KURAM VE UYGULAMADA EĞİTİM BİLİMLERİ EDUCATIONAL SCIENCES: THEORY & PRACTICE

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Research Article

Fourth-Grade Primary School Students' Thought Processes and Challenges Encountered during the Butter Beans Problem*

Neslihan Şahin[1]
Ondokuz Mayıs University

Ali Eraslan[2]
Ondokuz Mayıs University

Abstract

In parallel with mathematical modeling studies that have gradually drawn interest in recent years, the aim of this study is to investigate the thought processes of fourth-grade students in the Butter Beans Problem and to identify possible challenges in this process. For this purpose, a qualitative study was conducted at a university-foundation primary school in the city center of a large province in Turkey during the 2013-2014 academic year. After applying a four-week preliminary study to a fourth-grade classroom, three students included in the focus group were selected using the criterion sampling technique. A focus group of three students was videotaped as they worked on the Butter Beans Problem. The conversations of the group were transcribed, examined along with the students' written work, and then analyzed through the lens of Blum and Ferri's modeling-process cycle. The results showed that primary fourth-grade students can successfully work with model-eliciting problems; however, they did encounter some difficulties during the modeling process.

Keywords

Primary school students • Model eliciting problems • Mathematical modeling • Butter Beans Problem • Modeling

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1 Department of Primary School Education, Faculty of Education, Ondokuz Mayis University, Samsun, Turkey. Email: neslihan.sahin@omu.edu.tr

2 **Correspondence to:** Ali Eraslan (PhD), Department of Mathematics Education, Faculty of Education, Ondokuz Mayis University, Samsun Turkey. Email: aeraslan@omu.edu.tr

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While the need to access, use, and create knowledge has been continuously increasing in the 21st century, one of the main targets in education is to have a qualified work force be a citizen of the world, emphasizing world citizenship over individualism (Ministry of National Education [MoNE], 2013). Therefore, educating individuals to have skills like construction, hypothesis, identification, description, verification, prediction, manipulation, analytical thinking, and teamwork and who can effectively deal with problems and creatively develop solutions have become important educational goals (English & Watters, 2004). In this respect, mathematics education has greater importance in educating problem-solving individuals with analytical and creative-thinking skills. In line with these developments, Turkish curriculum was reshaped in 2005 to train individuals for these skills. Instead of emphasizing a step-by-step approach, memorization, or learning rules, the main focus of the current primary mathematics curriculum is to train individuals to use mathematics in their daily lives, to have problem-solving skills, to share their thoughts as a team, to have self-confidence in mathematics, and to develop a positive attitude towards mathematics (MoNE, 2009).

When considering the goals of mathematics education, it has become critically important that students understand and be able to explain mathematical concepts, test hypotheses, and analyze relationships, as well as learn how to reconstruct existing knowledge (Thomas & Hart, 2010). Behaviors related to mathematics appear in all levels of educational programs, from pre-school to higher education, with adaptations according to one's level of development. Lesh and Zawojewsky (2007) stated that memorizing mathematical processes and then applying these methods to similar problems is not enough. They emphasized the need for students to face complex and multifaceted problem situations and gain experience this way, thus allowing them to develop new skills and mathematical thinking to prepare them for their future life after school. At this point, primary education is an important period for developing these skills (English & Watters, 2004). Mathematical models and modeling approaches can be utilized to analyze complex problems that represent real-life situations students can actively participate in (Sriraman & Lesh, 2006). Therefore, model-eliciting activities that bring about situations where students can create solutions to problems that involve mathematical modeling should be used as early as primary school, allowing them to face complex, real-life situations like this at an early age (English, 2006b). However, model and modeling in Turkey's new primary-school mathematics education program make reference to solid materials such as cubes, cones, algebra tiles, pattern blocks, fraction sets, and ten-based blocks to help students easily understand abstract mathematical concepts (MoNE, 2009). The only parts that emphasize higher-level mathematical thinking in the new program are project and performance assignments, which are rarely used effectively by teachers.

Research studies conducted in primary schools have revealed that modeling activities enable students to express, test, revise, and change their thoughts several times (Eraslan, 2011a); improve the use of mathematical language, the ability to work in groups, social interactions, reading data from tables, and successful dealings with graphics (Watters, English, & Mahoney, 2004); enhance meta-cognition and critical thinking skills (English & Watters, 2004); contribute to overcoming some of young children's conceptual shortcomings (English & Watters, 2004); and discover the fundamental ideas and processes of problems, determine the priorities of basic elements according to interrelationships, and make mathematical calculations to transform qualitative data into quantitative (English, 2007). On the other hand, some students were observed to have difficulties interpreting and understanding a variety of representational formats of the presented data (English & Watters, 2004), converting data into different representational formats (English, 2012a), introducing created models systematically (English, 2003), and determining proper parameters (e.g., focusing on the amount of daylight rather than bean weight; English & Watters, 2004). In the Turkish literature, all research studies on model-eliciting activities have been conducted at the secondary-school level (Doruk, 2010, 2011, 2012; Kal, 2011; Kant, 2011; Sandalcı, 2013), while no study has yet to investigate primary-school students' modeling processes. Having no studies that show the extent to which primary school students are ready to solve the real-world problems encountered in secondary school, high school, outside of school, in their work life, or as a citizen is a shortcoming. Therefore, the aim of this study is to investigate fourth-grade students' model-eliciting processes and identify possible difficulties. For this purpose, the following research questions are asked: (a) what are fourth-grade students' thought processes while working on a model-eliciting activity, and (b) what difficulties do they encounter during these processes?

Theoretical Framework

While *modeling* is a process of constructing models; interpreting (identifying, describing, or creating) the problems and situations; coordinating, systematizing and organizing a pattern; and using different mental schema; *models* are conceptualized mental systems of both learners and problem solvers that require using equations, diagrams, computer programs, or other media contained in formalized representation (Lesh & Doerr, 2003a). In short, the relationship between model and modeling is similar to that between the product and process, respectively (Sriraman, 2005). *Mathematical modeling* in this context is a systematic process whereby a mathematical or non-mathematical condition of real life is expressed as best it can mathematically using numerous metacognitive activities such as analysis, synthesis, and interpretation (Swetz & Hartler, 1991). By engaging in mathematical modeling, students identify the underlying mathematical structure of complex phenomenon. Lesh and Doerr

(2003a) define mathematical modeling as a stage of model-eliciting activities or one of the processes that occur during modeling. Thus, model-eliciting activities are generally not traditional problems that give answers with numbers or words but non-routine, complex problems that express real world situations requiring mathematical interpretations and formulations that involve different possible solutions (Eraslan, 2011b; Lesh & Zawojewsky, 2007; Mousoulides, 2007). In parallel with Lesh and Doerr's (2003a) definition, the present study uses mathematical modeling as one of the modeling processes that mathematically explain the relationships among variables. This is also appropriately supported by Blum and Ferri's (2009) modeling cycle.

The aim of modeling activities is to help students mathematically conceptualize their thoughts and processes, as well as develop models that can be shared with others, so as to be able to apply them in other problem situations. Lesh and Doerr (2003b) suggested that developing children's mathematical definitions, explanations, and verifications can be achieved by model-eliciting activities. The models obtained at the end of these activities are founded on important mathematical structures, patterns, and multiple cycles of interpretations, definitions, assumptions, explanations, and implications (Lesh & Doerr, 2003a).

In the literature, Ferri (2006) discussed different modeling cycles that depend on various directions and approaches of how modeling is understood, and whether complex or non-complex tasks are used in certain cases. The author divided and explained four groups of modeling cycles in terms of differentiating real situations (RS), situation models (SM) and mental representations of the situation (MRS), real models (RM), and mathematical models (MM). These are then named and illustrated based on the first three phases. In the present study, Blum and Ferri's (2009) four-stage modeling cycle was used (Figure 1). According to Ferri (2006), researchers in this cycle do not distinguish between SM/MRS and RM. It is understood as a real model. As a result, the situation model is not a phase in this modeling cycle.

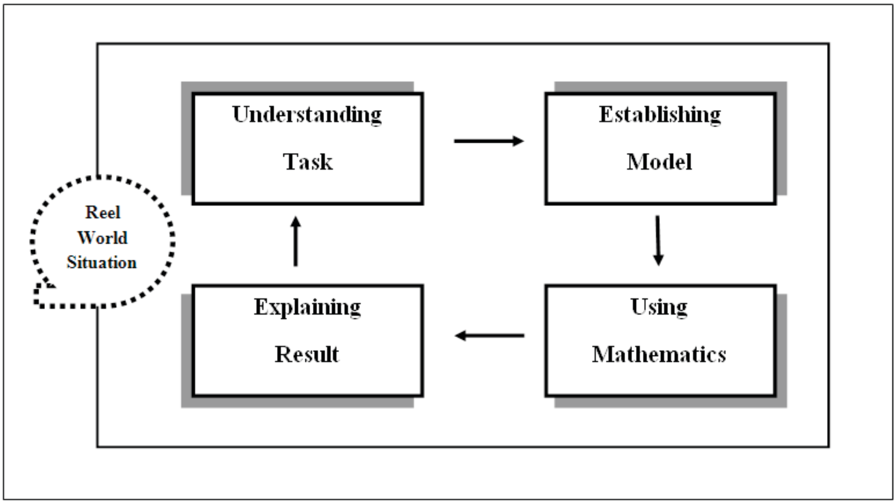


Figure 1. Blum and Ferri's (2009) Four-Stage Modeling Cycle.

Blum and Ferri (2009) emphasized that these stages do not have to be linear or in a given order. The four stages of this modeling cycle are as follows. The stage of *understanding the task* involves students' attempts at reading and understanding a problem adapted to their daily lives (visualizing, drawing, reading data tables, simplifying the problem). In e*stablishing the model*, students collect the required data, recognize and find the relationships and rules, and realize the patterns and make assumptions. In the stage of *using mathematics*, students are expected to determine the appropriate mathematical concepts, make the appropriate mathematical operations, and reach mathematical results at the end of their calculations. The stage of *explaining the result* ends with a cycle investigating whether what students have done is correct, whether the validity of the model has been verified, and whether the solution was reported by comparing the results with real-world situations.

Method

Research Design

This qualitative study was conducted to investigate the model-eliciting processes of fourth-grade primary school students and to identify challenges that may arise in the process. The study design is a case study, which aims to perform in-depth analysis in a group or case. The case presented in this study consists of a focus group of three individuals who were chosen in the effort to determine their thought processes.

Study Group

This study was conducted at a university-foundation primary school in the city center of a large province in the Black Sea region during the 2013-2014 academic year. The study included a total of 18 fourth-grade students in the classroom. A preliminary application was performed before the actual study commenced. The researcher took an active role as a practitioner-teacher in the process while the classroom teacher observed the whole process. Firstly, students in the six groups were asked every week to attempt different model-eliciting problems that require mathematical modeling. This period lasted for four weeks. Then, three students between the ages of 9 and 10 were selected as a focus group using the criterion sampling technique. The following criteria were used to select the group: care was taken to ensure that (a) the students would be able to work in harmony for four weeks, (b) the problem could be solved "as a group," and (c) the students had high self-confidence and the ability to freely express and verbalize their thoughts. Instead of selecting the students one by one, the actual application was carried out by choosing the group that fulfilled the criteria the best.

Data Collection Tools

After a four-week preliminary study, three students selected from the focus groups using the purposeful sampling technique were given the Butter Beans Problem (Appendix 1) and asked to work on this problem. The *Butter Beans Problem* is a model eliciting problem consisting of two parts (Doyle, 2006; English, 2004; English & Watters, 2005). In the first part, students are asked to determine which of the conditions is better for growing butter beans to produce the largest crop and then write a letter explaining their decision. In the second part, they are asked to predict the weight of butter beans produced in the 12^{th} week for each type of condition and explain how they made their prediction so that the farmer can use it for other similar situations. The Butter Beans Problem is a model-eliciting activity that enables students to gain skills at reading and interpreting mathematical and scientific knowledge presented in the form of text and diagrams; at reading and analyzing a simple data table, analyzing and representing the data, hypothesizing, and preparing written reports from the data; and at working in groups and sharing their solutions both verbally and in writing (Doyle, 2006; English, 2004; English & Watters, 2005). The focus-group study, which lasted 90 minutes, was recorded and then qualitatively analyzed alongside students' worksheets. Before conducting interviews with the students in the focus group, the students were informed about the study and were assured that would there would be no performance grade, nor would their actual names or drawings be used. In addition, the importance of the study is emphasized as it can provide important contributions to the development of primary-school mathematics programs.

Data Analysis

The mathematical thoughts and written responses of the fourth-grade students while working on the Butter Beans Problem were analyzed using descriptive analysis. Descriptive analysis comprises the following steps: (a) creating a framework for descriptive analysis, (b) processing the data based on the thematic framework, (c) identifying the findings, and (d) interpreting the findings (Yıldırım & Şimşek, 2011). Therefore, the fourth-grade students' thought processes on the model-eliciting problem were analyzed through the lens of Blum and Ferri's (2009) modeling cycle. In particular, worksheets and final reports written by the students in the study while solving the Butter Beans Problem were triangulated with transcripts made from the video recording. In order to improve the internal reliability (validity) of the study, the researcher observed the classroom, interacted with students, and joined class discussions for the two weeks prior to the preparatory activities. Moreover, a four-week training session that included four different applied model-eliciting problems was carried out before the main study to establish an environment of trust. In order to ensure the validity of the process, students' worksheets, reports, and video recordings were analyzed using data triangulation. In addition, two other faculty members with experience in qualitative research checked the modeling processes, and they fully agreed on the interpretations of the direct quotations used. On the other hand, in order to increase the transferability of the results to similar situations, detailed descriptions and purposive sampling methods were used (Yıldırım & Şimşek, 2011). Detailed descriptions include rich and extensive definitions, research procedure and design, data collection instruments and processes, participant descriptions, field notes, and documents (Merriam, 2013).

Results

The modeling processes of students in the focus group were created through mathematical thinking and written transactions; each stage in this process is presented below. The girls in the group were given the nicknames, Irem, Asya, and Demet.

Model Building Process

Understanding the task (first part). After delivering the modeling problem to the students, students preferred studying alone instead of working together. Asya, one of the group members, first questioned her friends after reading the problem and the table as follows:

> Asya: Which one is better, a heavy bean or a light one?
>
> Asya: But if it is too heavy, it may be not fresh. If it is very large, it cannot be fresh.
>
> Demet: What do you mean "not fresh?"
>
> Asya: Not fresh! I say, it turns out to be rotten when you open it. Disgusting!
>
> Asya: Let's decide. Now, in the daylight and in the shade... At Week 6 it is 5 kg in the shade; 9 kg in daylight.

From the above transcript, Asya first tried to comprehend the problem and then asked her friends whether better beans are "heavy" or "light." The next stage was a modeling process to be carried out by comparing the problems in the same columns and rows in the two tables.

Establishing the model (first part). During the modeling process, Asya first individually focused on solving the problem; she compared the data for daylight and shade and then asked the following question:

> Asya: This is (showing the Daylight Table, Row-1) higher than that (pointing to the Shade Table); this is (daylight data) higher than that (shade data). This is also (showing the data table in the shade) higher than that (showing the data table in the shade). In the 3rd week (for Week 10) it is 2 kg less than the other. This is equal; 18-13 (incorrectly showing row three), huh ... You have to know whether the man loves the heavy one or the light one. You understand when you see.
>
> Demet: How?
>
> Asya: You have to know which one does he love, heavy or light? When you look at this, you can understand. In Week 8 and Week 6 (showing the daylight table) it is heavier. This is (showing the daylight table) lower than that in the shade and it is equal to the other but I need to know which one he prefers.
>
> Demet: The largest is the best one...
>
> Asya: But I want to say something. If they are small, there are too many beans.

The quotes above show that the beans' weight for Weeks 6 and 8 is heavier in the daylight. On the other hand, the weight in the Daylight and Shade Tables for Week 10 are equal in one row; beans growing in the shade are heavier than those grown in the day light in another row. The student tries to make her decision considering the uncle's choice; whether he likes heavy or light beans. Although the beans' weight is given in kg in the table, having students discussing whether they should make a selection in terms of kilograms or the number of beans shows that they still had difficulty understanding the problem.

Using mathematics (first part). Students made the following mathematical comparisons to decide the most appropriate condition (daylight or shade); this is the first part of the problem:

> Asya: This is (showing the daylight table) heavier than that (showing the shade table)
>
> Researcher: How did you draw that conclusion?
>
> Asya: By looking at.
>
> Researcher: What did you look at?
>
> Demet: At kilograms and weeks.

> Researcher: How would you describe it to Uncle Ahmet?
>
> Asia: Well, Uncle Ahmet, 9 is greater than 5, 8 is greater than 5, 9 is greater than 6, and 10 is greater than 6. Twelve is greater than 9, 11 is greater than 8, 14 is greater than 9, 11 is greater than 10, 13 is less than 15, and 14 is equal to 14...

The above quotes show that students focused on *kilograms* and *week*, comparing the two tables according to the concepts of the beans' weight being *larger, smaller,* or *equal* (in the daylight and shade). By comparing the beans grown in the daylight and those in the shade, they were found to have considered only one piece of data while ignoring the others; therefore, their final decision was daylight. This is because they emphasized that the majority of the values in the daylight were greater than those in the shade.

Explaining the results (first part). Group members Demet and Irem stated their individual opinions as follows:

> Irem: My decision is in the daylight because in our garden, plants growing in the daylight yield more.
>
> Researcher: How did you demonstrate it from that table?
>
> Irem: I would show it as Asya did.
>
> Demet: I would just say... Look there are more products in the day light. If it weighs less, it may not be enough for this time, but if it produces more, you can put it in the freezer.
>
> Irem: One more thing, if you have more plants, you have more product.
>
> Demet: No! There are two plants, but one produces more.
>
> Irem: You see, the more plants there are, the more product.

The above quotations show that Irem, who expressed her opinions differently from the other group members, drew this conclusion by considering the plants grown in their garden, thus selecting the daylight option. Demet stated that the weight of the beans grown in daylight is greater (in kilograms) than those grown in the shade by pointing at the table. Also at this point, discussions about whether the weight or number of beans should be considered were understood to have ended and that the expression, "the more plants there are, the more product," should be considered. Irem, who inquired about the accuracy of results by questioning the situation in her daily life, expressed her opinions as follows:

> Irem: Teacher, I also know that there are a lot of plants in our garden. Some of them are always in the shade. They bear fruit once a year in July and some also in June. Tomatoes on the other side do not bear fruit, but those in the backyard have to be harvested constantly. I mean those grown in the daylight.

In the excerpt above, Irem stated that they harvest the produce growing in the daylight from their garden many times, but those grown in the shade yield once a year or not at all. Demet tried to explain that daylight is better for the plants as follows:

> Demet: Then I would say, "Look, Uncle Ahmet. In Week 6, Row 1 is at 9kg; Row 2 is at 8 kg; Row 3 is at 9 kg; and Row 4 is at 10 kg." I would also explain the rest by pointing at the table.
>
> Researcher: What are you going to tell us? What should Uncle Ahmet understand from this table?
>
> Demet: This table, um ...I would try to tell him that the beans growing in the daylight are heavier than those in the shade.
>
> Researcher: How did you draw that conclusion?
>
> Demet: We found out it by looking at the weeks and rows. For example, in Week 6 it was 9kg; in the 8th week it was 12 kg, in Week 10 it was 13kg ... We also looked at columns: 9kg, 8kg, 9kg, 10 kg (6th Week Column in the daylight). Here, we also looked at that (Shade Table, Week 6 Column). We then compared them.

From the quotes above, Demet was understood to think beans growing in daylight are heavier than those growing in shade, and she had to compare the tables. While her friends were explaining their opinions to the researcher, Asya completed the process by writing a report indicating how they had reached this conclusion (Figure 2).

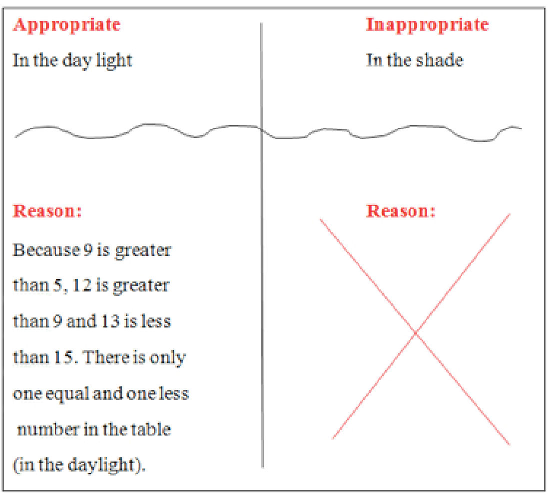


Figure 2. Part 1 Report.

Establishing the model and using mathematics (Second part). While guessing the weight of beans grown in the shade and daylight for Week 12, the following discussion appeared while the students were trying to find a pattern from the data in the table:

> Demet: Well, I think for example, if it is great in the daylight, the yield would be much more in kg.

Demet: If it is less in the shade, it will be less in kg, because the weight of those growing in the shade is greater than those growing in the daylight.

Researcher: OK! Do you have week 12 there?

Demet: No, weeks 11 and 12 are also missing. I think if we predict 11 first, we can also find Week 12 easily.

Irem: Teacher! I cannot solve the patterns. I could not figure out any pattern.

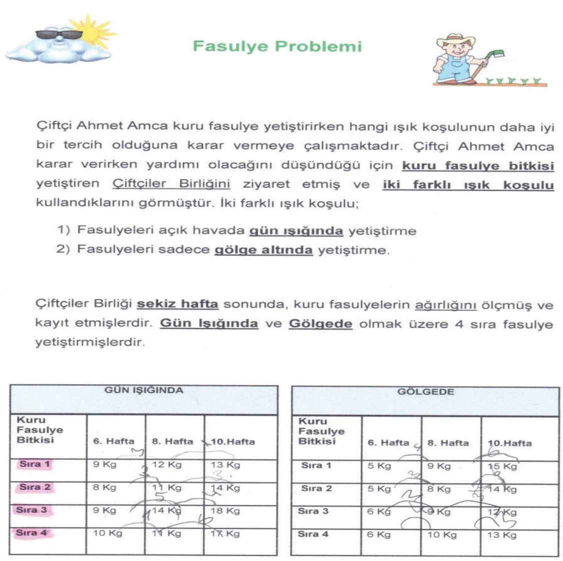
Demet: I found something more sensible. We have 9 kg now and it rose up to 12 kg, with an increase of 3 kg. Then, it increased from 12 kg to 13 kg, so it increased 1 kilogram. I thought that (showing daylight table) there may be 3 kg- increase. That's why I thought the line 11 is 16 kg.

Irem: But you put line 11 there....

Demet: Yes, because, line 11 is ... Look at the table (showing the table)! According to what I did it would increase 1 kg. Here (showing the results in Week 11 on an A4-size paper), If I found it as 16, this time in Week 12 it would be 17, as it is increasing.

Researcher: How much does it increase?

Demet: Well, look. It increased from 9 kg to 12 kg, and from 12 kg to 13 kg (an increase of 1 kg). If it increased 3 kg from 13 to 16 kg; it forms a pattern. According to what I did, it is 12 kg in the 17th week. These are the (showing the paper) estimated values. Let's look at the second row. It increased 3kg here, so it will increase 3kg. It means the increase will be 3 kg next time... 14 (she adds 3, and it becomes 17).



Fasulye Problemi

Çiftçi Ahmet Amca kuru fasulye yetiştirirken hangi ışık koşulunun daha iyi bir tercih olduğuna karar vermeye çalışmaktadır. Çiftçi Ahmet Amca karar verirken yardımı olacağını düşündüğü için **kuru fasulye bitkisi** yetiştiren Çiftçiler Birliğini ziyaret etmiş ve **iki farklı ışık koşulu** kullandıklarını görmüştür. İki farklı ışık koşulu;

1) Fasulyeleri açık havada **gün ışığında** yetiştirme
2) Fasulyeleri sadece **gölge altında** yetiştirme.

Çiftçiler Birliği **sekiz hafta** sonunda, kuru fasulyelerin ağırlığını ölçmüş ve kayıt etmişlerdir. **Gün Işığında** ve **Gölgede** olmak üzere 4 sıra fasulye yetiştirmişlerdir.

GÜN IŞIĞINDA			
Kuru Fasulye Bitkisi	6. Hafta	8. Hafta	10.Hafta
Sıra 1	9 Kg	12 Kg	13 Kg
Sıra 2	8 Kg	11 Kg	14 Kg
Sıra 3	9 Kg	14 Kg	18 Kg
Sıra 4	10 Kg	11 Kg	17 Kg

GÖLGEDE			
Kuru Fasulye Bitkisi	6. Hafta	8. Hafta	10.Hafta
Sıra 1	5 Kg	9 Kg	15 Kg
Sıra 2	5 Kg	8 Kg	14 Kg
Sıra 3	6 Kg	9 Kg	12 Kg
Sıra 4	6 Kg	10 Kg	13 Kg

Figure 3. Weekly increases in Bean Weight.

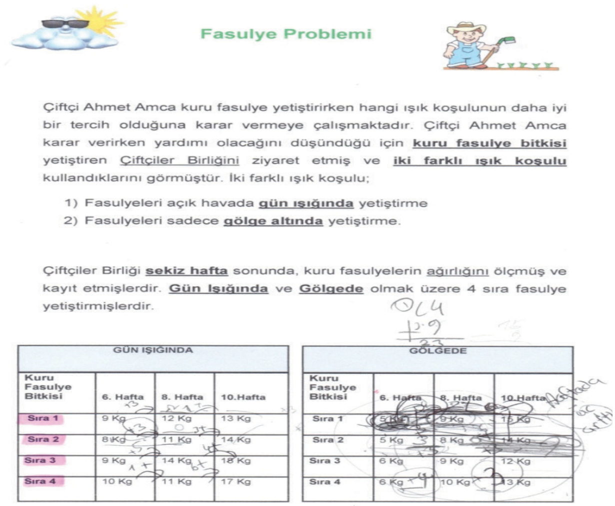
In the above quotes, based on the results of the first part of the problem in the modeling process, Demet stated that the weight of beans grown in daylight in Week 12 would be heavier than those grown in the shade. She expressed that Week 12 was missing from the table and they had to know the weight of the beans in Week 11 to estimate the values for Week 12. In this way, Demet said that it would be easier to estimate the values for Week 12, and she tried to establish a model. Demet investigated the weekly increase in the weights of the beans and noticed the patterns of a 3-kg increase for Weeks 6-8, and a 1-kg increase for Weeks 8-10 (Figure 3). Similarly, the increase in the 11th week should be 3 kg, and thus the number in line 1 for Week 11 should be 16 (by adding 3 kg to 13). Demet, who explained Week 11 to Irem, expressed that the value in the 12th week could be found by adding 1 to the value in Week 11. Demet, who expressed that the pattern developed by using the increases in Weeks 6 and 8 from the first row could be used similarly for the second row, calculated the values for Week 12 by adding 3 kg to Weeks 10 and 11, respectively. A new discussion arose between Irem and Demet about the pattern as follows:

> Irem: What I am doing at the moment is writing the numbers between them.
>
> Researcher: Numbers between them?
>
> Irem: I mean the increase in kilograms.
>
> Demet: I found the values for Weeks 11 and 12 by sequencing a pattern.
>
> Irem: +4, +4, 13, +3 (writing the weekly increase onto the chart).
>
> Demet: I found a pattern in Row 3 (showing Row 3 of the Daylight Table).
>
> Irem: Which pattern did you find?
>
> Demet: 5, 4 (writing the weekly increases on the table)
>
> Irem: It seems 3+, 3+ but it goes 6 (Daylight Table Row 2 weekly increase).

The above quotes show that the students tried to find a pattern by working on the increases from the data tables. By exploring the direction of the relationship between numbers, each row was thought to have a different pattern for finding the values for Weeks 11 and 12. Irem continued to find separate patterns both for the Daylight and Shade Tables, and then discussed them with Demet as follows:

> Demet: I'm trying to find out the values in the daylight. In the third row, it increases 5 kg, but the increase is 4 kg for the 8th and 10th week. Therefore, it will be ......if …..
>
> Irem: The exact calculation of that (showing the Daylight Table) is like that. The increase can be 4 kg (pointing at Row 1 from the Shade Table). Four can be divided into two. The increase in the 6th and 7th week was 2, but when you divide it by 2, it equals four. But here, (showing Daylight Table) dividing 3 by 2 equals…, it is not a whole number.
>
> Demet: Normally, daylight….I had better find one more pattern. Anyway! 23 kg (finding Row 3, Week 11) ... 23,
>
> Irem: Could you tell me for daylight?

Demet: Daylight, the pattern in the first row, not a pattern: Week 11, 16 kg; Week 12, 17 kg. Well, Row 2, 20 kg; Row 3, 27. Now, I am going to find Row 4… (Showing the weekly increase between the 6th and 8th weeks) It is 1, the increase is 6; 18 plus 6 = 24.



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Sıra 3	6 Kg	9 Kg	12 Kg
Sıra 4	6 Kg	10 Kg	13 Kg

Figure 4. Dividing the amount of weekly increase by 2.

The above quotes show that Demet continued to apply the pattern to Row 3 by adding the amount of increases from Weeks 10 and 11. Irem, who tried to develop a different assumption, struggled to develop a new model based on the missing values from the odd weeks (Weeks 7 and 9) that were not shown between Weeks 8 and 10. This model involved the following steps: dividing the amount of weekly increases by two and distributing these values to the 7th and 9th weeks (Figure 4). Although Irem tried to find the values for Weeks 7 and 9 by dividing the weekly increases by two, she abandoned this model due to insufficient pre-knowledge of division (since 3 is not divisible without remainder). Demet continued to apply the pattern and found the bean weights for Weeks 11 and 12 in the daylight; meanwhile, Asya worked individually and developed a pattern as follows and shown in Figure 5:

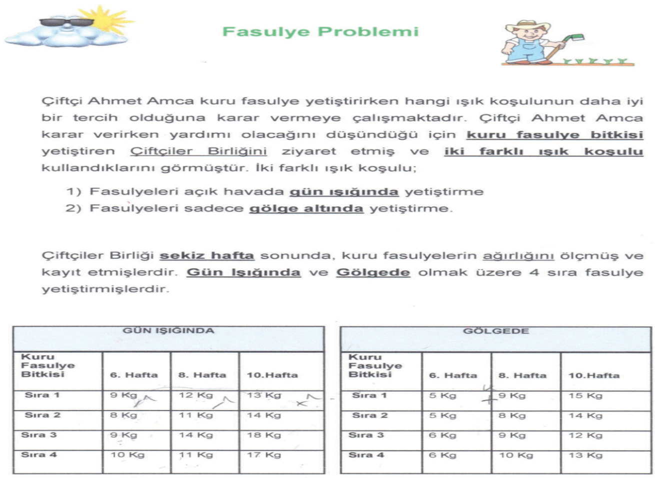
Asya: Now, I've calculated …, I was going to find the fifth row. But I did it with a different technique: these numbers include, -1, -1, +1. If I use a pattern, the value in the 11th week will be -1, for example…

Researcher: I did not understand.

Asya: Teacher, can I explain?

Asya: (Pointing to Daylight Table, Week 6, Rows 1 and 2), the decrease is 1. (Pointing to Daylight Table, Week 8, Rows 1 and 2) the decrease is 1 again. (Showing Daylight Table, Week 10, Rows 1 and 2) it increase from 12 to 13. If we follow this pattern, the table will be like this: (drawing a table on the back of the page) ... 11 and 12, it will be -1 (writing it under the 11th week), this will be -1 (writing it under the 12th week) in that -1, -1, +1. -1, -1, +1.

> Asya: I'll make use of my technique (as stated above, -1, -1, + 1). Daylight is seventeen! The pattern does not match!



Fasulye Problemi

Çiftçi Ahmet Amca kuru fasulye yetiştirirken hangi ışık koşulunun daha iyi bir tercih olduğuna karar vermeye çalışmaktadır. Çiftçi Ahmet Amca karar verirken yardımı olacağını düşündüğü için **kuru fasulye bitkisi** yetiştiren Çiftçiler Birliğini ziyaret etmiş ve **iki farklı ışık koşulu** kullandıklarını görmüştür. İki farklı ışık koşulu;

1) Fasulyeleri açık havada **gün ışığında** yetiştirme
2) Fasulyeleri sadece **gölge altında** yetiştirme.

Çiftçiler Birliği **sekiz hafta** sonunda, kuru fasulyelerin ağırlığını ölçmüş ve kayıt etmişlerdir. **Gün Işığında** ve **Gölgede** olmak üzere 4 sıra fasulye yetiştirmişlerdir.

GÜN IŞIĞINDA			
Kuru Fasulye Bitkisi	6. Hafta	8. Hafta	10.Hafta
Sıra 1	9 Kg	12 Kg	13 Kg
Sıra 2	8 Kg	11 Kg	14 Kg
Sıra 3	9 Kg	14 Kg	18 Kg
Sıra 4	10 Kg	11 Kg	17 Kg

GÖLGEDE			
Kuru Fasulye Bitkisi	6. Hafta	8. Hafta	10.Hafta
Sıra 1	5 Kg	9 Kg	15 Kg
Sıra 2	5 Kg	8 Kg	14 Kg
Sıra 3	6 Kg	9 Kg	12 Kg
Sıra 4	6 Kg	10 Kg	13 Kg

Figure 5. Vertical increases between rows for each week.

In the excerpt above, Asya developed a pattern by examining the vertical increases between rows for each week (Figure 5). She then recognized that she had estimated the fifth row; this was not the problem asked. She wanted to verify the validity of her pattern using the increase between rows for the sixth week. Using Demet's model, Asya added the amount of increases to the values for Weeks 10 and 11. In this way, she tested her model to support Demet's model with her own pattern; however, she figured out that it did not work.

Explaining the Result (Second part). For the second part of the problem, students developed a model that involved estimating the bean weights for Week 12. Demet and Irem wrote their reports about the Daylight and Shade Tables, respectively, and completed the modeling process (Figures 6 and 7), whereas Asya reorganized the written report from the first part (Figure 8) and read it to the researcher as follows:

> Asya: Can I read teacher? Heavy beans are better, when the tables are compared, except for one low and one equal value, it is the heaviest. We also chose it. Growing plants (beans) in the shade is not suitable. This is because Uncle Ahmet wants more product, so we didn't choose it.

The above quote shows that the students reached the desired results by comparing the weights in the tables. They correctly interpreted the problem (productivity) as more beans.

Explanation For Daylight

First of all: I found the numbers between the weeks.

Then: I formed a pattern to estimate the 11th and 12th row.

After that: Each row in the pattern has different numbers so I formed a pattern for each row.

Finally: I found the weights in the 11th and 12th rows and wrote down.

Figure 6. Report for the second part on the daylight table.

How we reached that conclusion (in the shade)?

We did the followings in order to find the values in the twelfth week;

There is a pattern here. As there are two weeks, we divided the amount of increase by two. For example: first row is 4, in this case, the increase is 2 in the 11th and 12th weeks.

Figure 7. Report for the second part on the shade table.

Appropriate	**Inappropriate**
In the day light	In the shade
Win	**Lose**
Reason: Because heavy beans are better. When compared the tables, there is only one equal and one less weight. So we chose this option.	**Reason**: Uncle Ahmet wants more products. That's why we chose this option.

Figure 8. Asya's revised report for the first part.

Analyzing the Modeling Process

The focus group, which consisted of the fourth-grade students, Asya, Demet and Irem, tried to understand the Butter Beans Problem and whether the better beans should be heavy or light. Group members tried to reach a conclusion by asking the researcher questions rather than discussing within the group. The group members who could not get help from the researcher compared the data tables (Weeks 6, 8, and 10) without understanding the problem. As a result of these comparisons, they concluded that the beans grown in daylight are heavier than those grown in shade for Weeks 6 and 8, but the beans grown in the shade are heavier than the others for Week 10. Therefore, they made their decision based on the farmer's request. Students' efforts to decide in accordance with the farmer's desire show that they had a hard time understanding the problem and making sense of the concept of greater yield. The researcher frequently intervened and asked the students questions. Afterwards, the students understood that the beans grown in daylight are heavier, and those grown in shade are lighter. Students only compared the weights of the beans in the same row according to the weeks in the Daylight and Shadow Tables, but they drew their conclusion without performing any mathematical operations. The researchers participated in group discussions at this stage, and encouraged students to express their thoughts more clearly. While one of the group members was writing their letter, the others expressed their thoughts individually and associated the problems with real life situations. Throughout the process, sometimes only one student worked on the problem while the others kept silent. Students also lost their attention and asked irrelevant questions. All these results show that the students had difficulty with teamwork and producing a common idea.

In the second part of the problem (estimating the weight of the beans for Week 12), the process started with focusing on the problem. Demet, who took charge, focused on the problem and pointed out that Week 12 is not included in the table. She then realized that the values for Week 12 could be found by using those from Week 11 and tried to develop a pattern in order to find values for Weeks 11 and 12. After finding the weekly increases from the tables, she found the value for Week 12 by adding the amount of increase from Weeks 10 and 11, respectively. The model they developed was applied to the Daylight and Shade Tables, and mathematical operations aimed at estimating the weight of beans for Week 12 were performed. In the meantime, Asya individually tried to develop her own model. Irem argued that the values for Weeks 7 and 9 could be estimated by averaging the amount of increase in Weeks 6 and 8. She did not use the model because odd numbers are not divisible evenly by two. She accepted the model developed by Demet and continued to apply it to the Table. Because this group of students had not been taught the concept of averages in previous grades, and Irem did not have prior knowledge on decimal fractions, she abandoned her model. On the other hand, it is important to note that they developed these new mathematical concepts themselves during the modeling process, even though they had not been taught beforehand.

In this process, Asya did not participate in group work and tried to develop a new model by herself. Unlike her friends, she found the weights of the beans in different rows; in other words, she examined the columns vertically and tried to develop a pattern. She abandoned her model because she realized that it did not work. Then, she continued the individual study of her model and applied her friends' model, expressing that she had developed a different strategy. In this way, she tried to verify her results by comparing them to those her friends had obtained. She abandoned this model because her results were not consistent with the results of Demet's model. In this process, the values in Week 12 were obtained using the model developed by Demet, and students completed the process by writing their letters to Uncle Ahmet.

Discussion

This study reveals that, as in English and Waters' (2004) study, the students in the focus group used cognitive or meta-cognitive thought processes while trying to draw a conclusion in a non-linear cycle for the Butter Beans Problem. On the other hand, they also encountered some challenges while dealing with the modeling process. In understanding the problem (the first stage of the modeling process), the students had difficulty making sense of some phrases in the problem. In this stage, students are expected to determine the factors of *greatest crop* and *best condition* in the given problem, but they focused on qualitative characteristics rather than quantitative ones. When they imagined the weight of the beans, they related them to the different situations they had encountered in their own daily lives (tin box, packaged, or opened box). These different interpretations prevented students from simplifying the problem. This result is also supported by the research of English and Watters (2004), who found that students had problems understanding and interpreting the problem at the very beginning of the process.

Another result is that they individually tried to find a quick answer rather than first fully understand the problem as a group. In other words, there was no group attempt to understand the problem, no questions were asked to each other, and no effective variables were identified while solving the problem. In the process, having students ask the researcher questions rather than perform in-group discussions could be attributed to the use of teacher-centered instruction, as in the study by Eraslan (2012).

In the stage of establishing the model, the students in the group made quantitative and qualitative data comparisons by using the tables in the first part of the problem. Although they figured out that beans growing in daylight are not always heavier, they ignored the situation where the weight of the beans was lighter; they accepted the final state as correct without making any changes. Students, by wanting to draw conclusions quickly without spending enough time verifying it, could be the cause of these difficulties (Blum & Ferri, 2009). On the other hand, they frequently related their models with situations they had encountered in daily life, trying to verify whether their choices were correct or not. In the second part of the problem, the group established four different models and attempted to predict the unknown data in the table by using patterns. They abandoned the first three models due to the fact that their previous knowledge was insufficient and that the new or revised model did not meet the desired valid results. English and Watters (2005) stated that students' informal knowledge could be helpful while solving the problem or it could constitute an impediment.

During group work, the students experienced focusing problems and often took breaks. Some students were also observed to be unable to focus on the subject during group discussion and wanted the group to be quiet; these ones left the group to work alone in different corners of the room. In addition, one student in the group often came to the forefront during group work while others just listened to her or studied on their own. In these cases, the researcher often interrupted the group work and asked them to focus on the problem, emphasizing the importance of group work and encouraging students to generate their own ideas and express their own thoughts more clearly.

In the stage of using mathematics, the students were successful at making different mathematical calculations for obtaining mathematical results. During the first part of the problem, the students made comparisons between the rows and columns using verbal expressions, not mathematical operations. During the second part of the problem, they made different mathematical calculations as a result of different assumptions. Students developed a pattern for Week 12 by calculating the increases from Weeks 6 to 10. Although the average concept was not learned previously in this process, the students constructed and used this concept. Similarly, English and Waters

(2004) stressed in their study that students had intuitively expressed the notions of averaging and aggregating. This means that these modeling problems, as indicated in Chamberlin's (2004) study, allow students to create new learning environments and develop new mathematical thinking.

In the stage of expressing results, the students wrote down what they had thought and how they had reached the result throughout the process. In the first part of the problem, except for the student responsible for explaining the result, the other group members expressed their individual thoughts to the researcher. The students tended to validate their results by relating them to real-life situations. On the other hand, instead of studying individually, they worked in a group and shared responsibility for the second part of the problem. They wrote their names and surnames at the bottom of the letter. This supports the idea that students are still willing to study individually rather than work together.

As in English's (2006) study, the current study's results show that students can successfully develop mathematical ideas, identify factors related to the problem, make and revise different assumptions based on newly generated ideas, create models and test them, develop new strategies, find patterns, and interpret them. Students were also observed to be ready to use mathematical language, interact socially, work with model-eliciting problems, question assumptions, and interpret the data given in tables. Moreover, the students tried to verify the validity of their models by relating them to real life-situations. Similar to the study of English and Watters (2005) and English (2002, 2009), students were shown to have related their models to real-world situations and to have accessed new mathematical ideas. On the other hand, this study also shows that students had several difficulties in understanding the problem, in developing appropriate models based on assumptions, in identifying the relationships among the components of a qualitative variable, in associating variables with each other, in using the appropriate mathematical operations, and in working in groups. This can be because of the limited number of activities involving student collaboration, interpretation, and new-idea generation in and outside of school (Kant, 2011). In general, in order to overcome the difficulties encountered by students, learning environments should be established where model-eliciting problems encourage students to use mathematical language and interpret real-life situations. For this reason, model-eliciting problems can also be included in primary-mathematics curriculum, as well as middle-school curriculum. Therefore, the elective, Mathematics Applications Course included in the fifth grade in middle school should be extended in such a way that it can be included in primary-mathematics curriculum beginning with the first grade. Implementing data-modeling problems in primary school can help students at this age be ready for model-eliciting problems (English, 2013a, 2013b). In addition, implementing interdisciplinary model-eliciting problems that allow students to realize the relationship between mathematics and other disciplines may help students develop a positive attitude towards mathematics (English, 2013b). Until this elective course (Mathematics Applications) is put into practice as a main primary-school course, at least one model-eliciting problem should be added to the end of each unit in primary school mathematics textbooks. This will provide a significant contribution to students' creativity, high-level thinking skills, communication skills, and social development.

The results of this study are limited to the thought processes of three 4th grade students in a focus group, the selected Butter Beans Problem, and the research method that was used. New research studies on model-eliciting activities should involve all pre-school, primary-school, and middle-school grades. In addition, investigating model-eliciting processes and the effect of modeling on changes in opinions and thoughts towards mathematics will contribute to the enrichment of the national literature.

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Appendix 1

The Butter Beans Problem (Doyle, 2006)

Uncle Ahmet the Farmer is trying to decide which lighting conditions are better for producing beans. Uncle Ahmet visits the Farmers' Association, which grows dry beans and offers help on deciding lighting conditions. He realizes they use two different lighting conditions:

1) Growing beans outdoors in direct sunlight

2) Growing beans only under shade.

The Farmers' Association measured and recorded the weight of dry beans at the end of 10 weeks. They grew beans in the light and in the shade.

DAYLIGHT			
Butter Bean Plants	**Week 6**	**Week 8**	**Week 10**
Row 1	9 Kg	12 Kg	13 Kg
Row 2	8 Kg	11 Kg	14 Kg
Row 3	9 Kg	14 Kg	18 Kg
Row 4	10 Kg	11 Kg	17 Kg

SHADE			
Butter Bean Plants	**Week 6**	**Week 8**	**Week 10**
Row 1	9 Kg	12 Kg	13 Kg
Row 2	8 Kg	11 Kg	14 Kg
Row 3	9 Kg	14 Kg	18 Kg
Row 4	10 Kg	11 Kg	17 Kg

Your first investigation

Using the data above, determine which lighting conditions are better suited to butter beans for produce the greatest crop. In a letter to Uncle Ahmet the Farmer, outline your recommended lighting condition and explain how you arrived at this decision.

Your second investigation

Predict the weight of butter beans produced in Week 12 for each light conditions. Explain how you made your prediction so that Uncle Ahmet the Farmer can use it for other similar situations.