KURAM VE UYGULAMADA EĞİTİM BİLİMLERİ EDUCATIONAL SCIENCES: THEORY & PRACTICE

Received: November 26, 2015
Revision received: August 9, 2016
Accepted: November 6, 2016
OnlineFirst: December 15, 2016

www.estp.com.tr
DOI 10.12738/estp.2017.1.0383 • February 2017 • 17(1) • 265–292

Research Article

Examining Social and Sociomathematical Norms in Different Classroom Microcultures: Mathematics Teacher Education Perspective

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Abstract

Each classroom has its own microculture with its own norms that belong to this microculture. It is these norms that characterize every kind of activity and discussion in the classroom. What makes a mathematics classroom different from any other classroom is the nature of norms, rather than their existence or absence. This study aims to identify the social and sociomathematical norms that belong to different mathematics learning environments within this framework as a multiple-case study based on the qualitative design. The data has been collected through observations of two different classrooms in a mathematics teacher education program at a state university in Turkey. The constant comparative method was used for data analysis. This study, with prospective teachers as participants, identifies the social and sociomathematical norms that regulate the classroom microcultures. The findings show how norms with different qualities can be established and sustained in two different courses within the same teacher training program, and their possible effects on learning and teaching are discussed in the context of teacher education.

Keywords

Social Norms • Sociomathematical norms • Classroom microculture • Teacher education • Mathematics education

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Citation: Güven, N. D., & Dede, Y. (2017). Examining social and sociomathematical norms in different classroom microcultures: Mathematics teacher education perspective. *Educational Sciences: Theory & Practice*, *17*, 265–292. http://dx.doi.org/10.12738/estp.2017.1.0383

Doing mathematics is not only an individual construction activity but also a social one (Bowers, Cobb, & McClain, 1999; Hershkowitz & Schwarz, 1999). Highly complex human interactions occur in mathematics classrooms. Furthermore, the process of teaching and learning mathematics involves a kind of collective and interactive relation (Bauersfeld, 1980). Investigating how math is learned and taught from a sociological perspective by generally analyzing classroom culture in general and mathematical culture in particular, scholars have drawn on some concepts such as classroom microculture and mathematical classroom traditions (i.e., Cobb, 1999; Cobb, Stephan, McClain, & Gravemeijer, 2001; Cobb, Wood, Yackel, & McNeal, 1992). Cultural constitution, which appears in a small group and makes more interactions possible among participants, can be defined as the system of knowledge, belief, behavior, and tradition shared by the members of a group (Fine, 1987, as cited in Fine, 2003). Like every community, each classroom establishes, sustains, modifies or eliminates various patterns such as norms, standards, obligations, rules, and routines (Sekiguchi, 2005). This process is called *culture building* (Fine, 2003). From this perspective, this study focuses on the social and sociomathematical norms embedded in the culture building process.

A norm is an important element of classroom microculture that is established by the teacher and students (Cobb, 1999). Norms can be defined as "ideas that determine manners; what is expected to be done by a group member, or a person under prescribed conditions" (Homans, 1951, p. 123). Similarly, Cobb et al. (1992) and Cobb and Yackel (1996a) use the concept of norm in the meaning of specifying and meeting the mutual expectations that arise in the classroom through the interaction between teacher and students. Norms characterize regularities in individual or collective classroom activities (Cobb et al., 2001). Norms are established and developed through constant student-teacher interactions and thus may differ significantly from one classroom to another (Cobb & Yackel, 1996b). This study, with prospective teachers as participants, thus aims to investigate which social and sociomathematical norms exist in different classroom microcultures.

Theoretical Framework

Classroom Microculture

A classroom is defined as a complex environment that accommodates individuals who come together with the aim of constructing a learning community (Levenson, Tirosh, & Tsamir, 2009). Like every community, a classroom constitutes and develops an association of social relations and its own microculture (Gallego, Cole, & The Laboratory of Comparative Human Cognition, 2001; Lopez & Allal, 2007). The microculture of a mathematics classroom contains social interactions and the

construction of mathematical meaning (Voigt, 1995). It does not exist separate from the mathematical activities of a classroom community (Cobb et al., 1992). Its characteristics depend on norms, patterns, and regulations that are difficult to change, such as students' attitudes (Voigt, 1995). Social and sociomathematical norms, together with a classroom's mathematical practices, constitute the classroom microculture where individual and collective mathematical learning occurs (Cobb et al., 2001).

Social and Sociomathematical Norms

Cobb and Yackel (1996a), who extended their studies from general classroom norms to the normative aspects of mathematical arguments regarding student activities, distinguished norms as social and sociomathematical. Social norms express the social-interaction aspects of a classroom that become normative (Yackel, Rasmussen, & King, 2000). These norms are common norms that can be enacted in any field (Cobb & Yackel, 1996b). For example, explaining and justifying solutions, identifying and stating agreement, trying to make sense of others' explanations, expressing disagreement on ideas, and so forth are social norms for discussions where the whole class participates (Cobb & Yackel, 1996a).

On the other hand, sociomathematical norms state normative understandings related to mathematical reality (Yackel et al., 2000). Although sociomathematical norms pertain to mathematical activities, they are different from mathematical content. They deal with the evaluation criteria of mathematical activities and discourses unrelated to any particular mathematical idea (Cobb et al., 2001). Normative understandings regarding things in classrooms that are mathematically different, complex, efficient, and elegant are sociomathematical norms. In addition, things that are accepted as a mathematical explanation and justification or regarded as a mathematically different, complex, or efficient mathematical solution are considered to be sociomathematical norms (Cobb, 1999; Cobb & Yackel, 1996a, 1996b; Yackel et al., 2000). Besides, sociomathematical norms are not obligations or regulations for student to meet (Voigt, 1995); they are established through interactions such as social norms (Yackel et al., 2000). As long as students participate in establishing sociomathematical norms, they develop mathematical beliefs and values that enable them to act as an autonomous member of the classroom community (Bowers et al., 1999; Cobb & Yackel, 1996b). Sociomathematical norms involve ways of making decisions, and they enable the classroom community to talk about and analyze the mathematical aspects of activities in math classes. For example, the sociomathematical norm *What provides mathematical difference?* supports a high level of cognitive activity (Cobb & Yackel, 1996b). Although these norms include normative understandings exclusive to mathematics, they transcend

mathematical content by dealing with the similarities, differences, complexities, effectiveness, and mathematical quality of solutions. Accordingly, constructing sociomathematical norms is pragmatically important and provides the basis for a classroom's mathematical microculture (Cobb & Yackel, 1996b).

Observing and Determining Norms

Both social and sociomathematical norms are methodologically identified by determining regularities in the patterns of social interactions (Cobb & Yackel, 1996b). Analyses that focus on the social norms of a classroom generally provide a portrayal of the participation structure within a classroom (Lampert, 1990). Analyzing four components of a mathematical activity (problems, solutions, explanations, and justifications) provides an empirically grounded way to describe and characterize mathematics classroom microculture (Cobb et al., 1992). Briefly, in order to determine norms, one must reveal implicit and explicit regularities in the patterns of social interactions by observing the class-participation structure apart from problems, solutions, explanations, and justifications during classroom discourse.

In order to consider discourses, behaviors, and thoughts as norms in classroom microculture, one must consider how they are enacted and how individuals participate. To do this, looking for explicitly expressed norms in discourses is not an indispensable prerequisite (Sánchez & García, 2014; Sekiguchi, 2005). For example, students who are satisfied with a teacher answering "Because it's a rule!" when they ask why yields the sociomathematical norm for what is accepted as a mathematical justification in the classroom (Yackel et al., 2000). On the other hand, a norm can also be identified by a teacher's explicit statement. For example, the sentence "We study collaboratively in this classroom and everybody must help each other" is a clear indicator of a social norm (Gorgorio & Planas, 2005). According to Sfard (2008), who stated that widely approved and enacted meta-rules can be interpreted as norms that facilitates discourse in a classroom community, a norm must be enacted and supported by the majority of the classroom community. Moreover, almost everyone in the community must approve it (Sfard, 2008). Additionally, observing the action in at least three different class sessions is enough to understand its repetitive nature (Park, 2015).

Considering this view, cases of dissonance with a conjectured norm must be noted, and whether or not the classroom community finds these cases to be acceptable needs to be analyzed while developing assumptions about norms. If the case of dissonance is acceptable, conjecture about establishing norms must be reviewed once again; if it is unacceptable, then this case must be treated as new evidence for the conjectured norm (Cobb et al., 2001).

The Purpose and Significance of the Study

From the most traditional to the most reformist, every classroom in general has its own social norms, and every mathematics classroom in particular also has its own social and sociomathematical norms. What makes one mathematics classroom different from another is the nature of its norms, not their existence or absence (Yackel et al., 2000). Moreover, one can suggest that because norms document the regularities in classroom activities as performed by the teacher and students (Cobb et al., 2001), the quality of norms influences the quality of individual or collective teaching activities in general, as well as the quality of mathematical activities in particular. Thus, the quality of norms becomes important in making classroom microculture appropriate for effective learning.

On the other hand, teachers are central in establishing norms (Bishop, 1985; Cobb et al., 2001), and teachers' ability to understand the importance and effects of norms on teaching and learning is the first step in establishing norms in class (Van Zoest, Stockero, & Taylor, 2012). It would be unrealistic to expect teachers to establish or develop a behavior or phenomenon with which they've no experience or understanding (McNeal & Simon, 2000). Teachers construct their professional knowledge by bringing their experiences to the teaching and learning environments (Tsai, 2007). The norms that prospective teachers acquire and internalize are persistent when they start their profession. Thus, establishing productive norms in teacher education can be stated as an investment, and this investment on one level can support further learning in subsequent levels (Van Zoest et al., 2012). Accordingly, the norms and microcultures that are established and sustained in prospective teachers' classrooms become important for future attempts at establishing productive classroom microcultures in their profession. From these perspectives, this study identifies the social and sociomathematical norms that have been established and sustained in two different classrooms of the same prospective teachers in a teacher education faculty. The study investigates a mathematics education course ("Methods of Teaching Mathematics II") and a mathematics content course ("Numerical Methods and Discrete Mathematics"), two of the three main types of courses in the national teacher education syllabus of Turkey.

Additionally, one cannot adequately explain teachers' developmental process without analyzing the pedagogical communities they've participated in (Cobb & McClain, 2001). By using an interpretative framework for analyzing individual and collective mathematical learning in a mathematics classroom, Cobb et al. (2001) considered social and sociomathematical norms in order to analyze classroom microculture from a social perspective. Thus, by investigating classroom norms, the current study investigates the collective mathematical learning in a teacher education faculty. Discussing the norms of two different courses and their possible outcomes

over prospective teachers, this study provides insight into the quality of teaching and learning in a teacher education faculty within the framework of norms.

The importance of establishing norms for different class levels and environments has been extensively reported in the literature like primary schools (Cobb & Yackel, 1996b; Lopez & Allal, 2007), elementary schools (Levenson, Tirosh, & Tsamir, 2006, 2009; Sekiguchi, 2005), high schools (Hershkowitz & Schwarz, 1999), upper secondary schools (Partanen & Kaasıla, 2015), universities (Stylianou & Blanton 2002; Yackel et al., 2000), teacher education (Dixon, Andreasen, & Stephan, 2009; McNeal & Simon, 2000; Sánchez & García, 2014; Van Zoest et al., 2012), and professional development (Clark, Moore, & Carlson, 2008; Elliott et al., 2009; Tsai, 2004, 2007). Considering the central role of the teacher in establishing and sustaining norms (Bishop, 1985; Cobb et al., 2001), limited research has focused on these aspects in teacher education.

Furthermore, specific activities have generally been the focus of previous studies. For example, social and/or sociomathematical norms have been investigated in mathematical activities focusing on problem solving (Lopez & Allal, 2007; Tatsis & Koleza, 2008), explaining (Elliott et al., 2009; Levenson et al., 2006), justifying (Partanen & Kaasila, 2015; Stylianou & Blanton, 2002), proving (Connelly, 2012) and defining (Sánchez & García, 2014). All class activities have been focused on in the current study.

Also, the intentional establishment and negotiation processes of productive norms for teaching and learning mathematics have been previously investigated (e.g., Dixon et al., 2009; Tatsis & Koleza, 2008; Van Zoest & Stockero, 2012). Portraying the current condition is the major concern of this research. Being in a classroom with the aim of negotiating norms may inherently cause changes in them and the observed microculture (Partanen & Kaasıla, 2015).

Accordingly, this study provides detailed insight into the current condition of teacher education programs by identifying which norms prospective teachers have in their undergraduate education. In this context, the study attempts to address the following questions:

1. What are the social norms in an undergraduate mathematics content course classroom and in mathematics education course classrooms?

2. What are the sociomathematical norms in an undergraduate mathematics content course classroom and in mathematics education course classrooms?

Method

Research Design

This study identifies the social and sociomathematical norms of an undergraduate mathematics content course classroom and mathematics education course classrooms. It is a multiple-case study based on qualitative design (Yin, 2003) and portrays the existing conditions of different classroom microcultures in terms of the social and sociomathematical norms over which researchers have little control in a real life context. Each microculture of a classroom community is considered as a case to be studied. The social and sociomathematical norms of microcultures are the units of analysis.

The Study Group

The participants of the study were two faculty members and 54 students of a secondary mathematics education department from a state university in Turkey. The study aims to investigate social norms as well as sociomathematical norms. Social norms can be researched in any classroom. However, because sociomathematical norms pertain to mathematical activities, they can only be researched in classrooms that conduct mathematical activities (Cobb & Yackel, 1996b). Therefore, the courses must include mathematical activities, discussions, and discourses. To this end, maximum variation sampling, a purposeful sampling method, was used to choose two different courses, a mathematical content course and mathematics education course. These are two of the three main course types from the national teacher education syllabus, educational sciences courses being the third.

Additionally, observing the differences and similarities that occur in classroom microcultures is intended for classes in which the same prospective teachers are participating. Accordingly, the students participating in the content course and mathematics education course should also be the same. "The Methods of Teaching Mathematics II" and "Numerical Methods and Discrete Mathematics" courses satisfy these criteria. The former is one of the main required mathematics education courses in the national teacher education syllabus, and the latter is the only content course attended by students who were taking the other course in the same semester. Indeed, the aim of "Methods of Teaching Mathematics II" is basically how to teach and learn mathematics. However, when the classroom community discusses teaching a mathematical concept, the instructor starts asking questions about mathematical aspects and often tests the mathematical content knowledge of prospective teachers. Hence, the classroom community involve in doing mathematics and talk about doing it in almost every class session. The researchers are familiar with the classroom microcultures from their prior observations and participation in the same course

conducted by the same instructor. The other instructor was included in the study only because he is the sole instructor of the content course, as each course was taught by only one instructor from this department. Both instructors had more than 20 years of professional experience. They studied mathematics in their postgraduate education. In addition, the instructor of the mathematics education course has been studying mathematics education professionally for 20 years and conducting this course for over 10 years. Prospective students were also included in the study because they were the only students who were participating in each course. The students were in their fourth year of a five-year-education program and had already completed most of their mathematical content, mathematics education, and educational science courses such as "Multivariable Calculus," "Algebra," "Measurement and Assessment," "Methods of Teaching Mathematics I," "Instructional Technology and Material Development," and "Developmental Psychology."

Lesson Contents and Teaching Arrangements

The students were divided into two classrooms in the mathematics education course because of a high number of students, as well as lesson content and teaching arrangement considerations. In the content course, students were taught in one class. The content course classroom is hereafter referred to as A1; mathematics education course classrooms are hereafter referred to as AE1 and AE2. There are no differences between AE1 and AE2 except the students and class schedule.

There were 26 and 28 students in AE1 and AE2, respectively, and the students studied in groups of three to four (groups were generally single-sex and formed by students). They sat around roundtables with their groups in the mathematics laboratory equipped with a variety of mathematical materials and instruments. The aim of the mathematics education course is to enable students to acquire the main skills needed for teaching mathematics. Students in this course focus on the activities of exploring and evaluating mathematics curriculum and analyzing concepts that children have difficulty understanding (considering related literature); they also explore the elements of an effective lesson plan and discuss how to teach a mathematical concept or relation. The classroom community was also involved in mathematical discussions while talking about teaching a mathematical concept or relation. Courses were conducted as two 40-minute sessions per week. During this course, students are responsible for designing a lesson for a learning outcome in mathematics curriculum, presenting it to their peers, and evaluating each other's studies. In a typical lesson, a group of students present their lesson design for about 20 minutes. Then, the instructor starts a discussion requiring the other students to evaluate their peers' presentations and share their ideas. Two groups of students present their lesson designs each week. The content course class included 54 students, and the physical conditions

of the classroom were standard. This course is rather teacher-centered, compared to the mathematics education course. Within the scope of this course, topics such as graphs, complements, Euler trails, and circuits were taught. In a typical lesson, the instructor presents the concepts directly; there is no routine and systematic classroom discussion or activity. Students listen to the lesson and take notes passively; they assume an active role in the lesson if or when they want. Both courses are mandatory.

Data Collection and Procedure

The data collection methods are the non-participant in-class observations and field notes taken by the researchers during and after the lessons. The data collection process was completed in less than two months. Important when collecting data was a focus on the individual and collective mathematical activities; planning, implementing, and evaluating processes pertaining to the activities; the patterns of behaviors; and the interaction methods of students and teachers (Cobb et al., 2001).

The A1 instructor did not allow video recording and opted for only audio recording. Audio and video recordings ranged from seven to eight hours. Similar time periods have been reported in the literature (i.e., Lopez & Allal, 2007). Each class observation was conducted simultaneously for six and seven weeks. Two sections for each classroom were observed in a week. Thus, 12 different class sessions were observed on average. This is enough to observe an action at least three different times (Park, 2015). The time differences among recordings resulted from the researchers' decision about whether they had reached theoretical saturation or not (Arber, 1993). Structured observation protocol was not used for field notes. The actions and discourses that could be considered a norm or an indicator of a norm were field-noted during the observations and compared with the recording transcripts during data analysis.

Data Analysis

The methodological recommendations of Bowers et al. (1999), Cobb et al. (2001), and Lopez and Allal (2007) on identifying norms were considered in the data analysis (i.e., in the section Observing and Determining Norms). The constant comparative data analysis method was used to reveal the qualitative differences of classroom microcultures. A microanalysis producing categories and offering relationships between categories was conducted, and some of the processes of constant comparative analysis such as open, axial, and selective coding were conducted. Thus, a detailed, line-by-line analysis was provided by means of separating data (Strauss & Corbin, 1998). Each video and audio recording from the three classes were transcribed in chronological order. Then the transcripts and field notes from each classroom were analyzed separately for interaction patterns by considering some points given in a sample analysis:

Actions and discourses that lead to establishing new norms or that indicate an established norm. Two students presented their lesson plan on logarithm functions to their classmates in the first presentation in AE1. After this, the presentation was evaluated by their peers through class discussions. An example of a conversation that took place between the teacher (T) and students (B#) follows. In the excerpt below, B3 wants to comment first on her peer's representation. Before B3 starts talking, the instructor warns the classroom community to listen to each other carefully:

> T: Friends! [addressing the class] … Say it [your comment] loudly [addressing B3], and you [addressing the class] … While you are listening, maybe you will realize things that your colleague has not asserted yet… in this respect… yes [indicating to B3 she could start] …
>
> B3: I like this activity very much. […] They didn't give the logarithm function *directly*; they gave it as the reverse of the exponential function. You know… they gave *the steps* very well. I mean they showed the equality.

Considering the natural developmental sequences of the logarithm function, prospective teachers based their lesson design for teaching logarithmic functions on secondary-school students' possible prior knowledge of exponential functions. They designed tasks to remind and organize children's prior knowledge on exponential functions. The subsequent tasks that were designed or selected were based on first investigating the reverse of the exponential function and then noticing the equality between the exponential function and logarithmic function by referring to their graphs. As the first speaker, B3 noted this point, emphasized the word "step" and agreed with the idea of constructing concepts based on children's prior knowledge of exponential functions rather than introducing the concept "directly". The developmental process of logarithms historically took place as the reverse of exponential functions. In this context, following the hierarchy in learning and teaching mathematics could be expected or considered for this community (open coding). Now, more discourses and actions proving that this is a common expectation or consideration of the classroom community must be researched in the data. It is also possible that the data may show this is not a common expectation or consideration for this classroom community.

Actions and discourses proving or disproving the existence of conjectured norms. After a while, a similar emphasis was made by another student in the same class session:

> B1: Sir, there are tables. Their [exponential and logarithmic functions'] graphs were drawn previously. First for the exponential [function] then for the logarithmic function... The values of these functions were identified in the tables and then their graphs were drawn before…

B1 remarked on and agreed with the idea of reaching the logarithm concept from the investigated graph through tables by having children who know the exponential function draw its graph and investigate the graphs of an exponential function and its reverse. This is another indicator of the same expectation.

Additionally, the same considerations were seen in other students' lesson plans and evaluations in other class sessions. For example:

> B5: Students must relate rational numbers to real numbers. We tried to relate the properties of real numbers to rational numbers in our lesson plan.
>
> Or,
>
> B10: […] when I teach integral, I say if you know derivative well, you won't have any problem with integral. I think this lesson plan is exactly like this and is very good, I mean… It is constructed on students' prior knowledge.

In these statements, there is an emphasis on relating a concept first to a familiar concept in the discipline to make it more understandable and then on readiness level for learning mathematics (open coding). Basically, the current use of relating a discipline, considering the readiness level for learning mathematics or prior knowledge, and following the hierarchy in learning and teaching mathematics originates from considering the cumulative nature of mathematics. Accordingly, the norm *The cumulative nature of mathematics should be considered in this classroom* can be conjectured. This conjectured norm is also an axial coding because it can be accepted as a category that presents a clearer explanation for the case and involves some subcategories obtained from open coding (Pitney & Parker, 2002). Additionally, negative reactions to conforming to, violating, or ignoring a norm must be researched in the data. No precedence had been given in the last two analytical phases.

Negative reactions to conforming to, violating, or ignoring a conjectured norm. In this phase of analysis, controlling reactions helped understand whether or not this behavior was accepted by the classroom community. Negative reactions to conforming to conjectured norms identified by researchers were not seen in the data.[3] However, negative reactions were only seen when ignoring a conjectured norm. For example, the norm *Providing one or two examples is not accepted as sufficient for mathematical abstraction* was widely enacted in the classroom. In case this norm had not been considered in one of the lesson plans, students reacted immediately:

> B9: Is one example enough for abstraction? You tried to abstract using just one example!

These reactions were accepted as another indicator of the norm. After the classrooms' norms were identified by axial coding, they were compared with themselves through selective coding. In this way, when norms emphasizing the same actions or ideas were seen in findings, they were expressed using a more inclusive one. For example, when the conjectured norm *Students should explain their way of*

3 Consequently, some of the conjectured norms were not accepted as a classroom norm not because of negative reactions but because of insufficient enactment and participation from the classroom community.

thinking was examined with its representative excerpts, it was seen to be fundamental to the norm *An idea should be justified and explained by its justifications*.

Study Reliability

Preserving the nature of classroom microculture during data collection is important for the study. Both teachers and students were accustomed to the same researcher's participation in their classroom from their prior experiences. The instructor of the AE classrooms videotaped lessons the semester before this study with students' permission. Thus, they were accustomed to being videotaped. Audio recording is also thought to not threaten the natural microculture of A1. Teachers and students' words were transcribed verbatim. The transcripts and field notes were examined a few times to categorize the data and determine common statements.

In A1, there was apparently far less communication or student talk because of the instruction, and gleaning insight into the norms was thus more difficult in A1 compared to both AE1 and AE2. This situation was therefore considered while discussing the classrooms' norms.

At the beginning of data analysis, a part of the conducted analysis was submitted to an expert who was about to complete her doctoral dissertation on discourse analysis. After completing the data analysis, expert opinions were sought again from two research assistants who were also PhD candidates. One of the experts wrote a master's thesis investigating the reflections of teachers' mathematical values on their classroom practices. When one regards that values involve some indication of norms (Ernest, 2009), one can suggest norms and values are reflexively related (Cobb & Yackel, 1996b). Furthermore, the experts had attended these courses and classrooms for more than a semester. Thus, they know quite well the lesson content, classroom environment, and community. They were asked specifically to examine whether the norms and their excerpts really represented and matched each other. In addition, they were asked to identify the existence of repeated norms and to check the norm classification and statement forms.

In light of the expert opinions, similar or overlapped norms were determined, and the statements of some norms were corrected. For example, the conjectured norm in A1 (*The most effective mathematical solution is the most easily obtained one*) was revised to *The most effective mathematical solution is the shortest one*, in line with the expert opinions. Thus, peer review as an external control mechanism (Lincoln & Guba, 1985) was practiced accordingly for the reliability of research data. As a result of examining all the data and categories, the agreement-correlation coefficients between the researchers and experts were calculated to be 0.94 and 0.85, respectively.

Results and Discussion

In this section, the identified social and sociomathematical norms of AE1, AE2, and A1 are presented and compared by discussing the possible reasons for similarities and differences among the classrooms.

Social Norms in Classroom Microcultures

Although there were different conjectured norms for both AE1 and AE2, the classroom communities accepted and sustained the same social norms except for one of AE2's. The identified social norms can be seen in Table 1.

Table 1
Social Norms in Classroom Microcultures

Classroom	Social Norms
AE1 & AE2	An idea should be justified and explained by its justifications. Everybody should share their ideas in this classroom. Every effort must be appreciated in this classroom. Students should apply the knowledge they have acquired in their undergraduate education in this classroom. The ideas shared in this classroom should be questioned. Language should be used carefully in this classroom.* Steps and expectations should be clear and detailed in every study. Every effort must have a clear and realistic aim, and should be explained with its justifications.
AE2	Everybody should place value on learning not the grade.
A1	An idea should be justified and explained by its justifications. Students should express their ideas in this classroom. Importance should be placed on getting a good grade rather than learning. Different solution methods should be sought for solving problems. Alternatives should be interrogated when conflicts are seen in interpretations.

* Turkish language and educational/mathematical terminology should be used correctly; statements, explanations, and comments should be spoken clearly and loudly.

As seen in Table 1, different social norms, except for the table's first one, were enacted in AE1, AE2, and A1 microcultures. The explanations about and representative excerpts of some norms are presented in Table 2 for clarification.

Table 2
Explanations and Representative Excerpts of Some Social Norms

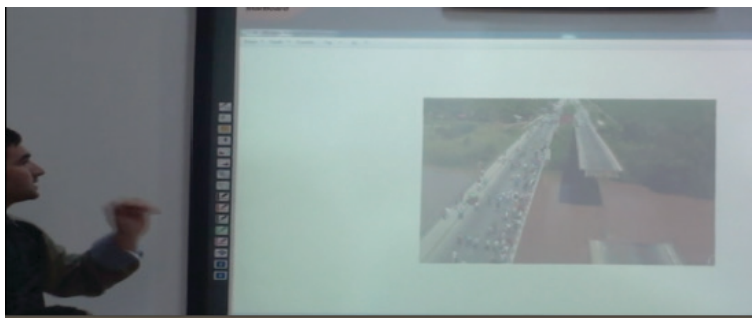
Social norms in AE1 and AE2	Explanations and representative excerpts
Language should be used carefully in this classroom	• This norm states that Turkish, as well as lesson terms, should be used correctly. B6: Now, you know, I'm trying to transfer something to children... T: No, look, let's remove [the use of the words such as] "transferring, giving, telling" in these classrooms. You are studying to construct something in children's minds. Or  *Figure 1.* An image for the continuity concept Showing an image (Figure 1), student: B7: This, for example, the most popular relationship between continuity and the bridge… whereas people can walk on one of them, they cannot walk on the other; so, it will try to explain the concept of continuity … T: Let's put it like this, one can walk on both of them but can't... maintain walking… They can maintain walking on one of them… B7: Yes. T: They can't on the other B7: Or, one can't cross over on the other one T: Yes • Statements, explanations, and comments should be said clearly and loudly. T: Don't interrupt the sentence till you reach whatever your goal is. You said "determine it" and he or she determined it and then what? Go as far as you can. Ask clearly!
Every effort must have a clear and realistic aim, and should be explained with its justifications.	Students were often in harmony with this norm in both classrooms. The examples for negative reactions to violating the norm can be given as follows: T: You used this but we don't know anything about the purpose it serves. Or T: Friends, let's repeat this part of feedback to all. You must know the justifications for what you use in your studies … I don't want ambiguous sentences. Got it? Or B9: You made us watch a video. I didn't… quite… you know, ellipse is drawn... I didn't understand well […] what is the use of that material? It made me curious.

Table 2
Explanations and Representative Excerpts of Some Social Norms

Social norms in A1	
An idea should be justified and explained by its justifications.	• Students' answers are justified only in problem-solving activities. B11: It can be a trail but cannot be circuit. T1: Why? B11: It must pass from *c* or *e* but it does not; so, it is not. • In other situations, students' explanations are not considered. The teacher asks question, then answers it without waiting for the students. Despite this, students (try to) explain their ways of thinking and ideas. Students expect the teacher to explain and justify his way of thinking and ideas in this classroom. T1: […] it does not go [continue]. Now, how many points does it have, friends? B12: 16 T1: The degree of every point… ok. B13: Sir, why does it not go? Why didn't it?
Students should express their ideas in this classroom.	This norm is not expressed like "idea sharing". Idea sharing must include the majority of the classroom, but there is no such interaction in this classroom community. Students just express their thought out loud; they may not always be able to get feedback from their ideas.
Importance should be placed on getting a good grade rather than learning.	Two exams are conducted in one semester because of the teaching arrangement. Grades were a main issue for this community. T1: Good! Drawing is important. I'll give extra points for beautiful drawings. Or T1: Is there a trail in this graph? [Students nod yes] Tell me then… B2: e k l d … I want an extra 10 points!
Different solution methods should be sought for solving problems.	B18: It would be better if he entered on that right side. T1: […] okay. You see, this, you don't have to find one and only way. My friends, by starting another point, what can you do? You can construct a trail.

There were variations among norms in the microcultures of the same students. The teacher and lesson content (teaching arrangement) is thought to cause this. For example, the norm *Students should apply the knowledge they have acquired in their undergraduate education* mostly originated from lesson content and teaching arrangements in AE1 and AE2. However, establishing other norms in A1 can also be expected.

The norms *Students should apply the knowledge they have acquired in their undergraduate education* and *Steps and expectations should be clear and detailed in every study* were not been encountered in the literature (i.e., Cobb, 1999; Cobb & Yackel, 1996b; Elliott et al., 2009; Sánchez & García, 2014) probably because classrooms with similar lesson content have not been investigated. Although Dixon et al. (2009) investigated the classroom community with lesson content similar to the current study's, they did not identify a similar norm. However, this is not surprising because they focused on the processes of intentionally establishing previously determined norms.

Although the teaching arrangements of AE1 and AE2 were based on group study, norms like *Everybody should help each other* (Gorgorio & Planas, 2005), *It's important to reach group consensus* (Hershkowitz & Schwarz, 1999), and *Learning more mathematics requires collaboration* (social norm of proving process; Connelly, 2012) were not identified in the present study. Disregarding collaborations in the current classrooms could be a reason or a result of this aspect. Despite these, the norms in AE classrooms could be accepted as productive norms for a classroom microculture to have prospective teachers improve some basic skills such as critical thinking, communication, and research- inquiry. *An idea should be justified and explained by its justifications* was identified in all classrooms. However, this norm is not thought to be such a productive norm for supporting the development of basic skills in A1. Students enacted it mostly with their active contribution; it did not have enough contribution or guidance from the teacher. This was the same for the norm *Students should express their ideas* (see Table 2).

Accordingly, one can infer that the norms of A1, with the exception of *Different solution methods should be sought for solving problems* and *Alternatives should be interrogated when conflicts are seen in the interpretations*, are not sufficient to regulate classroom activities for effective teaching and learning. On the other hand, the norms of AE1 and AE2 can be inferred as useful for prospective teachers' learning. Prospective teachers can acquire important skills for planning lessons and regulating classroom activities through the social norms *Steps and expectations should be clear and detailed in every study*, *Every effort must have a clear and realistic aim*, *and should be explained with its justifications*. They could gain useful insight about classroom management in a democratic and inclusive way through the norms *Everybody should share their ideas in this classroom* and *Every effort must be appreciated in this classroom*, contrary to their experiences in A1. While the norms "*Even inefficient attempts could contain important ideas*" (Sekiguchi, 2005, p. 156), *"Meaningful activity is valued more than to correct answers"* (Hershkowitz & Schwarz, 1999, p. 150), and *Errors are accepted as a part of learning* (Elliott et al., 2009) can be seen in the literature, they were absent in this study. Conversely, the norm about getting a good grade identified in both A1 and AE2. While A1 emphasized the importance of this norm, AE2 was guided to remove this idea. Even though these norms were established with different aims, the need for establishing such norms in classrooms could originate from the Turkish education system's design with large-scale central exams (Yıldırım, 2008) and that decisions are made over students' success or failure mostly based on the results of these exams for almost all of their educational life. Indeed, big social constitutions like the education system have an important effect on the classroom microculture with their tacit messages (Gorgorio & Planas, 2005; Sánchez & García, 2014).

Sociomathematical Norms in Classroom Microcultures

Only the norm *Mathematics should be related to everyday life* was common among the three classrooms. Seven of the same sociomathematical norms were sustained in AE1 and AE2. The identified sociomathematical norms are listed in Table 3.

Table 3
Sociomathematical Norms in Classroom Microcultures

Classroom	Sociomathematical Norms
AE1 & AE2	The cumulative nature of mathematics should be considered in this classroom. The representation (modeling, visualization, representing by graph, representing by table) is accepted to be important because it makes abstraction, generalization, conclusion, and problem-solving easier. Mathematical claims should be questioned in this classroom.* While doing mathematics, discourses** should be mathematical. Providing one or two examples is not accepted as sufficient for mathematical abstraction. A mathematically convincing claim must be justified with a proof that explains its justifications. Mathematics should be related to everyday life.
AE1	Terms and properties used in defining a concept should be known beforehand. A concept should be used by knowing its history of development. Intuitive approaches are necessary in mathematics but should not circumvent algebraic proofs.
AE2	Definitions are necessary for differentiating mathematical concepts.
A1	Providing one or two examples is accepted as sufficient for mathematical abstraction. Mathematics should be related to everyday life.

* Justification, examination, and thinking are important while doing mathematics.

** Discourses are different ways in which people unify the language with nonlinguistic things such as different ways of thinking, behaving, interacting, valuing, feeling, and believing, as well as using symbols, tools, and objects (Gee, 1999).

Explanations and representative excerpts are given in Table 4.

Table 4
Explanations and Representative Excerpts of Some Sociomathematical Norms

Classroom	Sociomathematical norms	Explanations and/or Representative Excerpts
AE1 & AE2	While doing mathematics, discourses should be mathematical.	Students must talk and act as a mathematician* when doing mathematics. Using the language of mathematics correctly is important. Furthermore, a mathematically acceptable justification must be done through the methods of mathematical proof. Representative excerpts: T: if *a* is an irrational number and *b* is a rational number, then is the sum of them a rational number? B5: *a* plus *b*. Let's look... *a* is irrational, *b* is rational… No, it can't happen. T: What if it happens? B13: If it happens, then the intersection is disjointed… T: I mean I do not… I do not … the answer "it cannot happen"…I do not accept it. […] B14: But the Closure property of addition… it [Closure property of addition] is present in rational numbers and also in irrational numbers, but both of them cannot be... B5: Sir, Closure property... T: Please, give me an answer like a mathematician. Or T: You say that "continuous points in the graph, in light of the knowledge that you learned presently." You say "points". I mean are points split in half as continuous points and discontinuous points? What you want to say is different there. B14: Sir, I get what you mean… B15: It is continuous on the graph. B16: The continuity of the function T: The continuity in the point is there, there is no such thing as a continuous point. Or B17: The motion of a variable is related to the changes in its received value. And we... T: "Received value" is not a good expression. [It should be] The number that is represented…
	Providing one or two examples is not accepted as sufficient for mathematical abstraction.	Representative excerpts: T: Your question is an abstraction question… how can we say it as an abstraction question? (Student shows the options; in fact there is just one example there.) T: If it is an abstraction -you know, we studied abstraction before- there must be a set of objects and these objects must have common properties. When students study these common properties, what needs to be done ... on the particular common property by ignoring differences? You need to have students come up with an idea, an entity. B14: Yes, first… T: Now, how can you expect an abstraction? … But there is not a set of objects, just one. I mean there are u and v. There is one example. Or B16: After *enough examples*, we want students to make an abstraction.

Table 4
Explanations and Representative Excerpts of Some Sociomathematical Norms

AE1	A mathematically convincing claim must be justified with a proof that explains its justifications.	Interpretations and ideas about mathematics must be proved and justified mathematically. To be convincing, an acceptable mathematical claim, conclusion, or thought must be based on logic and must be confirmed with an explanatory proof. Representative excerpts: T: A number such as this one is irrational, but next I want this also, … subsequent to this…, we need to show why this is an irrational number, why it can't be written in the form of a/b […] unless we show why it can't be written in the form of a/b, it cannot convince. […] You are going to bring me the proof for the reasons why *e* and π are irrational numbers. And please when you bring the proof, we should see what it's based on. I mean, why it can't be rational. What is going on, and then why it can't be rational number? This is more important.
	Terms and properties used in defining a concept should be known beforehand.	For example, while the concept of exponential function is defined, for the understandability of this concept, these all must be known: The exponential function is a function, what is a function, and the domain and range (co-domain) of an exponential function.
	Intuitive approaches are necessary in mathematics but should not circumvent algebraic proofs.	Representative Excerpt: T: Deciding intuitively is not rational because eyes can be fooled. Algebra comes into play when the eyes are not enough. And this is algebra, you know, vector algebra […] that, you know, there are the parts which we can constitute intuitively in an easy way. We should not mess around there; we should focus on these [showing algebraic proofs and algebraic explanations which are written on the whiteboard].

* Mathematical discourse is not just speech, behavior, interaction, thinking, reading and writing; it also includes mathematical values, beliefs, and perspectives (Moschkovich, 2003).

The different sociomathematical norms between AE1 and AE2 microcultures are remarkable, unlike their social norms. The reasons focus on lesson content and even more so on the role of students. Lesson content was thought to be a reason because learning outcomes, which students considered while designing a lesson, changed with every class session. Thus, the mathematical activities differed according to the content of learning outcomes. Herein, various conjectured norms with fewer repetitions were revealed, and one couldn't find any opportunity to enact the norm again.

Students were the other determinant of AE1 and AE2. The norms about making connections within the discipline and using a concept by knowing its development history were emphasized by the students, approved by the teacher, and sustained interactively in AE1. On the other hand, the norm *Intuitive approaches are necessary in mathematics but should not circumvent algebraic proofs* was established from reactions to students because of their mistakes in lesson designs. Hence, the AE1 community was more successful in negotiating sociomathematical norms.

According to Cobb and Yackel (1996b), establishing sociomathematical norms is pragmatically important because they are fundamental aspects of the classroom's mathematical microculture. However, in the literature, no study presenting criteria that specify which sociomathematical norms are important for a mathematics classroom and which qualifications these norms should meet has been encountered. Nevertheless, if current classrooms are compared in terms of their sociomathematical norms, the norms of AE1 are productive in developing classroom's mathematical microculture and regulating mathematical activities.

Generally, sociomathematical norms of the current classrooms are related to cumulative nature of mathematics, representations, questioning mathematical claims, mathematical discourse, abstraction, proofs, definitions, and relations with everyday life, as well as consideration of the developmental history of mathematical concepts. In AE1 and AE2, the norms related to associating mathematics with everyday life, considering the developmental history of mathematical concepts, and the cumulative nature of mathematics may originate from the lesson content based on mathematics education. However, these norms can productively support students' reorganization of beliefs about the general nature of mathematical activity for their profession in terms of teaching and learning mathematics.

In A1, two sociomathematical norms were investigated. One of them is about relating mathematics to everyday life, which is common among the three classrooms. Because graph theory originates from everyday-life problems (e.g., Seven Bridges of Konigsberg, Four Color problem), using similar problems and examples might lead to establishing A1's norm. Although there are different motivations for establishing this norm in classrooms, it can foster students' belief in mathematics related to everyday life. The other norm of A1 (*Providing one or two examples is accepted as sufficient for mathematical abstraction*) seems to contradict that of AE1 and AE2 (*Providing one or two examples is not accepted as sufficient for mathematical abstraction*).

The former norm does not meet the principles of familiarity and similarity recognition that are critically important for abstraction (White & Mitchelmore, 2010). It is not a productive experience for prospective teachers' future attempts at utilizing abstraction to improve children's learning. Maybe the teacher of A1 is not aware of or does not agree with this. However, the norm is implicit, and the mathematical activities in A1 are characterized by this norm. It could be from lack of talking and discussion in the community or unawareness about norms. This example puts emphasis on the explicit negotiation of norms for the regulation of mathematical activities, which are the bases of mathematical learning. To this end, raising teachers' awareness about social aspects of classroom activities is crucial.

While what is needed to understand the definition of a concept has been emphasized (*Terms and properties used in defining a concept should be known beforehand*) in AE1, the current study does not have the norm *A definition should be economic* (Sánchez & García, 2014). It was observed only once in AE1 and was not accepted as a norm because of insufficient enactment. The norm *Terms and properties used in defining a concept should be known beforehand* is similar to the norm *"In mathematics, you cannot write what you have not shown to be true yet"* (Sekiguchi, 2005, p. 157), in terms of using terms and properties that are already known.

AE1 and AE2 emphasized that a proof should provide an explanation about the justifications for a claim (*A mathematically convincing claim must be justified with a proof that explains its justifications*). Putting forward reasons by questioning claims was important in these classrooms. Considering that proofs and justifications are central to the discipline of mathematics (Knuth, 2002), these norms are important for making sense and for developing a deep understanding of mathematics. Additionally, intuitive approaches were accepted as important but insufficient, and the algebraic proof was expected from a mathematics teacher candidate in AE classrooms. The norm *While doing mathematics, discourses should be mathematical* supports this idea of classrooms. Additionally, when considering that mathematical discourse is not just speech, behavior, interaction, thinking, reading, and writing but also includes mathematical values, beliefs, and perspectives (Moschkovich, 2003), this norm and norms of the cumulative nature of mathematics, the role of intuition in mathematics, and considering the history of mathematical concepts are considered to enhance each other. As such, they help to improve prospective teachers' "knowledge about the nature and discourse of mathematics" (Baturo & Nason, 1996, p. 237).

Studies into sociomathematical norms (e.g., Cobb, 1999; Elliott et al., 2009; Hershkowitz & Schwarz, 1999; Lopez & Allal, 2007; Partanen & Kaasıla, 2015; Sánchez & García, 2014; Sekiguchi, 2005; Stylianou & Blanton, 2002; Tatsis & Koleza, 2008; Yackel et al., 2000) show how the current study differs somewhat. For example, the norms "*Confusion and error are embraced as opportunities to deepen mathematical understanding*" (Elliott et al., 2009, p. 396), *When investigating mathematics, one should approach the topic in a creative way* (Partanen & Kaasıla, 2015), and other similar norms weren't identified in this study. The behavior *Students should use their errors as an opportunity to review perceptions about their idea* was a conjectured norm that was enacted just once by a student in AE2. However, although this conjectured norm was seen appropriate by the classroom community, it was not sustained. Thus, it was not accepted as a classroom norm. Additionally, the norms *A mathematically convincing claim must be justified with a proof that explains its justifications* and *Intuitive approaches are necessary in mathematics but should not circumvent algebraic proofs* identified in this study have not been encountered in the

literature. At this point, one should realize that the norms of mathematical activities are established by the community (Cobb & Yackel, 1996a); they are related to the beliefs and values given to these activities. Stating that sociomathematical norms are restricted by beliefs, Cobb and Yackel (1996b) and Bowers et al. (1999) remarked on the interactive relationship between them. Likewise, while Ernest (2009) approached values as evidence for sociomathematical norms, Voigt (1995) used norms to define a criterion of values for mathematical activities. Accordingly, when considering the nature of norms mentioned above, different mathematical beliefs and values that stem from different study groups (teacher and students) could be one of the reasons for differences in sociomathematical norms.

Conclusions and Implications for Mathematics Teaching and Teacher Education

If norms are important for a classroom community's individual and collective mathematical learning, and if teachers have a central role in improving the mathematical quality of the learning environment as well as initiating, guiding, and organizing the establishment of norms for mathematical aspects of the classroom activities, how can one help them be competent in performing this role? When attention is given to teacher education, considering that teachers tend to act the same way as they were taught (McNeal & Simon, 2000), it is useful to investigate first the norms experienced by teachers in their own education. Accordingly, this study identified the social and sociomathematical norms that characterize prospective teachers' classroom microcultures, and it discussed their qualities. Social and sociomathematical norms are two of the three aspects of classroom microculture indicated under the social perspective of interpretative framework by Cobb et al. (2001), who used the interpretative framework as a conceptual tool "to understand what is going on in a classroom" (p. 121). Thus by investigating the norms of prospective teachers classrooms, the current study provides an insight into what is going on in teacher education in Turkey in terms of two aspects of social perspective.

One should remember that the members of AE1, AE2, and A1 communities are prospective teachers. They are going to be responsible for having their students acquire basic skills and knowledge aimed by the curriculum and to intentionally negotiate establishing productive norms that make effective teaching and learning possible in classrooms. Tsai (2007) found a positive reflective relation between the teaching norms of teachers' professional development community and their learning norms, which were established by the teachers in their classrooms. This result demonstrates that teachers' acquisition of productive norms helped them have their students acquire productive norms. Therefore, considering the importance of norms for individual and collective mathematical learning in a mathematics classroom,

establishing norms that make effective teaching and learning possible should be a key point of teacher education (Van Zoest et al., 2012). Classroom microcultures with norms that contribute to effective learning and teaching, and that enable them to develop some basic skills for their profession, is desired for prospective teachers. From this perspective, one can infer that when compared with A1, the norms of AE1 and AE2 can be seen as productive norms that provide prospective teachers opportunities for establishing productive classroom microcultures, learning how to teach mathematics in an effective way, and improving knowledge about the nature and discourse of mathematics. However, norms related to creativity and learning from mistakes as seen in the literature (e.g., Elliott et al., 2009; Partanen & Kaasıla, 2015) were absent in the current classrooms. Considering the importance of creativity and using mistakes as potential avenues for learning mathematics (National Council of Teachers of Mathematics, 2000), negotiating relevant norms in classrooms is significant for prospective teachers. Many teachers are reported to not have the abilities needed for conducting activities that develop students' creativity, generally because of a lack of prior experience or proper teacher education (Shriki, 2005, as cited in Shriki, 2010).

There were some norms from A1 that could influence teaching and learning negatively, such as *Providing one or two examples is accepted as sufficient for mathematical abstraction* and *Importance should be placed on getting a good grade rather than learning*. When considering prospective teachers' future roles, having them participate in the microculture of A1 is not desired. The implicit nature of norms, insufficient negotiation, and especially instructor's unawareness about norms are considered important reasons for this result. Efforts to teach mathematics in the best way would lead to establishing productive norms, as in AE1 and AE2. However, most of the productive norms of AE1 and AE2 could also be established in A1 (e.g., most of the sociomathematical norms). In A1, there could be unidentified norms because of less systematic classroom discussion and communication. However, widely enacted and sustained unidentified norms did not occur, particularly because of this.

As students participate in negotiating norms, they develop mathematical beliefs and values that make them more autonomous in participating in mathematical activities (Cobb & Yackel, 1996a). Accordingly, the findings show that explicit negotiation of norms is important for students, as well as for the productivity of norms. No explicit norm negotiation occurred in A1 due to norm unawareness and a lack of explicit conversation or discussion. Hence, there were no negative reactions to conforming to unproductive norms in the classroom community or the community-sustained norms. For example, if somebody in A1 stated explicitly that getting a good grade is more important than learning or that one or two examples is sufficient for mathematical abstraction, the A1 community would have disagreed

with and reacted negatively to this because they would have been enacting norms that contradicted those in AE1 and AE2. Thus, norms being spontaneously established should not be expected; the intention should be to conduct activities that establish productive norms (Dixon et al., 2009). Otherwise, "as long as discursive norms are tacit, their grip on our thinking is particularly strong, and they are particularly difficult to change" (Sfard, 2000, p. 171).

Looking at the differences in microcultures, one can say there was a lack of coordination among the content course and mathematics education courses in terms of social perspective. The classroom norms identified in this paper were different. Thus, the nature of classroom activities and interactions was different. Accordingly, the emphasized beliefs and values were also different. To have prospective teachers acquire the necessary skills, perceptions, beliefs, values, and habits just from a few lessons is not effective or enduring. For example, getting a good grade in class was obviously important for the participants of AE2, but their current pedagogy knowledge said that understanding is more important than getting a good grade and that the learning process is more important than the final exam. However, they had a habit from previous experience. Thus, the students and the teacher established and maintained a norm *Everybody should place value on learning not the grade* to change it. In the meantime, the same students enacted the norm *Importance should be placed on getting a good grade rather than learning* in A1 classroom. Such a gap or conflict[4] between the mathematics content course and the mathematics education course in the same teacher education program does not contribute to prospective teachers' behaviors, habits, beliefs, or values regarding mathematics or mathematics education. To expect to be able to educate prospective teachers just through some courses as planned by a program is not realistic. Therefore, designating all teacher education program courses without exception in order to enable prospective teachers to have experience with productive norms not only in mathematics education courses but also in mathematics content courses is important from social perspective. Similarly, unifying, coordinating, and connecting content courses in mathematics with education courses is recommended for teacher preparation in mathematics by the committee of Educating Teachers of Science, Mathematics, and Technology (National Research Council, 2001).

This study investigated the existing condition of prospective teachers' classroom norms in a natural context, differently from previous studies that had researched intentional establishment and negotiation processes of productive norms. According to Yackel et al. (2000), revealing and interpreting social and sociomathematical norms of university-level classroom microcultures is important for faculty members to conceptualize social aspects of the classroom. This is particularly important for

4 Remember that the sociomathematical norms of abstraction are another example.

teacher education faculties. This conceptualization provides a framework for adopting and developing an approach to mathematics teaching (Yackel et al., 2000). At this point, the findings on norms from AE1, AE2, and A1 illustrate notable examples of productive and unproductive norms for teaching and learning mathematics.

Some requirements have appeared in this study's results that show teacher training programs should identify classroom norms in such a way that they enable prospective teachers to acquire the knowledge, skills, and competencies needed for effective teaching and learning mathematics. In this respect, there is a need to reconstruct teaching approaches and adopt these approaches in all undergraduate courses. Moreover, establishing intentionally social and sociomathematical norms in teacher education will enhance prospective teachers' knowledge of productive norms and awareness about all processes of establishing norms through their ongoing practices and experiences (Van Zoest et al., 2012). Additionally, faculty members' awareness of norms and their effects should be raised to adopt these approaches in teacher education and make teacher education more effective. Teachers and faculty members should be aware of the implicit messages about doing, learning, and teaching mathematics that are delivered to their students through established classroom microculture.

Focusing solely on what kind of mathematical explanations could be normative, some studies have determined which social and sociomathematical norms could be productive at supporting teacher learning (Elliott et al., 2009; Van Zoest & Stockero, 2012). Additionally, the current study reported two productive sociomathematical norms for a proof activity different from the literature: *A mathematically convincing claim must be justified with a proof that explains its justifications* and *Intuitive approaches are necessary in mathematics but should not circumvent algebraic proofs*. Upon seeing the need to interpret this study's findings, a new study is suggested that presents which sociomathematical norms should be established in mathematics classrooms and how they should be established, as well as which specific criteria about norms' properties should be considered; it could fill a gap in the literature and also guide teacher education to adopt new approaches.

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