piagetian principles highlighted in different sources (e.g., Gallagher & Reid, 1981; Piaget, 2001; von Glasersfeld, 1995). Learning is an internal process and people construct and (re)design their experiential world through reflecting over their own actions not through direct transmission of knowledge. In addition, students need to be at the necessary developmental stage in order to be able to learn new concepts. Experience is important but not sufficient for learning in this regard. So the students need to have the necessary prior knowledge and skills in order to understand what is being taught. In addition, students organize their knowledge and actions at a higher level and this is only done through reflection over what has been done. Problems 4, 5 and 6 are designed to enable such reflection and organization.