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Research Article

Analysis of Sixth Grade Students' Think-Aloud Processes While Solving a Non-routine Mathematical Problem*

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Abstract

Problem solving is highlighted in many mathematics curricula and has recently become one of the most investigated topics in the field of mathematics education. Extensive studies report that developing students' problem solving skills enhances their understanding of mathematics. Therefore, there is a focus on investigating problem solving processes of students. These processes are analyzed using various measurement techniques where data are collected in written or verbal forms. One of the techniques using verbal data is the "think-aloud method." This study investigates sixth grade students' think-aloud processes while solving a mathematical problem verbally. The study group consists of 24 sixth grade students selected according to the results of a problem solving test developed by the researchers. The think-aloud process of each student was videotaped and transcribed. Collected data were coded and categorized. Frequencies of categories were determined and compared according to students' performance at solving mathematical problems. Results indicate that students have difficulty in expressing their thoughts during problem solving. Most students attempted operations with the numbers given, spending almost no time on understanding the problem. Some of the students who did the necessary computations correctly could not reach the solution because they could not interpret the result of the operation.

Keywords

Mathematical problem solving • Verbal data • Think-aloud method • Mathematical thinking process • Problem solving performances

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It is not unusual to face unfamiliar situations in our daily lives and to have to come up with new ways to overcome problems. Thus it is essential to develop the ability to solve problems. In the broadest sense, a problem is a situation that includes open questions such that a person does not have the necessary algorithms, procedures, and methods to answer these questions (Blum & Niss, 1991). Problem solving, on the other hand, is a process where we synthesize our existing knowledge and transfer it to a new situation (Schoenfeld, 1992). While problem is a static state, problem solving refers to a dynamic process. Lester and Kehle (2003) define problem solving as an activity that includes various cognitive tasks such as reaching and using prior knowledge and experience.

Problem solving is also one of the most important research topics in the field of mathematics education (Sawada, 1999). As the problem solving process involves understanding mathematical knowledge and forming relationships (Polya, 1973), many studies emphasize that students must develop problem solving skills in order to better understand mathematical concepts (e.g., National Council of Teachers of Mathematics [NCTM], 2000; Sawada, 1999; Schoenfeld, 1985; Soylu & Soylu, 2006; Zhang, 1996). In recent years, mathematical problem solving has become one of the main goals of mathematics curricula. The current Turkish mathematics curriculum considers mathematical problem solving as an indispensable part of learning mathematics (Turkish Ministry of National Education [MNE], 2013).

Mathematical problems are classified into two major categories: *routine* and *non-routine* problems. While routine problems can be solved by rules and operations familiar to problems solvers, non-routine problems are not familiar and require alternative strategies, thinking processes and creative thinking (Mahlios, 1988). Although most students can easily perform mathematical operations, they may have difficulty in transferring mathematical knowledge to other areas and tasks that need higher-order thinking skills (Monroe, 1996, as cited in Brennan, Rule, Walmsley, & Swanson, 2010). Students are usually unable to work out answers in a straightforward way while solving problems (Schoenfeld, 1985) and difficulty of the task increases when mathematical operations are embedded in the context of a problem. Thus the problem becomes a non-routine one. A second grade student, for instance, can easily compute $12:3$ and $12:4$. However, things get more complicated when the student is faced with a word problem that requires carrying out these operations, such as “Naz has 12 cookies. She wants to give her cookies to Alp, Oğuz, and Toprak and share them equally. Alp eats his cookies while Oğuz and Toprak keep their cookies. How many cookies are not eaten?” When such a mathematical problem is given, most students either do not attempt to solve those problems at all or they only try to pull the numbers out of the problem and perform some sort of operation on them and come up with an answer. Therefore, successfully carrying out operations is not enough to

become good problem solvers; students need to understand the problem and be able to develop necessary strategies to come up with an acceptable solution. In order to better understand students' difficulties with problem solving, it is very important to examine in detail how students solve problems.

In literature there are various strategies to investigate students' problem solving processes, and data are collected in written or verbal forms. One type of verbal data collection is called the think-aloud method, where participants verbalize their cognition concurrently as they are performing a given task (Ericsson & Simon, 1993).

The think-aloud method is considered as one of the most effective ways to assess higher-level thinking processes (Kraemer & Ummelen, 2004; Olson, Duffy, & Mack, 1984). Thinking aloud during problem solving means that the subject keeps on talking, speaking out loud whatever thoughts come to mind, while performing the task at hand. Unlike the other techniques for gathering verbal data, there are no interruptions or suggestive prompts or questions. The subject is encouraged to give a concurrent account of their thoughts and avoid interpretation or explanation of what they are doing; they only have to concentrate on the task (Ericsson & Simon, 1993; Someren, Bardnard, & Sandberg, 1994). One of the major advantages of using the think-aloud method is being able to capture the individual's immediate thoughts (Ericsson & Simon, 1993; Jacobse & Harskamp, 2012; Wade, 1990; Young, 2005), which is important in understanding students' problem solving processes. The think-aloud method is considered a valid form of verbal data collection (Ericsson, 2006; Ericsson & Simon, 1993) and researchers in the field of social sciences have been using this method for quite some time. With the help of new technology (recording audio and video is becoming more practical), it is easier to collect and analyze data; there are now studies in the field of mathematics education that use this method (Brennan et al., 2010; Delice, 2003; Gunhan, 2014; Montague & Applegate, 1993; Muis, 2008).

In order to investigate problem solving processes of students in detail, it is important to consider that students vary in terms of performance level. In the literature there are studies that compare problem solving processes of different groups using the think-aloud method. For example, Montague and Applegate (1993) examined the verbalizations of middle school students as they thought aloud while solving mathematical problems. There were three types of student groups: learning disabled, average achieving, and gifted. The results indicated that students with learning disabilities used different approaches to the students in the other groups. Brennan et al. (2010) also used the think-aloud method to describe fourth grade students' problem solving in mathematics. Students were divided into three groups: those who solved the given problem correctly, those who made computation errors and those

who could not solve the problem due to process errors. These groups were compared according to their thinking processes while solving problems. It was concluded that the gifted group made the most comments in the think-aloud process but few of these comments were facilitating.

In Turkey, while there are research studies on students' mathematical problem solving processes (e.g., Aydın & Özmen, 2012; Bayazit, 2013; Delice & Yılmaz, 2009; Karataş & Güven, 2004; Yazgan & Bintaş, 2005), there is almost no research that examines thinking processes in solving mathematical problems by the think-aloud method. In addition, it is important to investigate problem solving processes of students in the first years of middle school. It is an important age group because students at this age are in a transition stage from concrete to abstract thinking required for developing strategies to solve problems. In order to fill the gap in literature, this study was conducted to investigate sixth grade students' think-aloud processes while solving a mathematical problem verbally, and aims to answer the following questions. (i) How do students perform while solving a non-routine mathematical problem by thinking aloud? (ii) How do students' think-aloud processes differ with respect to their performance at solving a non-routine mathematical problem?

Method

This is a qualitative study, aiming to investigate sixth grade students' think-aloud processes while solving a mathematics problem. In order to answer the research questions of this study, the case study method was used. This method is best applied when research involves descriptive or explanatory questions and aims to understand people and events (Yin, 2003).

Sample

The purposive sampling (maximum variation) method is used in this study. Participants were selected from a public school in Maltepe, İstanbul. The school is located in a busy central area where mostly middle-income people live. Authorities of the school have reported that the educational levels of most parents are intermediate. According to results of the mathematical problem solving test prepared by the researchers and the mathematics teachers of the participants, 69 sixth grade students were grouped into three categories: low, moderate and high achievers. Eight students from each group were selected so a total of 24 students (9 female (38%) and 15 (62 %) male) participated in the study. The mean age of the participants was 12 years.

Instruments

Think-aloud problem. Before conducting the research, a pilot study was carried out in order to test the data collection process using the think-aloud method with a separate group of sixth grade students from the same school. The following mathematics problem was used in the pilot study: *Vice principal Ahmet Bey has a box of pencils. There are 100 pencils in the box and he wants to give these pencils to all students from classes 6A and 6B. He decides that each student should get equal number of pencils and as many pencils as possible. There are 20 students in 6A and 25 students in 6B. The class teachers of 6A and 6B will be given one pencil each. After this distribution is done, how many pencils will be left in the box?*

It was observed that students immediately tried to solve the problem and they barely spoke. Although it was a non-routine problem it was familiar at first sight, hence the students used familiar procedures without thinking. So, instead of this problem, the following one was chosen for this study: *Two friends are traveling to Ankara by car. There is a gas station every 55 kilometers. After 196 kilometers they run out of gas. Which gas station is the nearest—the last one or the next one?*

This problem, adapted from [Jacobse and Harskamp's \(2012\)](#) study, can be solved in different ways and students need to interpret the result of their calculations. The mathematics teachers of the participants reported that the students had not solved this kind of problem before, and hence this problem was non-routine for them.

Data Collection

While collecting data with the think-aloud method, conditions stated by [Someren et al. \(1994\)](#) were taken into consideration. Think-aloud sessions were conducted individually by one of the researchers in a quiet room in the school. Each session lasted about 15 minutes. Before the think-aloud session began, the researcher first made small talk with the students to introduce herself, made them comfortable and then explained the think-aloud process by demonstrating an example of the process, providing a correct and an incorrect solution of the following problem: *I have 20 Liras. If I give a quarter of my money to my sister, how much money do I have left?*

This simple problem was chosen so that every student could easily follow the given solutions. The purpose for giving both correct and incorrect examples was to encourage students to participate and assure them that the researchers were only interested in their think-aloud process and it was not important whether they arrived at a correct solution or not. Students were informed beforehand that they could leave the session anytime they wanted. Each think-aloud process was videotaped and then transcribed. Videotapes and transcriptions were only available to the researchers and were used only for the purposes of this study.

Procedure

Students were categorized into three groups according to their performance of the think-aloud problem, and their solutions were examined to see common mistakes. In this study, students' comments during the think-aloud process were described according to types of events. Brennan et al. (2010) described these kinds of problem solving events by categorizing them as “paraphrasing,” “elaborating,” “monitoring,” “identifying a problem” and “other.” This categorization was adopted from the work of Cote (1998).

Paraphrasing, elaborating, and monitoring events were coded as facilitating or non-facilitating, where identifying a problem was coded as resolved and unresolved. The descriptions and coding of these events are given in Table 1.

Table 1
Types, Description and Coding of Events During The Think-Aloud Process

Event	Description	Coding	Sample Quotes
Paraphrasing	Repeating, rewording, substituting similar words	Facilitating: Leads to correct solution of the problem	“...after 196 kilometers they run out of gas...”
		Non-facilitating: Does not lead to correct solution of the problem	“...after they travel 55 km, they see gas station...”
Elaborating	Adding additional information, making logical or pragmatic inferences, forming cause and effect relationships	Facilitating: Leads to correct solution of the problem	“...to reach 196 I will multiply 55 with a number that gives me the nearest result...”
		Non-facilitating: Does not lead to correct solution of the problem	“... I will multiply 196 by 55...”
Monitoring	Evaluating the comprehensibility or familiarity of the problem	Facilitating: Leads to correct solution of the problem	“... I understand the problem...”
		Non-facilitating: Does not lead to correct solution of the problem	“...my math is not very good...”
Identifying the problem	Indicating difficulty of understanding the problem	Resolved: Able to overcome the difficulty	“This is wrong... I will try it again...”
		Unresolved: Not able to overcome the difficulty	“... I cannot do anything...”
Other	Making ambiguous comments or being silent		“...umm...,” “...I do not know what to think...”

Transcriptions of the students' think-aloud processes were coded independently by all researchers according to the descriptions of event categories given in Table 1. Then the coding of the researchers was compared and 95% agreement was reached. For the instances where disagreement occurred, three researchers reached a consensus and coding was adjusted accordingly.

Trustworthiness of the Study

In order to strengthen trustworthiness of this qualitative research, the following four aspects were considered: credibility, transferability, dependability, and confirmability (Lincoln & Guba, 1985).

To obtain credibility, analyst triangulation was used in this study (Patton, 2002). Three researchers analyzed data independently and compared the findings. In addition, an expert in qualitative research methodology was present during all stages of data analysis.

Including transcripts of think-aloud process of a student from each group and showing how the data was coded and categorized ensured transferability. Using purposive sampling method in this study also strengthened the transferability.

In this research, to ensure dependability, each think-aloud session was conducted under the same conditions, each researcher coded the data consistently, and researchers' coding was compared and agreement was reached. Furthermore, the data collection and analysis processes were reported in detail.

As all video tapes, transcripts and codings have been preserved the results can be confirmed if necessary. Hence, confirmability was also obtained.

Findings

To answer the research questions in the study, students were divided into three groups according to their performance at solving the think-aloud problem: those who solved the problem correctly (correct group), those who used the correct process but made an error in computation (incorrect computation group) and those who solved the problem incorrectly (incorrect process group). Table 2 shows the number and percentages of students in these groups.

Table 2

Distribution of Students According to Their Performance at Solving the Think-Aloud Problem

Group	f	%
Correct	5	20.8
Incorrect Computation	8	34.3
Incorrect Process	11	45.8

It can be seen from Table 2 that almost 80% of the students were unable to solve the problem correctly. In order to determine students' error patterns in think-aloud processes while solving the problem, their errors were categorized as shown in Table 3.

Table 3
Error Categories for Incorrect Groups

Group	Error Categories	f	Example of Category
Incorrect computation (n=8)	Cannot interpret the result	4	"... 55 is far from 196... so, the previous one..."
	Calculation error	3	"... I multiply 55 by 3... So, it is 166..."
	Cannot reach the result	1	"... I multiply 55 by 3 ... It is 165 ... Umm... I don't know..."
Incorrect process (n=11)	Aimless operation	6	"... could the answer be 3? I tried everything but I couldn't solve..."
	Cannot divide 196 by 55	3	"... I divide 196 by 55... Umm... I cannot do this... It doesn't mean anything to me..."
	Cannot any do any operations	2	"... I cannot do this... really..."

As can be seen from Table 3, the most frequent error observed in the “incorrect computation group” is students’ inability to interpret the outcome of the operation. So, students in this group could not provide the desired answer (finding out which gas station was closer) even though they had a correct approach to the solution; they either could not successfully carry out the operation or they could not interpret the result of the operation.

Looking at the error categories for the incorrect process group, half of the students attempted to do random operations with the numbers given in the problem. Some of them had no idea about what to do with the given numbers while the rest started with the correct idea of dividing 196 by 55 but gave up quickly without attempting any computation.

The aim of the second research question was to compare students’ think-aloud processes with respect to their performance at solving the mathematical problem. Transcripts of the students were coded according to the categorization given in Table 1. Coding examples of the transcriptions for each group are given below. It must be kept in mind that in the think-aloud sessions students spoke in their native language (Turkish) and original transcriptions were used in the coding process. These transcriptions were then translated to English in order to provide examples for the think-aloud process.

An example of coded transcription of a student who solved problem correctly is given below:

STU: First... after 196 kilometers they run out of gas. (Paraphrasing Facilitating)

STU: According to problem... every 55 kilometers... (Paraphrasing Facilitating)

STU: How close is 55 to 196? (Elaborating Facilitating)

STU: If I multiply 55 by 5, it is 275. (Elaborating Facilitating)

STU: This is not applicable.

STU: Then I will do it by subtracting 2. 55 times 3... 3 times 5 is 15... carry 1... 3 times 5 is 15 and carry 1... result is 165 (Elaborating Facilitating)

STU: 165 plus 55 equals 220. (Elaborating Facilitating)

STU: So, how close to the next. (Elaborating Facilitating)

STU: Now I subtract 196 from 220 (Elaborating Facilitating)

STU: 10 minus 6 is 4... 11 minus 9 is 2... (Elaborating Facilitating)

STU: 24 kilometers. Next, one. (Elaborating Facilitating)

STU: To find the last one if we subtract 165 from 196... (Elaborating Facilitating)

STU: 6 minus 5 is 1... 9 minus 6 is 3... 1 minus 1 is 0... (Elaborating Facilitating)

STU: If they stop at the last station, there are 31 kilometers to it; the next one is 24 kilometers away. (Elaborating Facilitating)

STU: So, the next one is the nearest. (Elaborating Facilitating)

An example of coded transcription of the student who solved problem incorrectly is as follows:

STU: If I subtract 55 from 196 kilometers. (Elaborating Facilitating)

STU: I cannot find by this way. (Identifying Problem Unresolved)

INT: What do you think now?

STU: I am reading the problem again.

STU: There is a gas station in every 55 kilometers and the car breaks down after 196 kilometers. (Paraphrasing Facilitating)

STU: I will subtract 55 from 196 kilometers (Elaborating Non-facilitating)

STU: I found 141 kilometers (Elaborating Non-facilitating)

STU: If I divide 141 by 2, I find the nearest station. (Elaborating Non-facilitating)

STU: I didn't understand. (Thinks again) (Identifying Problem Unresolved)

INT: So what are you thinking?

STU: I divided 141 by 2, I found 70 and I add 1 to 70 (Elaborating Non-facilitating)

STU: One is 71 and the other is 70. So the second one is closer. (Elaborating Non-facilitating)

STU: This is wrong, isn't it? (Identifying Problem Unresolved)

INT: I don't know. Thank you very much.

An example of coded transcription of the student who was in a correct process but the solution is incorrect is given below:

STU: I will subtract 55 from 196 (Elaborating Facilitating)

STU: I will find the nearest station. (Elaborating Facilitating)

STU: Umm... (Other)

STU: (trying to calculate, does not speak)

INT: Please think aloud

STU: Umm...(Other)

STU: I couldn't find anything. (Identifying problem unresolved)

INT: Don't you understand the question? If you want, you can read the problem again.

STU: (Reads the question again)

STU: It says every 55 kilometers, I will divide 196 by 55. (Elaborating Facilitating)

STU: It is not divisible. (Elaborating Facilitating)

STU: Umm... It is closer to the last one. (Elaborating Non-facilitating)

INT: How did you find out?

STU: Remainder is 31... (Elaborating Facilitating)

STU: This is lower than 55... Not higher... So it is closer to the last one (Elaborating Non-facilitating)

After all transcripts were coded, the percentage of the number of events in all categories was calculated for three groups as shown in Table 4.

Table 4
Percentages of Number of Words per Paraphrasing, Elaborating, Monitoring and Identifying Problem Events by Each Group

		Group		
		Correct (n=5)	Incorrect Computation (n=8)	Incorrect Process (n=11)
Paraphrase Facilitating	%	16.66	13.95	18.6
Paraphrase Non-facilitating	%	0	1.16	2.32
Elaborate Facilitating	%	68.18	36.05	19.37
Elaborate Non-facilitating	%	7.57	34.88	37.2
Monitoring Facilitating	%	0	0	1.55
Monitoring Non-facilitating	%	0	4.65	7.75
Identify Problem Resolved	%	4.55	1.16	1.55
Identify Problem Unresolved	%	3.03	8.13	11.62

Table 4 shows that during the think-aloud process, elaboration was the most frequently occurring event in all groups. Facilitating elaboration was seen most in the correct group followed by the incorrect computation and incorrect process groups. The students following appropriate processes were better at making inferences or forming cause and effect relationships that led them to the correct solution. Non-facilitating elaboration, however, was most common in the incorrect process group. The elaboration observed in this group of students did help them to solve the problem correctly.

For each type of event other than elaboration, there are almost no differences between numbers of events for all groups. However, the incorrect process group has higher percentages of non-facilitating monitoring and identifying unresolved events than the other groups. While facilitating paraphrasing is rarely used in all groups, almost no monitoring and identifying problem are observed. Low occurrence of paraphrasing events suggests that students immediately started to concentrate on the required operations.

Discussion

In this study, students' think-aloud processes while solving a non-routine mathematics problem were examined and compared with respect to their performance at solving this problem. First, the research question was related to the students' performance. According to the think-aloud protocols of the students in the incorrect process group, the following were observed: students mostly do operations aimlessly.

As stated in the literature (Aydin & Özmen, 2012; Bayazit, 2013; Verschaffel, Greer, & De Corte, 2000; Verschaffel, Greer, Van Dooren, & Mukhopadhyay, 2009), students approach word problems in a superficial way, and they execute arithmetic operations with the numbers given in the problem and do these operations without thinking about the context of the problem. Some of the students did no calculations, indicating that they were unable to solve the problem, while others tried to do division. As there is a remainder in the division, they thought they were doing something wrong and stopped. The expectation that there will be no remainder in the division indicates a misconception. When students first learn division, they work on examples with whole numbers and there is no remainder. Then they move on to working with division remainders and decimals. Thinking that principles related to a specific topic are applicable to other topics is a misconception called “over generalization” (Zembar, 2008).

Students in the incorrect computation group either made calculation mistakes or could not reach the solution. Some of them made the correct calculation—they did the division correctly—but they could not interpret the results because of the remainder. Similar findings were obtained in two other studies (DeFranco & Curcio, 1997; Reusser & Stebler, 1997): when students were given problems that required making division with remainders, almost all had difficulties in interpreting the results.

Another aim of this study, as stated in research question 2, was to find connections between students’ think-aloud processes and their performance at solving this problem. In this study, as stated in the procedure section, students’ comments during think-aloud processes were described according to types of events. Results of this study reveal that paraphrasing was rarely used by students in any of the groups. Brennan et al. (2010) found a similar result. Restating the problem in one’s own words, and thinking about what are given and what are asked for, are strong indicators of understanding the problem, which is the first step of problem solving. Therefore, lack of paraphrasing indicates that students omit this important stage. It was also observed in Jacobse and Harskamp’s (2012) study that students did not use paraphrasing during mathematical problem solving while thinking aloud. One of the reasons for this is tendency to solve the problems as quickly as possible, probably due to pressure of succeeding in national exams; hence it may not be very surprising that all groups skipped this part quickly. The reason for students in the correct group for not using paraphrasing is probably that they easily understood the problem and they passed straight to the other stages. As long as a student can solve the problem correctly, there may be no harm in skipping this stage. As Sweller (1988, as cited in Jonassen, 2000) indicated, people who can solve a problem easily have better developed problem schemas that can be used automatically. The reason for not using paraphrasing words for students in the two other groups could be problematic. As they could not solve

the problem correctly, it may indicate that they have difficulties in understanding the problem, which is one of the important stages of problem solving.

For all groups, elaboration (adding additional information, making logical or pragmatic inferences, forming cause and effect relationships) either facilitating or non-facilitating, was the most-used type of event. The number of elaborating facilitating events observed in the correct group was more than twice that of both the incorrect computation and incorrect process groups. Conversely, while elaborating non-facilitating events were frequently used in both incorrect computation and incorrect process groups, almost no elaborating non-facilitating events were observed in the correct group. These results are also parallel to the study of Brennan et al. (2010). As they said, this trend suggests that children can not necessarily arrive at a solution only by talking too much about the problem. Talking must be facilitating. In Montague and Applegate's (1993) study, which examined the verbalization of middle school students as they thought aloud while solving mathematical problems, it was also concluded that students who failed to solve the problem correctly used significantly more cognitive verbalization (reading, paraphrasing, visualizing, hypnotizing, estimating, computing and checking) than students who solved the problem correctly. To overcome this problem, teachers may create an environment in class where students think aloud about the problem and scaffold them to think aloud in a facilitating way.

Neither facilitating nor non-facilitating monitoring events were observed in any of the groups, which means that they did not evaluate comprehensibility or familiarity of the problem. Stating familiarity or non-familiarity reflects awareness about the problem, which is one of the important steps in problem solving. Similar results were observed in both identifying problem resolved and identifying problem unresolved events. Students usually consider comprehensibility and familiarity of problems when they try to solve non-routine problems. Even though the curriculum emphasizes the importance of non-routine problems, usually only routine problems are given in the mathematics lessons (Özmen, Taşkın, & Güven, 2012). Comprehensibility and familiarity of the problem are important metacognitive skills while solving problems (e.g., Efklides, Kiorpelidou, & Kiossoglou, 2006; Mevarech & Kramarski, 1997), hence, focusing on non-routine problems during mathematics lessons is important for developing these skills. Another reason for students' hesitation to declare they could not understand the problem may be to avoid revealing their failure.

Think-aloud data provides information inaccessible through written solutions (Rosenzweig, Krawec, & Montague, 2011). By verbalizing their inner speech as students think their way through a problem, researchers can model how expert thinkers solve problems. Thus, using the think-aloud method to investigate students'

mathematical problem solving processes empowers the results of this study. This method may be useful to determine specific areas of weakness in students processing skills, different types of errors and strategies used during problem solving. For this reason, thinking aloud could also be used as a teaching method. As results indicate, the students who could not reach the exact solution used lots of non-facilitating comments. At this point it is important to convert these non-facilitating comments into facilitating ones. Teachers can design class activities where students think aloud individually or as a group. Further studies can be conducted to test effectiveness of using the think-aloud method as an instructional tool. Moreover, this study can be done with bigger samples or different age groups. Students' written work can be examined in addition to their think-aloud process and results can be compared.

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