Research Article

A New Problem-Posing Approach Based on Problem-Solving Strategy: Analyzing Pre-service Primary School Teachers' Performance

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Abstract

This study examined pre-service primary school teachers' performance in posing problems that require knowledge of problem-solving strategies. Quantitative and qualitative methods were combined. The 120 participants were asked to pose a problem that could be solved by using the find-a-pattern a particular problem-solving strategy. After that, task-based interviews were conducted with 5 of the 120 participants who had answered in different ways. The data obtained were analyzed using semantic, descriptive, and content analysis methods. It was determined that 55% of the participants could pose a word problem that can be solved using the desired strategy. The other participants displayed three forms of difficulty: some posed problems that required an irrelevant strategy, some were unable to offer any answer, and some suggested problems that involved simply finding a general rule of a pattern. Because of the many pedagogical benefits associated with problem posing, pre-service teachers should be educated in problem-solving strategies and problem and types so that they can apply problem-posing skills effectively in primary schools.

Keywords

Problem posing • Word problems • Problem-solving strategy • Find-a-pattern strategy • Pre-service primary teachers

Problem posing has been described as the creation of new problems or the reformulation of a given problem (Tichá & Hošpesová, 2009) in order to explore and solve a given situation (Silver 1994). Stoyanova and Ellerton (1996, p. 518) similarly defined problem posing as "the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems." According to Christou, Mousoulides, Pittalis, Pitta-Pantazi, and Sriraman (2005, p. 149), "Problem posing is an important aspect of both pure and applied mathematics and an integral part of modeling cycles which require the mathematical idealization of real-world phenomena." Problem posing is one of the many components of problem solving. For the last two decades, researchers have considered the relationship between problem posing and problem solving (Cankoy & Darbaz, 2010; Christou et al., 2005; Işık, 2011; Kılıç, 2013a, Lowrie, 2002; Luo, 2009; Stoyanova & Ellerton 1996; Tichá & Hošpesová, 2009; Toluk-Uçar, 2009), and the results of previous studies have generally shown that problem posing aids problem solving (Cankoy & Darbaz, 2010; Lowrie, 2002; Stoyanova & Ellerton 1996).

Problem posing involves many skills, such as formulating everyday problems and mathematical situations, selecting a suitable approach to a mathematical situation, and recognizing relationships among different mathematics topics (Abu-Elwan, 1999). In recent years, many educators have focused on research related to problem posing because of the benefits it provides.

Benefits of Problem Posing

Problem posing is an effective mathematical activity that can help people to construct mathematical knowledge through integrating their existing structures of knowledge. Van-Harpen and Presmeg (2013) found a relationship between students' mathematical knowledge and their problem-posing abilities. Posing problems benefits students, pre-service teachers, and teachers in multiple ways. First, it can serve as a measure of the curriculum's effect on students' learning (Cai et al., 2012). Second, it is a tool for studying cognitive processes (Mestre, 2002) and helps students to expand their understanding of mathematics, as well as to explore the nature of problems rather than focusing only on arriving at solutions (Stoyanova, 2003). Further, it develops and strengthens students' critical thinking skills (Nixon-Ponder, 1995) and demonstrates deep understanding of a concept (Rizvi, 2004). For these reasons, problem-solving instruction that is based on problem posing can foster understanding (Cankoy & Darbaz, 2010).

As for the benefits gained by teachers, problem-posing tasks provide insights into how students construct their mathematical understanding and can thus be a useful assessment tool (Lin, 2004). Problem posing in elementary classrooms positively

influences teachers' beliefs about mathematics itself and mathematics instruction (Barlow & Cates, 2006). Stoyanova (2003) further stated that problem posing develops students' understanding of mathematics and that their ability to understand depends on teachers' ability to incorporate problem-posing activities into mathematics lessons.

Problem posing also contributes to the development of mathematics knowledge during the pre-service training of primary school teachers (Tichá & Hošpesová, 2009). It positively affects their concept of what it means to know mathematics (Toluk-Uçar, 2009), which might in turn help to improve their mathematical knowledge (Kılıç, 2013a). Furthermore, this approach motivates pre-service teachers to be active learners and to rethink mathematical objects and concepts without explicit instruction (Lavy & Shriki, 2010). Finally, Lowrie (2002) indicated that students' problem-posing actions can be nurtured by teachers' actions; therefore, it is important to educate teachers in this regard during their training.

Problem-posing Frameworks

The literature classifies problem-posing situations in several different ways (Christou et al., 2005; Kılıç, 2013b; Silver & Cai, 1996; Stoyanova & Ellerton, 1996). These different classification frameworks can be applied to analyze problem-posing performance and understand problem-posing processes. Silver (1994, p. 523) classified problem-posing situations according to whether they take place before, during, or after problem solving. Silver's three categories were (a) presolution posing, in which one generates original problems from a presented stimulus situation; (b) within-solution posing, in which one reformulates a problem as it is being solved; and (c) postsolution posing, in which one modifies the goals or conditions of an already solved problem to generate new problems.

Stoyanova and Ellerton (1996) offered a different three-category framework that distinguishes forms of problem posing by task:

- In *free problem posing,* students are asked to pose a problem based on a natural situation, such as "make up a difficult or a money problem."

- In *semi-structured problem posing,* students are given an open situation and are invited to explore the structure or to finish it. Examples of this type can include posing problems based on pictures or equations.

- *Structured problem-posing* situations occur when a well-structured problem or problem situation is given and the task is to construct new problems. Restating a problem based on its solution and presenting a problem in different informational formats are frequent examples of this form of problem posing (Stoyanova, 2003).

Christou et al. (2005) created a problem-posing model, based on the findings of Stoyanova and Ellerton (1996), that includes four processes: editing, selecting, comprehending, and translating. In editing processes, quantitative information is mostly associated with tasks that require students to pose a problem without restrictions. In selecting processes, quantitative information is associated with tasks that require students to pose problems or questions that are appropriate for specific answers. In comprehending processes, quantitative information refers to tasks in which students pose problems starting from given mathematical equations or calculations. In translating processes, quantitative information requires students to pose appropriate problems or questions based on graphs, diagrams, or tables.

Kılıç (2013b) presented a problem-posing model that combined the frameworks of Stoyanova and Ellerton (1996) and Christou et al. (2005) and encompassed the different forms of representations (symbolic, tables, pictures, etc.). According to Kılıç, problem-posing frameworks could be defined as free problem posing (posing a difficult problem and a given topic such as fraction problems), semi-structured (editing and translating), and structured (comprehending and selecting). The frameworks related to problem posing include mathematical topics, representations, and mathematical situations, but studies of problem posing based on particular problem-solving strategies have been lacking. In the present study, a new approach that relates problem posing to the problem-solving process was proposed and analyzed.

Problem Types

The many different types of mathematical problems can generally be divided into two characteristic groups: story or word problems and process problems (Souviney, 1994). Word problems can be solved immediately by selecting and applying one or more operations, whereas solving process problems requires more flexible thinking and better organizational skills (Souviney, 1994). Olkun, Şahin, Akkurt, Dikkartın, and Gülbağcı (2009) further classified story problems into standard and non-standard problems. A standard problem is as follows: "Sinem has three dolls. She received four more dolls for her birthday. How many dolls does she have now?" Such a problem can be solved using one operation (i.e., $3 + 4 = 7$). By contrast, a non-standard problem is as follows: "A counter clerk bought tickets with serial numbers starting at 12. He sold the 52nd ticket and then handed in the ticket counterfoils. How many tickets did he sell?" The solution is 41 tickets (i.e., $52 - 12 = 40$, $40 + 1 = 41$).

Cankoy (2003) classified problems as symbolic equations, story problems, and word equations. $(102 - 66)/6 = ?$ is an example of a symbolic equation. "What number is multiplied by six and added to 66 to equal 102?" is a word equation, whereas "I have 20 pencils and my mother gave me 30 pencils. How many pencils do I have?" is an example of a story problem.

4-Step Process for Solving Problems and Problem-solving Strategies

The problem-solving process consists of four sequential phases (Cathcart, Pothier, Vance, & Bezuk, 2003; Holmes, 1995; Souviney, 1994): The process is explained as below;

- *Understanding problem:* this phase is crucial for exact solution and involves grasping the problem situation, determining and deciding facts and the intended goal.

- *Devising a plan:* this phase occurs after understanding a problem.

- *Carrying out the plan:* after devising a plan for problem solving in this phase that plan must be carried out carefully.

- *Looking back:* in this phase the solution is assessed and computation is checked.

Knowledge of problem-solving strategy affects all parts of the problem-solving process. Such strategies can be specifically taught, and when they are taught, not only are they used more often, but students also achieve correct solutions more frequently (Holmes, 1995; Reys, Suydam, Lindquist, & Smith, 1998). Teaching problem-solving strategies gives students a wider array of tools to solve problems.

The literature contains many common problem-solving strategies: construct a table or chart, find a pattern, draw a picture or diagram, solve a simpler problem, guess and check, work backward, write an open sentence, logical reasoning, make a systematic list, construct a general rule, or add something to the problem situation (Cathcart et al., 2003; Souviney, 1994). Among these strategies, finding a pattern is particularly important due to its relation to patterns as a mathematical topic, and it is one of the most frequently used strategies. Smith (1997) categorized patterns as numerical (involving numbers) or non-numerical (involving shapes, sounds, or other attributes such as color and position), whereas Lin et al. (2004) classified geometric patterns as linear and quadratic.

In some studies, patterns are classified as repeating or growing (Cathcart et al., 2003; Reys et al., 1998; Van de Walle, 2004; Warren & Cooper, 2006). Zazkis and Liljedahl (2002) described the classifications of numerical patterns, pictorial or geometric patterns, patterns in computational procedures, linear and quadratic patterns, and repeating patterns. Repeating patterns have a recognizable cycle of elements, referred to as the "unit of repeat" (Zazkis & Liljedahl, 2002). This kind of pattern can have one attribute such as the color, size, shape, or orientation of objects (Threlfall, 1999). Repeating patterns can be found in alphabetic letters such as A-B-A-B-A-B, geometric shapes such as ▼●▼●▼● and actions such as stand, sit, stand, sit, stand, sit, stand, sit (Warren & Cooper, 2006). Growing patterns, meanwhile, change over time (Cathcart et al., 2003) and may be linear such as Y B B Y B B B B Y B B B B B

B B (in this example, only the B's are growing; Reys et al., 1998) or quadratic such as n2 squares. Hence, repeating and growing patterns are systematic configurations that include any types of representations such as shapes, symbols, and real-world objects according to their cognitive structures or schemas. Souviney (1994) asserted that looking for numerical and geometric patterns often provides clues to the structural relationships in a problem situation. One problem that encourages use of the find-a-pattern problem-solving strategy is the question, "How many squares of different sizes are there in a 7 x 7 square?" During that problem-solving process, people will discover patterns among the number and size of various squares such as 1×1, 2×2, etc. (Holmes, 1995).

Studies have shown that pre-service teachers prefer different types of problem-solving strategies and face various challenges when attempting to solve problems (Duru, Peker, Bozkurt, Akgün, & Bayrakdar, 2011; Van Dooren, Verschaffel, & Onghena, 2003). In the study by Van Dooren et al. (2003), the solutions applied by pre-service teachers were quite diverse; one subgroup tended to apply exclusively arithmetic methods (which led to failure on the most difficult word problems), whereas another subgroup was more adaptive in its strategy choices. Duru et al. (2011) found that pre-service primary school teachers' strategy preferences when solving word problems included arithmetic and algebraic approaches, guess and check, find a pattern, and modeling. Moreover, they observed that pre-service teachers who preferred the find-a-pattern strategy made more mistakes than those who preferred using a model.

Problem Posing and Solving in the Turkish Mathematics Curriculum

In all Turkish primary schools, teachers follow the same standards, contained in the national mathematics curriculum established by the Ministry of National Education. Since 2005, the mathematics curriculum has emphasized problem-posing applications, especially from first to fifth grade. The curriculum's approach is based on the constructivist philosophy of learning and asserts that problem-posing abilities should be developed daily using mathematical situations (Milli Eğitim Bakanlığı [MEB], 2009). In this curriculum, there are learning areas, sub-learning areas, objectives, samples of activities, and explanations of activities. Learning areas are subdivided into number, data, geometry, and measurement; problem posing is incorporated into the number and measurement learning areas. The Turkish mathematics curriculum asserts (MEB, 2009) that learning mathematics includes basic concepts and skills as well as thinking about mathematics and the acquisition of general problem-solving strategies. It highlights appreciation of mathematics as an important tool in real-life situations. In the curriculum, it is expected that students will solve and pose problems.

Problem-posing Activities with Pre-service Teachers

Studies related to problem posing have explored many aspects of the issue. The substantial body of research that has focused on structured problem-posing situations has been used to determine pre-service teachers' problem types (Goodson-Espy, 2009; Işık, 2011; Luo, 2009; Rizvi, 2004; Toluk-Uçar, 2009). Those studies examined pre-service teachers' problem posing on mathematical topics such as fractions and operations using fractions. Furthermore, considering that the problem-posing actions of students can be nurtured by teachers' actions (Lowrie, 2002) and that teachers will have to teach their students the practice of problem posing, it is important to understand pre-service teachers' performance in problem posing as a means of solving problems, so as to educate them effectively during their years of training. Moreover, pre-service teachers should be capable of posing meaningful mathematical problems and correcting students' problem-posing efforts (Lavy & Shriki, 2010; Tichá & Hošpesová, 2009; Toluk-Uçar, 2009). It has been asserted in several studies that pre-service teachers often have difficulties related to problem-posing activities (Korkmaz & Gür, 2006; Luo, 2009; Toluk-Uçar, 2009). For example, pre-service primary and mathematics teachers were found to pose word problems that were mainly derived from mathematics textbooks and rarely reflected creativity (Korkmaz & Gür, 2006), pre-service elementary teachers were unable to construct appropriate word problems when given particular symbolic expressions (Luo, 2009), and pre-service teachers had difficulty in generating a conceptually correct representation of statements presented to them (Toluk-Uçar, 2009). In previous studies, structured problem-posing situations were used to determine pre-service teachers' preferred approaches (Goodson-Espy, 2009; Işık, 2011; Luo, 2009; Rizvi, 2004; Toluk-Uçar, 2009).

Most studies on problem-posing frameworks have focused on mathematical topics or representations (Christou et al., 2005; Kılıç, 2013b; Stoyanova & Ellerton, 1996) or on the time relationship between problem posing and problem solving, i.e., before, during, or after (Silver, 1994). No studies have directly examined the role of problem-solving strategies within the problem posing activities.

The Aim of the Research

This study analyzes pre-service primary teachers' ability to pose problems that can be solved by a specific problem-solving strategy. There is insufficient research on how pre-service teachers perform in posing problems that require knowledge of problem-solving strategies. The present study seeks to fill that research gap by examining the performance of pre-service teachers in posing problems that call for the find-a-pattern problem-solving strategy.

This approach was chosen because of the belief that pre-service primary teachers' posed problems could reflect their mathematical knowledge of problem-solving

strategies, patterns, and problem posing. Moreover, the study was expected to facilitate understanding of participants' knowledge of problem-solving strategies, so that this skill could be assessed and then more effectively strengthened. The goal is to find ways to help teachers focus not just on mathematical topics, contexts, or processes when posing problems, but also on what kind of problem-solving strategies could be used to solve the problems posed.

The study addresses the following research questions: (1) What kinds of problems that can be solved by using the find-a-pattern problem-solving strategy are posed by pre-service primary school teachers? (2) What kinds of issues do pre-service primary school teachers encounter when attempting to pose problems that involve knowledge of problem-solving strategies? The study also provides insights into the similarities and differences between pre-service primary teachers' performances.

Method

This study engaged in data triangulation by using qualitative research methods to support the quantitative findings. In a triangulation-based research design, "the researcher simultaneously collects both quantitative and qualitative data, compares the results, and then uses those findings to see whether they validate each other" (Fraenkel & Wallen, 2005, p. 443). Data were collected using a two-step process. In the first step, all participants were asked to pose problems related to the find-a-pattern problem-solving strategy. In the second step, five volunteers who responded in different ways in the first phase of the research participated in task-based interviews. These interviews allowed the researcher to find out how problems were generated and to understand participants' process of integrating mathematical knowledge through analysis of their posed problems.

Participants

To prevent bias, participants were selected using a two-step sampling process. In the first sampling process, 120 participants who had taken Mathematics Teaching Course I and were attending Mathematics Teaching Course II participated in the study. In all courses, participants were taught problem-solving processes and strategies, patterns, problem types, and problem posing as separate topics. It was assumed that all participants already had basic knowledge of these ideas and could use their knowledge to create problems. Of these participants, 65 were female and 55 were male; all were 20 or 21 years old.

In the second sampling process, five volunteers were selected, using the maximum variation sampling technique, for task-based interviews to probe their posed problems in greater depth. The volunteers had posed different types of problems or displayed

difficulties that emerged frequently during the study. To protect confidentiality, the five interviewees were coded as P1, P2, P3, P4, and P5. P1 posed a repeating pattern problem, P2 offered a growing pattern problem, P3 posed an irrelevant problem, P4 did not give any answer, and P5 posed a situation that involved finding the general rule of a pattern. In the interactions quoted below, the researcher is identified as "I."

Data Collection

Participants were asked, "Could you pose a standard word problem that can be solved using the find-a-pattern problem-solving strategy?" Posing a word problem was chosen because of the frequent appearance of word problems in the mathematics curriculum. To confirm the suitability of this problem-posing task, a mathematics educator who has extensively studied problem posing, problem solving, and patterns was consulted. This educator indicated that the problem-posing situation used in this study was suitable for pre-service teachers. Furthermore, to understand the conformity, validity, and reliability of the task-based interview questions and problem-posing task, a pilot study was conducted with one pre-service primary school teacher. Had this pilot activity not been deemed sufficient to refine the problem-posing task and interview questions for the main study, additional pre-service teachers would have been engaged in the pilot study. As a result of the pilot study, as suggested by Goldin (2000), the questions were revised to prevent mathematical misconceptions and uncertainties as well as unexpected situations.

The questions used in the individual task-based interviews were open-ended to allow for the assessment of participants' thinking processes (Hunting, 1997). Questions included the following: "I asked you to pose a standard word problem that can be solved using the find-a-pattern problem-solving strategy, and you posed such a situation. Could you explain how did that?" "Do you think the problem that you posed is in accordance with the problem-posing task? Do you think it could be solved using the find-a-pattern problem-solving strategy? Why? Could you explain?" The interviews took 20 to 25 minutes and were tape-recorded.

Data Analysis

The data obtained from the problem-posing task were analyzed at two levels. The first level entailed semantic analysis; at the second level, descriptive analysis provided an overall picture of the problem types posed by participants. In the semantic analysis, the structure of the problems suggested by participants was analyzed in accordance with the find-a-pattern problem-solving strategy, to determine whether the problems could indeed be solved by this strategy. The problems and statements produced by participants were first listed and classified according to their semantic structures. The generated problems and mathematical statements were then coded in the categories

shown in Figure 1. A mathematics educator's opinions were considered in developing this framework and constructing Figure 1. As the figure explains, appropriate problems were of two types (repeating and growing patterns); the main issues affecting participants who posed inappropriate problems were irrelevant strategy, inability to answer at all, or selecting problems that involved finding a general rule.
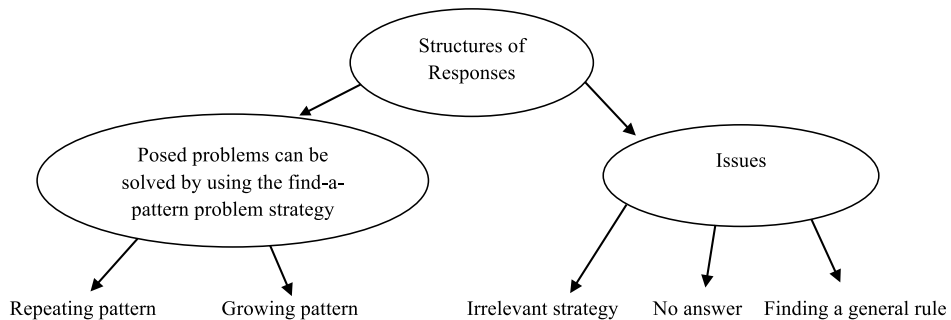

Figure 1. Framework for analyzing strategy-based problem-posing activity.

After the semantic analysis of the posed problems was complete, the frequencies and percentages were calculated for each category (Kılıç, 2013a). Next, the transcripts of task-based interviews were transcribed verbatim, and the data obtained from the interviews were analyzed using Miles and Huberman's (1994) data analysis model, which consists of three phases: data reduction, data display, and conclusion drawing/ verification. In the data reduction phase, the researcher coded the concepts and patterns considered important. Content analysis was used for coding data. Raw data were coded and categorized to capture the relevant characteristics from the interview transcripts. The posed problems were analyzed using a problem-posing diagram, including categories and subcategories developed by the researcher. Excerpts from the interviews were selected to illustrate each category and subcategory of response.

Validity and Reliability

To increase the reliability and validity of the study, the member checking technique was used as suggested by Lincoln and Guba (1985). Furthermore, the researcher asked for the opinion and assessment of one colleague who was unfamiliar with the data and unbiased regarding the code list and research findings. To examine interrater reliability, a colleague independently classified the posed problems. Agreement between the two raters occurred in 95% of cases. The pilot study also contributed to ensuring the validity and reliability of the problem-posing task.

Findings

The structures of the responses produced by participants are given in Table 1.

Table 1
The Situations Posed by Participants and Their Relationship to the Find-A-Pattern Problem-Solving Strategy

Categories	Subcategories	Frequency (N = 120)	Percentage %
Posed problems can be solved using the find-a-pattern problem-solving strategy	Growing pattern	66	55
	Repeating pattern	17	14.17
Total		83	69.17
Issues			
Posed problems or situations cannot be solved using the find-a-pattern problem-solving strategy	Irrelevant strategy	17	14.17
	No answer	13	10.83
	Finding a general rule	7	5.83
k		37	30.83

Among the responses, 69.17% could be solved using the find-a-pattern problem-solving strategy and 30.83% could not.

Posed Problems that could be Solved Using the Find-a-pattern Problem-solving Strategy

Growing pattern. This pattern occurred in 55% of all cases. One example follows:
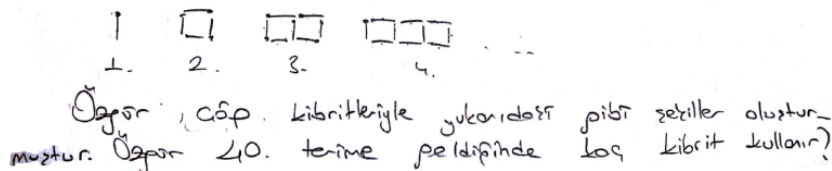
I: I asked you to pose a standard word problem that could be solved using the find-a-pattern problem-solving strategy, and you posed such a situation. Could you explain how you did that?

P2: I needed a problem that could be solved using this strategy. For that reason I imagined problem types and patterns. I thought I could write a problem including a situation that grows in an orderly way. I drew a figural pattern consisting of matchsticks. First I drew one matchstick, for the second term of the pattern I drew 4, for the third term I drew 7, and so on. Then I wrote a problem based on those figures, asking the student to produce the 40th term of the pattern.

I: Do you think that the problem can be solved using the find-a-pattern problem-solving strategy? Why? Could you explain?

P2: As I mentioned before, that problem can be solved using a find-a-pattern problem-solving strategy because there is an order in the terms of the pattern and I asked for the 40th term of the pattern. Also, I am sure that it is a word problem.

Repeating pattern. In 14.17% of all cases, repeating pattern problems were offered. A problem example and an excerpt from the related interview follow.

I: I asked you to pose a standard word problem that could be solved using the find-a-pattern problem-solving strategy, and you posed such a situation. Could you explain how you did that?

P1: The task asked us to pose a problem that can be solved using the find-a-pattern problem-solving strategy. For that reason, I wrote a problem that can be solved using the strategy mentioned in the task.

I: Do you think that the problem you posed is in accordance with the problem-posing task? Why? Could you explain it?

P1: I am sure that the problem that I posed is related to the task. I wrote a word problem that can be solved by the find-a-pattern strategy. Moreover, there is a repetitive situation. I asked, "My brother went into the army. Today is Sunday, and he will come back 57 days from now. On which day of the week will my brother come back?" To solve that problem, the problem solver should be considering the repetitive situation in the problem.

Posed Problems or Situations that could not be Solved Using the Find-a-pattern Problem-solving Strategy

On the other hand, 30.83% of the participants did not produce a problem that could be solved using the desired strategy. Some could be solved with other strategies, and a few responses involved simply finding a general rule. Furthermore, some participants could not come up with any answer at all.

Irrelevant strategy. In 14.17% of all cases, problems were offered that could be solved with other strategies but not with the find-a- pattern strategy. An excerpt from the interview with P3, which contains the problem, follows:

I: I asked you to pose a standard word problem that can be solved using the find-a-pattern problem-solving strategy. You proposed this problem: "In a town all houses' numbers will be painted. It costs 2 liras to paint each digit on the house numbers. Six hundred liras will be used for painting. So, how many houses are there in the town?" Could you explain how you posed this problem?

P3: I posed a word problem and I tried to write a solvable problem.

I: Do you think the problem that you posed is in accordance with the problem-posing task? Why? Could you explain?

P3: In fact, I am not sure that the problem can be solved.

I: What do you think about that problem? Do you think that the problem can be solved using the find-a-pattern problem- solving strategy?

P3: I think that the problem cannot be solved using that strategy. The problem can be solved by dividing the problem into sub-problems.

No answer. Thirteen participants (10.83%) were unable to write any problem in response to the task assignment. An interview with one such person elicited the following information:

I: I asked you to pose a standard word problem that could be solved using the find-a-pattern problem-solving strategy, but you did not produce any situation. Could you explain why?

P4: First of all, I imagined that I would write a problem, but that problem was not an ordinary problem. To develop a word problem that could be solved using the find-a-pattern problem-solving strategy was a very difficult task for me.

I: Why do you think so?

P4: In fact, I know the pattern-based problem-solving process and how to pose a problem. Of course I could produce many problems, but I could not write any situations involving what you asked. I could not do that although I thought through all the topics that we have learned. So I did not write anything.

Finding a general rule of a pattern activity. Seven responses (5.83%) were related to finding a general rule of a pattern, instead of posing a word problem that could be solved using the desired strategy. Following is an excerpt from the interview with participant P5:

I: I asked you to pose a standard word problem that could be solved using the find-a-pattern problem-solving strategy, and you posed such a situation. Could you explain how you did that?

P5: I wrote a number pattern. I generated the first five stages of the 2, 10, 30, 68, 130, ...? number pattern and I asked for the general rule of that number pattern.

I: Do you think the problem that you posed is in accordance with the problem-posing task? Why? Could you explain it?

P5: I think it is related to the question that you asked but I think it is not a word problem. It is a problem only, or just a pattern of numbers.

I: Is that situation that you produced a word problem or not?

P5: It seems to involve producing a number pattern rather than writing a word problem. It is related to just finding a general number pattern.

Discussion and Conclusions

Problem posing is a crucial skill and an effective mathematical activity that has many benefits for students as well as for pre-service and in-service teachers. Furthermore, it can reveal individuals' ability to integrate mathematical knowledge and serves as a good approach to analyzing mathematical knowledge and displaying the (mis)conceptions of participants. It has been indicated that teachers' ability to incorporate problem-posing activities into mathematics lessons affects students' understanding of mathematics (Stoyanova, 2003) and that the problem-posing actions of students can be nurtured by teachers' actions (Lowrie, 2002). Since teachers' problem-posing performance can affect their students' level of achievement and since content knowledge is an important knowledge component (Shulman, 1986), it is crucial to educate pre-service teachers on problem posing.

Since problem-posing activity is a good tool to assess the knowledge and skill of pre-service teachers, in this study 120 such persons were asked to write a word problem that called for use of the find-a-pattern problem-solving strategy. More than half of the participants successfully posed word problems representing growing and repeating pattern types. In this way, they integrated their mathematical knowledge regarding patterns and problem solving. Problems involving a growing pattern were much more common than those containing a repeating pattern. The choice of problems can be explained by pre-service teachers' knowledge of patterns and problem-solving strategies, their imagination or creativity, their experience with problem solving (Chapman, 2012), and their general educational experience (Tichá & Hošpesová, 2012).

In this study, pre-service primary teachers faced several common difficulties. Some posed problems that required other strategies to solve them, some were unable to produce any problem, and some wrote problems that involved simply finding a general rule for a pattern. Of these three difficulties, the first (i.e., posing problems that cannot be solved by using the find-a-pattern strategy) was the most frequent. It can be concluded that these pre-service teachers could produce problems but lacked sufficient knowledge of problem-solving strategies. Moreover, pre-service primary school teachers might have never tried or been asked to pose problems related to problem-solving strategies in their academic or professional lives, and thus they may prefer to pose problems that can be solved by using other strategies or by finding a general rule. These results suggest that in methods courses, more time should be devoted to teaching problem types, problem-solving strategies, and problem posing.

Understanding how pre-service primary school teachers respond when asked to pose problems related to a specific problem-solving strategy could make an important contribution to the improvement of teacher education courses. In this vein, during

the training provided to pre-service teachers, many problem-posing activities that include various problem-solving strategies could be incorporated. Previous studies have asserted that pre-service teachers should participate in problem-posing activities (Abu-Elwan, 1999; Contreras, 2007; Işık, 2011; Kılıç, 2013b; Lavy & Bershadsky, 2003; Luo, 2009; Toluk-Uçar, 2009) and that problem posing should be a main activity in teacher education courses within undergraduate education programs (Abu-Elwan, 1999; Barlow & Cates, 2006; Kılıç, 2013a; Korkmaz & Gür, 2006; Rizvi, 2004; Tichá & Hošpesová, 2012).

Korkmaz and Gür (2006) went further, contending that a course related to problem posing should be built into teacher education programs to enhance these skills in pre-service teachers. Since subject matter content knowledge includes the structure of the subject matter, it is important to include information on how the basic concepts and principles are organized (Shulman, 1986). As Quinn (1997) indicated, teachers who have inadequate meaningful mathematical content knowledge and/or poor attitudes toward the subject often exacerbate students' difficulties in learning mathematics. Chen, Van Dooren, Chen, and Verschaffel (2011, p. 923) stated, "The completeness, correctness and coherence of both teachers' subject matter knowledge and their pedagogical content knowledge impact the nature and quality of their actual teaching and, consequently, of students' mathematical learning processes and outcomes." Therefore, training of pedagogically effective mathematics teachers is very important. For this reason, different problem-posing approaches should be taught to pre-service teachers as part of their training programs.

One limitation of this study was that it measured pre-service teachers' problem-posing performance based on only one problem-solving strategy. Future studies could ask pre-service teachers to write problems associated with other strategies such as constructing a table or chart, drawing a picture or diagram, solving a simpler problem, guess and check, working backward, writing an open sentence, logical reasoning, making a systematic list, constructing a general rule, or adding something to the problem situation. Furthermore, different problem-posing tasks, including digital image photos (Nicol & Bragg, 2009), computer programs, or spreadsheets (Abramovich & Cho, 2008), could be applied during teacher education programs to assess and develop pre-service teachers' problem-posing performance related to various strategies. Also, studies similar to the present one could be conducted with middle- or high-school students to assess their problem posing performance that require their knowledge of problem-solving strategies.

As indicated by Chen et al. (2011), intervention studies that compare how various instructional approaches influence pre-service teachers' topic-related cognitions and beliefs could also be investigated. Given that interventions improve problem

posing and provide a richer understanding of what makes a good problem (Crespo & Sinclair, 2008), teacher educators should consider new approaches, such as the posing of problems associated with particular problem-solving strategies, to improve how pre-service teachers integrate their mathematical knowledge into the context of problem posing. In view of the fact that problem-posing activities are useful in helping teacher educators to assess their students, these activities should be placed within methods courses so that pre-service teachers will be equipped with both problem-solving strategies and problem-posing skills. In summary, the findings of the present study suggest that pre-service teachers should be educated in problem-solving strategies and problem and types so that they can apply problem-posing skills effectively in primary schools.

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