# Examining Changes in Preservice Mathematics Teachers' Technological Pedagogical Content Knowledge from their Microteaching[*]

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## Abstract

Microteaching is regarded as an effective method that allows preservice teachers to gain the experience of instructional processes and is used to improve teaching skills in an environment similar to a real classroom. Microteaching, which is utilized as a method for increasing the quality of a teacher's education, is also used in research studies conducted within the scope of Technological Pedagogical Content Knowledge (TPCK). Accordingly, the purpose of this research is to observe changes in preservice teachers' TPCK from their microteaching practices conducted under a conceptual TPCK framework. The participants are 52 third-year preservice teachers. An analysis was performed based on specific themes from the application, which had been developed to reveal the changes in preservice teachers' conceptual framework of TPCK components. Content analysis was used to analyze the observation forms, self-evaluations, and transcriptions from the interviews on the self-evaluations for the effects over their procedural changes within the context of the themes. The study's findings indicate that preservice teachers use the technological tools to attain what they had specified in their instructional plans. Also when considered in terms of the TPCK components, preservice teachers made remarkable progress.

## Keywords

Technological pedagogical content knowledge • Mathematics education • Microteaching •
Preservice teacher • Technology integration

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Technology becomes more widespread day by day in the field of education, as in every area of life. Each advance in technology results in a new technological tool, and these tools generally guide educators' research methods, practices, and questions, particularly those of mathematicians (Artigue, 2002; Baki, 1996). The use of technological tools is significant, especially in terms of ensuring the visualization of mathematical concepts (Metaxas & Karagiannidou, 2010; Sacristra & Noss, 2008), and allows for access to multiple representations of mathematical concepts (Brenner et al., 1997; Kieran, 1994; O'Callaghan, 1998). With technology use becoming popular in mathematics education, the mathematics curricula of several countries have emphasized the necessity and importance of mathematical instruction through technology (Milli Eğitim Bakanlığı [Turkish Ministry of National Education], 2013; National Council of Teachers of Mathematics [NCTM], 1989, 1991, 2000). Integrating computer-assisted education with mathematics education depends on numerous factors such as a school's technological infrastructure, whether it has the required educational software and student/teacher resources, students' and teachers' attitudes towards technology, and teachers' education. When considering teachers' reluctance to use technology in classroom practices (Baki, 1996) or how they feel incompetent when including technology within the instructional process (Hugnes, 2004; Russell, Finger, & Russell, 2000; Yıldız, Sarıtepeci, & Seferoğlu, 2013), teacher education is considered the most crucial element for accurately and efficiently integrating technological tools within the instructional process, and much research has focused on this (Lawless & Pellegrino, 2007). Indeed, teachers can integrate technological tools accurately and efficiently by understanding the pedagogy of using these tools and using them according to the aim of the class (Harris, Mishra, & Koehler, 2009). In the literature, teachers' knowledge of technological pedagogy (i.e., how a concept is learned and taught using technology) is defined as Technological Pedagogical Content Knowledge (TPCK; Niess, 2005; Pierson, 1999).

TPCK is defined as the type of knowledge that emerges by adding technological knowledge to the concept of pedagogical content knowledge. TPCK is the interaction and intersection of technological, pedagogical, and content knowledge (Koehler, Mishra, & Yahya, 2007; Mishra & Koehler, 2006). Taking pedagogical content knowledge as the base, TPCK involves different components such as instructional strategies and knowing how to assess/evaluate and understand students and the curricula (Angeli & Valanides, 2008).

The information structures described in the TPCK model define diverse types of technological integration and provide a basic theoretical construct. TPCK's theoretical construct aims to help develop better techniques that define how theoretical information on technology can begin to be put into practice. Studies have concentrated on investigating teachers' knowledge and competences using the TPCK framework

(Angeli & Valanides, 2008; Mishra & Koehler, 2006). Teachers are said to come to a better point in understanding technological integration's level of potential when they gain the necessary practicality (Harris et al., 2009). Teachers need to consider all components rather than handling just one aspect so that instruction can be performed by simultaneously combining technological, pedagogical, and content knowledge in the TPCK construct. According to Harris et al. (2009), teachers should be able to navigate through content, pedagogical, and technological knowledge, as well as their complex relations within certain contexts. To sum up, providing teachers with technologically equipped classrooms, access to technology, and positive attitudes in accordance with the technological integration model will not guarantee technological integration in classrooms (Perkmen & Tezci, 2011). In addition to sufficient content, technological, and pedagogical knowledge, integrating the knowledge and performance of practical applications is what's actually important for achieving technological integration. As mentioned above, TPCK requires thinking about this knowledge in multiple ways, not just one. Hence, as stated by Niess (2005), preservice teachers need a well-developed information base and ability to apply the subject to related areas.

Microteaching, utilized as a method for increase the quality of teacher education (Keser, 2007), has also been used in TPCK research studies (Cavin, 2007; Cavin & Fernandez, 2007; Kafyulilo, 2010; Taşar & Timur, 2010). Microteaching is regarded as an effective method that allows preservice teachers to gain experience in instructional processes (Görgen, 2003; Kavas, 2009) and is used to improve teaching skills in environments similar to real classrooms (Kpanja, 2001; Taşpınar, 2007). In microteaching, preservice teachers instruct in front of other preservice teachers for 5-20 minutes. In this sense, microteaching is a limited, artificial (Taşpınar, 2007), and condensed practice course compared to real classroom instruction (Doğanay, 2009; Keser, 2007). Through microteaching, preservice teachers gain experience by observing other preservice teachers' instruction alongside their own (Görgen, 2003).

According to the TPCK model, teachers need to have general knowledge on pedagogical content and technological subjects, as well as understand the interactions and relations among these knowledge types. This is why having the relative knowledge and skills is not enough on its own for being an effective teacher of technological use (Koehler et al., 2007; Mishra & Koehler, 2006). According to Koehler et al. (2007, p. 741), "At the heart of TPCK is the dynamic, transactional relationship between content, pedagogy, and technology. Good teaching with technology requires understanding the mutually reinforcing relationships between all three elements taken together to develop appropriate, context-specific, strategies and representations." This approach addresses the TPCK theoretical model in preservice education, whereas emphasizing the importance of microteaching allows for the development of strategies and presentations specific to the learning environment. When reduced

to content, preservice teachers should have an extensive understanding of TPCK in order to be prepared for teaching mathematics. Mathematics teaching states that if teachers and preservice teachers can answer how to teach a mathematical subject using a certain technology, provide approaches on how one can teach mathematical concepts to students so they can perform experiments with their ideas, concepts, and hypotheses, and generalize about technology-aided instruction, they can succeed in technology integration (Niess, 2005; Richardson, 2009; Schmidt et al., 2009).

In studies in the literature investigating the effect of microteaching on teachers/ preservice teachers' TPCK in the literature, preservice teachers have been revealed to acquire awareness of: the details necessary for performing instruction with technology, how to use traditional instruction methods along with technology in student-centered learning environments, and how microteaching positively impacts their general TPCK development (Cavin, 2007; Cavin & Fernandez, 2007; Kafyulilo, 2010). Yet in the literature, the aspect of content seems to be generally neglected in terms of technological pedagogical content knowledge and applications that preservice teachers are to be provided with. Based on this consideration, this research is thought able to fill a gap in the field by examining preservice mathematics teachers' change in TPCK from microteaching practices, as well as provide suggestions along a theoretical framework integrated with the aspect of content.

## Conceptual Framework of the Research

Within the conceptual framework of this research, the components of TPCK achieved as a result of addressing the components of pedagogical content knowledge are examined within the context of technology. These components are:

• Technology and multiple representations of concepts.

• Technology and student difficulties regarding concepts and misconceptions.

• Technology and the methods and strategies for concept teaching.

• Technology and concept assessment-evaluation.

• Teaching concepts in the curriculum using technology. (Özmantar, Akkoç, Bingölbali, Demir, & Ergene, 2010)

Özmantar et al. (2010) set forth the TPCK components they used as the framework in their study by identifying the components of pedagogical content knowledge based on those introduced by Grossman (1990) and expanded by Magnusson, Krajcik, and Borko (1999). They also added knowledge of multiple representations as a component in consideration of the knowledge of representation that Shulman (1986) defined for pedagogical content knowledge. Because content is involved as an aspect and has

been ignored in the current TPCK literature, Özmantar et al.'s (2010) conceptual framework has been addressed for identifying TPCK components as the objectives with which to provide the preservice teachers.

## TPCK Components' Importance in Mathematics Teaching and Learning

**Technology and multiple representations of concepts.** Multiple representations of a concept in mathematics teaching and learning provide different perspectives on students' understanding of problems and providing solutions, thereby reinforcing their conceptual interpretation (NCTM, 1989). As using multiple representations in teaching emphasizes various aspects of the same concept, by using the relationships established among representations, one can actualize a more conceptual understanding and reach more students through different learning styles (Berthold, Eysink, & Renkl, 2009; Mallet, 2007). With technology becoming more popular, new opportunities have been offered in terms of instruction through multiple representations. Providing multiple representations of mathematical concepts (i.e., algebra, graphs, tables) using technologies allows one to emphasize different aspects of each concept and bring a more extensive perspective to the mathematical concept (O'Callaghan, 1998; Zbiek, Heid, Blume, & Dick, 2007). Yet studies on multiple representations highlight that, when the use of technologies in instruction is not planned carefully, they may actually pose a hindrance to learning instead of supporting it (Ainsworth, 1999; Berthold et al., 2009; Mallet, 2007). In these respects, the importance of preservice and in-service teacher education on the efficient use of multiple representations in teaching becomes apparent. Teachers and preservice teachers, as educated individuals, can efficiently teach multiple representations by providing them with technological facilities that support their conceptual understanding.

**Technology and student difficulties regarding concepts and misconceptions.** Developing conceptual understanding, being a main concern of mathematics education, can be difficult to achieve because the strength of the prerequisite relationship affects interpretation, and a student who has difficulty/misconceptions with a concept can have difficulty succeeding later on in related concepts (Yetkin, 2003). Therefore, these difficulties need to be eliminated right away when identified among students (Duval, 2002). Structuring an efficient learning environment has been suggested for eliminating student difficulties. By using technology, many concepts can be visualized through multiple representations, positively supporting conceptual learning and mathematical thinking, and thus addressing the difficulties students frequently experience (O'Callaghan, 1998; Selden, Dubinsky, Harel, & Hitt, 2003; Yerushalmy, 1991; Zimmermann, 1991). Meanwhile, handling student difficulties efficiently with the help of technology is possible by providing these knowledge and skills to teachers and preservice teachers.

**Technology and the methods and strategies for concept teaching.** The methods that can be used in mathematics teaching include direct instruction; question & answer; discovery; discussion; instruction though analysis and demonstration; instruction using scenarios; and instruction using games, projects, cooperative learning, and problem-solving (Altun, 1998; Uğurluel, 2003). Using technological aspects to shape the component of instructional methods and strategies addressed within the scope of pedagogical content knowledge (Shulman, 1986) is important. Considering that technology has become an inseparable part of education (and more specifically instruction), how the component of instructional strategies and methods should be handled in the presence of technology, how these strategies and methods are shaped by the presence of technology, and the role of the teacher in this process are all very critical for efficient education and instruction. Aside from providing appropriate strategies and methods for ensuring students' meaningful conceptual learning, appropriate technologies should also be used for an effective process. In this context, a teacher's preferred strategy/method can impact how, for what purpose, and at what level technology is used (Hughes, 2005). For instance, teachers who adopt presentation as a method can use the technology at hand to this end. Thus for students' conceptual learning, preservice teachers and in-service teachers need to also use technological tools to establish the relationships among concepts or representations by utilizing different ones. Because technology-usage levels and purposes are influenced by preferred instructional methods/strategies, those in preservice and in-service teacher education need to be provided with these usage levels and knowledge.

**Technology and concept assessment-evaluation.** Paralleling the changes in curricula, instructional methods, and techniques, assessment and evaluation styles have also seen changes (Uğurlu, 2009). In recent years, two different assessment types (i.e., summative and formative) have been debated (Linchevski, Kutscher, & Olivier, 1999). While summative assessment-evaluation is used for ranking students according to certain criteria in order to identify those who have successfully completed a given instructional process (Bloom, Hastings, & Madaus, 1971), formative assessment-evaluation aims to monitor students' progress within the instruction to ensure adjustments in teaching and learning in accordance with students' failures or successes (Gronlund, 2006). For technological integration, various knowledge and skills on assessment-evaluation have come into play. Research on technology has shown assessment and evaluation to be ignored (Kissane, Bradley, & Kemp, 1994; Kissane, Kemp, & Bradley, 1996). Considering its summative and formative aspects, TPCK gives importance to how assessing/evaluating with technological tools should be done. Because students shape their learning styles by how they are assessed and evaluated, having students be evaluated through the learning outputs of normal teaching unassisted by technology is insufficient for the end of technology-assisted teaching. How to evaluate students who are taught with the help of technology should be addressed in preservice and in-service teachers' education.

**Teaching concepts on the curriculum using technology.** Curriculum knowledge is handled under pedagogical content knowledge (Grossman, 1990; Park & Oliver, 2008). This knowledge offers guidance on how deep and how far to address the concepts being taught (Rasinen, 2003). The depth to which a concept should be taught is important, especially in terms of teachers' in-class decisions and insight into what students need to know about the concept being taught for the academic year and the years ahead (Magnusson et al., 1999). Several studies on mathematics teaching have drawn attention to the key role of depth in teaching concepts (Graeber, 1999; Manouchehri, 1997; Stump, 2001). This type of knowledge becomes even more important when integrating technology. Many studies have argued that addressing advanced conceptual relations in simple concepts particularly affects the degree to which advanced concepts are handled. Teachers superficially utilize educational technologies and students actualize low-level learning when instruction is neither deep nor diversified enough for them to succeed (Groff & Mouza, 2008; Levin & Wadmany, 2008; Russell et al., 2000; Zhao, Pugh, Sheldon, & Byers, 2002). The frequent exclusion from the curriculum of application examples on the use of educational technologies in teaching indicates a significant problem. The concepts addressed in mathematics education studies have often been observed to be outside the curriculum (Metaxas & Karagiannidou, 2010). Based on this point, emphasis should be place on teachers and preservice teachers to not perform activities in technology-assisted instruction that exceed the curriculum; otherwise, this situation can cause failure in preservice and in-service teachers' educational learning outputs.

### The Purpose of the Research

The main purpose of the research is to examine the changes preservice mathematics teachers experience within the context of TPCK alongside the components of microteaching as structured within the theoretical framework of TPCK.

### Method

This study is a product of an application shaped through the TPCK conceptual framework for providing preservice elementary mathematics teachers with TPCK. The study uses mixed methods that aim to combine or blend qualitative and quantitative research techniques, methods, approaches, or concepts in a single study (Creswell, 2006; Johnson & Christensen, 2008; Johnson & Onwuegbuzie, 2004). Because the qualitative and quantitative phases were conducted in multiple stages (sometimes one, sometimes more than one), the study utilizes the fully mixed sequential dominant status design, with its greater focus on the qualitative stage (Leech & Onwuegbuzie, 2009).

## Participants

The study's participants are 52 third-year preservice teachers attending a state-university elementary mathematics education program. The study focuses on the TPCK changes preservice teachers exhibit from microteaching. In accordance with participants' infrastructures within the scope of pedagogical content knowledge and TPCK, the components which are: multiple representations of concept, student difficulties regarding concepts and misconceptions, concept assessment-evaluation, and teaching concepts in the curriculum were handled in the elective Mathematics Curriculum course. The preservice teachers performed basic computer-software functions in programs such as Graphical Analysis, GeoGebra, Excel, and Cabri3D in the courses of Applications of Technology in Education and Instructional Technologies and Material Development courses. After addressing the methods and strategies for concept instruction in the Teaching Methods I course where the application was performed as the study topic, the preservice teachers performed microteachings regarding the goals specified in the curriculum.

## Application Process

Before performing the first microteaching within the TPCK framework, the Self-Efficacy Scale in Relation to Computer-Based Education (20 items; developed by Arslan, 2006) and the 5-point Likert-type Computer-Assisted Mathematics Instruction Questionnaire (30 items; developed by Yenilmez & Sarıer, 2007) were applied to the participants. After briefing them on the application process, the preservice teachers were asked to report the goals they had specified for their microteachings and the software they would use in their instruction. The participants who preferred the same learning topics agreed to aim for different subtopics. Table 1 shows the distribution of participants by computer software used in the microteaching.

Table 1

*Computer Software used by the Preservice Teachers in the Microteaching*

| Computer Software Used | Frequency ($f$) | Percentage (%)[*] |
|---|---|---|
| Graphical Analysis | 12 | 23 |
| GeoGebra | 41 | 79 |
| Excel | 15 | 29 |
| Cabri3D | 9 | 17 |

*Percentage values may exceed 100% because some preservice teachers used multiple software packages.

The participants delivered their prepared instructional plans on their selected goals prior to microteaching in front of the other preservice teachers for 5 to 20 minutes. The other preservice teachers evaluated each peer's microteaching using an observation form the researchers had prepared along the lines of TPCK. Each microteaching was recorded with two cameras, one focusing on the preservice teacher's communication and the other focusing on their lecture. The preservice teachers examined the

evaluations from their peers as well as their post-lecture microteaching video records and wrote their own self-evaluations using the same TPCK components as in the observations. Semi-structured interviews were also performed with the preservice teachers on their micro-teaching self-evaluations. The preservice teachers continued to gain experience by observing the other preservice teachers' microteaching. Having revised and delivered their instructional plans in parallel with the self-evaluations from the first microteaching, each preservice teacher performed a second microteaching. Following the second microteaching, the Self-Efficacy Scale in Relation to Computer Based Education and the Computer-Assisted Mathematics Instruction Questionnaire were once again applied. The application process is shown in Figure 1.
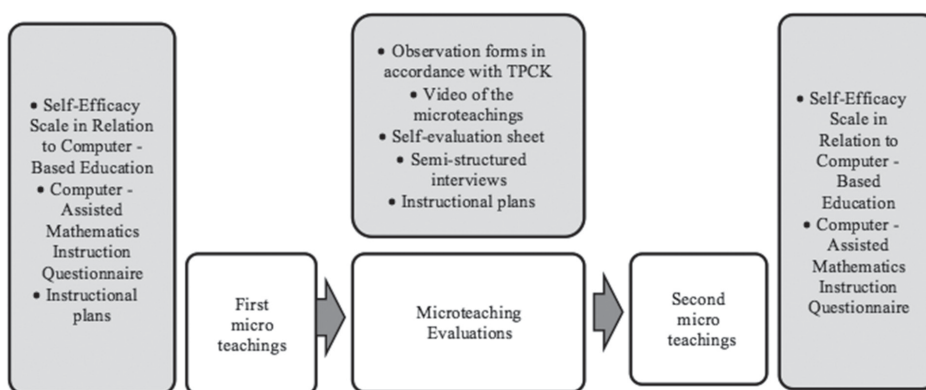


*Figure 1.* Application process.

### Data Collection Instruments

**Self-Efficacy Scale in relation to computer-based education.** This self-efficacy scale in relation to computer-based instruction was applied before and after the microteachings in consideration of the key role of preservice teachers' self-efficacy perceptions in computer-based education. This is a 20-item Likert-type scale developed by Arslan (2006). Reliability of the self-efficacy scale in this study was found to be .89.

**Computer-Assisted Mathematics Instruction Questionnaire.** This survey was applied before and after the microteachings in consideration of the key role of preservice teachers' opinions in computer-assisted education. This survey was developed by Yenilmez and Sarıer (2007) as a 5-point Likert-type scale composed of 30 items. This study's reliability of the Computer-Assisted Mathematics Instruction Questionnaire was found to be .91.

**Observation Form.** Each preservice teacher's first performed microteaching was evaluated within the context of TPCK by the other preservice teachers using the researchers' observation form. The point here is to increase the effectiveness of

microteaching by having the preservice teachers adopt the criteria within the context of components while evaluating each preservice teacher. Three experts, two specializing in mathematics and one in computer education and instructional technologies, prepared the questions on the observation form in consideration of the TPCK components, and five field experts reviewed them to achieve content validity. The observation form involves questions on technology and multiple concept representations, technology and student difficulties regarding concepts and misconceptions, technology and the methods and strategies for instructing concepts, technology and concept assessment-evaluation, and the suitability of teaching concepts in the curriculum using technology. The questions also require evaluating the general ways and limits technology offers for instructing. With three questions for each of the six aspects, the form is composed of 18 questions. As an example, the following three questions were asked on technology and the methods and strategies in concept instruction:

> Which methods and strategies were used in the instruction? To what purpose and level was technology used for the preferred methods and strategies? How did the preferred technology affect the methods and strategies used in the instruction?

**Microteaching videos.** Each preservice teacher's microteaching was recorded with two cameras, one focusing on the preservice teacher's communication and the other focusing on the teacher's lecture. The videos were recorded by the researchers to examine the change of each preservice teacher along the axis of TPCK, and to allow the preservice teachers to review their own microteaching videos so they can self-evaluate more objectively and in more detail.

**Self-evaluation form.** The preservice teachers examined their peers' evaluations from the observation forms and microteaching videos following the first microteaching. They evaluated themselves using a self-evaluation form. This form was applied to ensure that the preservice teachers would see their own instruction in a more detailed and focused way by evaluating their instruction from the perspective of the other preservice teachers. As on the observation form, the three experts prepared the questions on the self-evaluation form in consideration of TPCK components, which the five field experts reviewed to achieve content validity. The self-evaluation form involves questions on the same areas as the observation form with the additional requirement of evaluating the general ways and limits that technology offers for instructing. This form is composed of 22 questions that were created by shaping the observation form to allow for self-evaluation. For example, the following questions were asked for technology and the methods and strategies in concept instruction:

> Which methods and strategies did you prefer for instruction? Please explain your reasons in detail. To what purpose and level were you able to use technology for your preferred methods and strategies? Please explain in detail how the purpose and level of use affected your instruction

and what you would change if you could perform it again. How did your preferred technology affect your methods and strategies? If you had the chance to perform another instruction, would you change your technology, methods, or strategies? Please explain your reasons in detail.

**Semi-structured interviews.** Semi-structured interviews were performed with the preservice teachers on their microteaching self-evaluations. The interviews were conducted face-to-face for explaining parts from the preservice teachers' self-evaluations that weren't clear or that needed to be interpreted.

**Instructional plans.** The preservice teachers prepared their instructional plans according to the goals they set for their microteaching. A specific instruction-plan format was given to the preservice teachers. Following their first microteaching, the preservice teachers examined the evaluations made by their peers on the observation forms and microteaching videos and then wrote self-evaluations on the same TPCK components as on the observation form. The preservice teachers revised their instructional plans in parallel with their self-evaluations after the first microteaching. The revised instructional plans were then received from the preservice teachers.

## Data Analysis

Data obtained from the Self-Efficacy Scale in Relation to Computer Based Education and the Computer Assisted Mathematics Instruction Questionnaire, which had been applied before and after the microteachings, were analyzed by SPSS software using the paired samples *t*-test.

The preservice teachers were encoded as PT1, PT2, PT3, and so on in order to distinguish them when analyzing the qualitative data. An analysis based on TPCK themes was performed in the application to reveal preservice teachers' TPCK changes. The component of *technology and multiple representations of concept* focuses on how preservice teachers will use multiple representations for their concept instruction and how they plan to benefit from the software to correlate these representations. *Technology and student difficulties with concepts and misconceptions* focuses on the theme of planning with the help of computer software to overcome students' difficulties regarding the selected concept, while *instructing concepts on the curriculum using technology* focuses on the theme of preparing a concept-instruction plan at level appropriate to the curriculum. The videos of the first and second microteachings and instructional plans were analyzed using content analysis within the framework of these components' topics. In the application developed to reveal preservice teachers' TPCK conceptual component changes, analysis was performed based on these themes. Coding using the pre-specified concepts was used in the content analysis (Strauss & Corbin, 1990). The observation forms, self-evaluations, and interview transcriptions of the self-evaluations affecting this change were also analyzed using content analysis

within the framework of these same themes. The analysis was handled as a cyclical process that involved rereading and reorganizing the data, as well as reconsidering the themes specified for the components within the TPCK conceptual framework. To ensure reliability, the study's raw data were encoded separately by three experts, and then the consistency of the coding was checked for any necessary adjustments before the themes were finalized. Because different content analysis techniques were also used for the components of *technology and the methods and strategies for concept instruction* and *technology and concept assessment-evaluation*, detailing was needed apart from the framework of the overall data analysis.

The component of *technology and the methods and strategies for concept instruction* addressed the theme of planning technological instruction for the methods and strategies preservice teachers had chosen to be gained. In the conducted analyses, the levels of technology use introduced by Hughes (2005) were used to determine the level of technological integration. Explanations of these levels and their reinforcement with examples of preservice teachers' applications are given Table 2 below.

Table 2

*The Framework of Technology Usage Levels Introduced by Hughes (2005) and Examples of Preservice Teacher Applications*

| Levels | Level Information | Exemplary Preservice Teacher Application |
|---|---|---|
| Level 0 | No technological tool used by the preservice teacher. | [Level 0: No technological tool use] PT19 reminded the area of a rectangle by drawing it on the whiteboard before moving on to the area of a triangle. |
| Level 1 (replacement) | Use of technological tools by the preservice teacher only for changing the environment. | [Level 1: GeoGebra was used only to change the environment.] PT19 changed the environment by drawing a rectangle using GeoGebra. |
| Level 2 (amplification) | Utilization of the technology by the preservice teacher to help perform certain operations faster rather than ensuring conceptual and deep learning. | [Level 2: The preservice teacher used GeoGebra to perform area operations.] PT19 drew different rectangular fields on the grid in GeoGebra and asked the students to find the areas of the rectangles with the help of unit squares. |
| Level 3 (transformation) | Performance of activities by the preservice teacher to change learning-teaching routines and to cause students to have deep understanding. | [Level 3: The preservice teacher used GeoGebra while performing an activity to cause students to have deep understanding.] PT19 presented the prepared activity using the grid feature in GeoGebra before the course. The presentation reminded that the area of a rectangle equals the sum of unit squares. Next, two triangles were obtained by dividing this rectangular area in half from its diagonal. Using the rotation function of GeoGebra showed the triangles to be matches. Consequently, students were made to discover the area of the triangle to be half the area of the rectangle. |

Accordingly, each preservice teacher's instructions recorded in the scope of the microteaching were examined, and the ranges of the various levels of technology use were specified. Next, total duration was determined for each level and the durations

were proportioned to the total duration of the course, thus quantifying the levels from which technology had been benefitted. Quantification examples in the application of preservice teachers' technology-usage levels and explanations shown in the tables are presented in the Findings (see Figure 4 & Table 6).

The component of *technology and concept assessment-evaluation* takes into account the theme the preservice teachers made planning how to benefit from computer software while applying several aspects (i.e., formative, summative) of assessment and evaluation. The summative and formative aspects of assessment-evaluation were taken into consideration under the objectives specified in the literature. These aspects and examples of their reinforcement from preservice teacher applications are given in Table 3.

Table 3

*Formative and Summative Aspects of Assessment-Evaluation and Examples of Their Reinforcement through Preservice Teacher Application Examples*

| Various aspects of assessment-evaluation | Details of aspects | Technology-Assisted Exemplary Preservice Teacher Application |
| --- | --- | --- |
| Summative | Summative assessment-evaluation is used for ranking students according to certain criteria and identifying those who have successfully completed a given instructional process (Bloom, Hastings, & Madaus, 1971). | A computer-assisted homework used by PT38 for summative purposes is given in Figure 6. PT38 assigned applications similar to those in the microteaching as homework to the students. |
| Formative | Formative assessment-evaluation aims to monitor students' progress throughout the instruction to ensure adjustments in teaching and learning in accordance with the successes and failures of the students (Gronlund, 2006). | PT8's dynamic activity and questions for the format throughout learning are shown in Figure 5. |

The analyses examined in-depth how the preservice teachers used computer software in their instructional plans and their assessment-evaluation tools to serve formative and/or summative purposes during their microteaching, as well as how they benefited from the technology in this stage.

## Findings

In this study aimed at observing preservice teachers' TPCK changes from microteaching, the Self-Efficacy Scale in Relation to Computer-Based Education and the Computer-Assisted Mathematics Instruction Questionnaire were applied to the preservice teachers before and after the microteaching. Preservice teachers' perceptions of self-efficacy regarding computer-assisted education ($t = .508$, $p > .05$) and opinions on computer-assisted mathematics teaching ($t = .544$, $p > .05$) were examined, and no statistically significant difference was found between the pretest and posttest mean scores. According to the pretest (= 3.85) and posttest ( = 3.91) mean scores, preservice teachers' opinions on computer-assisted mathematics teaching were found to have improved positively, even though no statistical difference is present among their opinions.

The qualitative findings of the study are given in detail under separate headings within the scope of TPCK components.

## Findings Regarding the Component of Technology and Multiple Representations of Concept

When examining preservice teachers' instructional plans from the first microteachings, most were seen to have not associated representations, even though they had used different ones. The rate of technology use in different representations was found to be low for the first microteaching. Following these and after examining the peer and self-evaluations, participants were seen to have increased the specified representations for attainment in the second microteaching, as well as the rate technology was used in their correlations among representations. Although the preservice teachers benefitted from technology and the means it offers for different concept representations, most participants were seen to have not benefitted from technology for correlating the representations, instead trying to have the students explore these correlations through verbal discussions. The semi-structured interviews with the preservice teachers on their self-evaluations of the first microteaching also revealed that most of them not benefitting from technology for correlating representations cannot be explained by incompetent technology use, ignorance of how to present course content using technology, or lack of being able to see technology's contribution to concept presentation. The finding is supported by the preservice teachers' opinions obtained from the interviews.

> [W]hen showing my representations, I benefitted from technology but I preferred asking students questions rather than using technology in the correlations among representations. ... I don't know why I didn't use it. It would have been more comfortable and understandable, I could have presented all of them in one window and have the students discuss it, but it didn't occur to me while planning. (PT7)

> [I] realized the importance of correlating representations in my second instruction. In my first instruction, even my friends said they couldn't do it. If they can't correlate, my students won't either. (PT16)

## Findings Regarding the Component of Technology and Student Difficulties Regarding Concepts and Misconceptions

According to the examples preservice teachers presented on their instructional plans' goals from the first microteaching, difficulty, misconception, error, and their interrelations were seen to confuse them. In terms of the methods for addressing these concepts, the fact that most participants did not benefit from computer software when planning is also intriguing. PT31's mix-up of the concepts of misconception

and error for a given student's difficulty and the use of investigating questions in the instructional plan are exemplified in Figure 2.
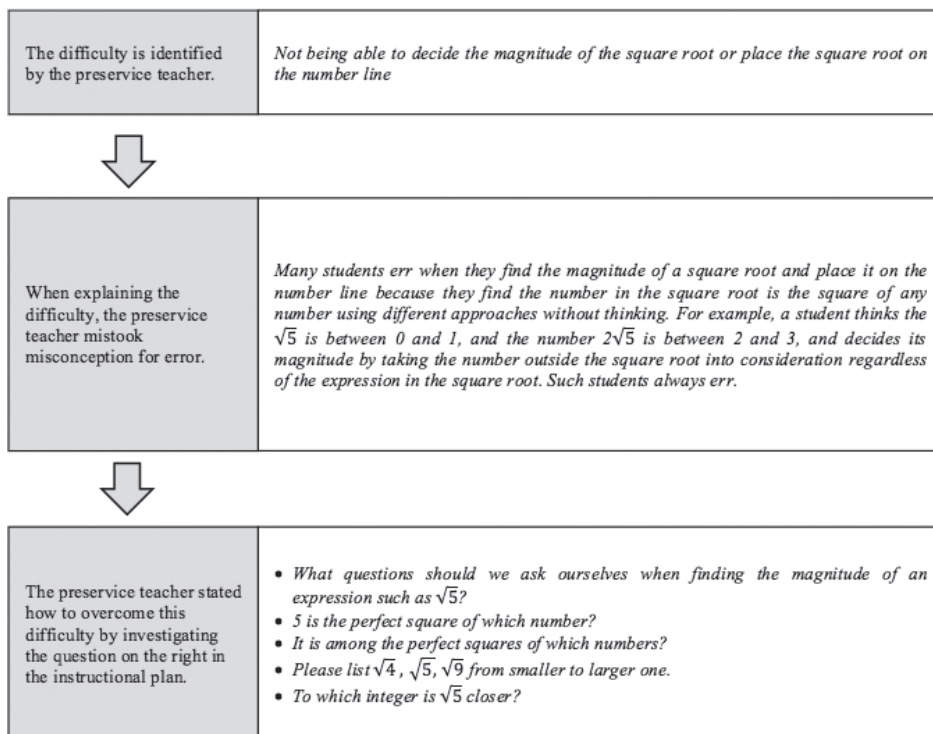
| | |
|---|---|
| The difficulty is identified by the preservice teacher. | *Not being able to decide the magnitude of the square root or place the square root on the number line* |

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| | |
|---|---|
| When explaining the difficulty, the preservice teacher mistook misconception for error. | *Many students err when they find the magnitude of a square root and place it on the number line because they find the number in the square root is the square of any number using different approaches without thinking. For example, a student thinks the $\sqrt{5}$ is between 0 and 1, and the number $2\sqrt{5}$ is between 2 and 3, and decides its magnitude by taking the number outside the square root into consideration regardless of the expression in the square root. Such students always err.* |

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| | |
|---|---|
| The preservice teacher stated how to overcome this difficulty by investigating the question on the right in the instructional plan. | • *What questions should we ask ourselves when finding the magnitude of an expression such as $\sqrt{5}$?*<br>• *5 is the perfect square of which number?*<br>• *It is among the perfect squares of which numbers?*<br>• *Please list $\sqrt{4}$, $\sqrt{5}$, $\sqrt{9}$ from smaller to larger one.*<br>• *To which integer is $\sqrt{5}$ closer?* |

*Figure 2.* A portion of PT31's first microteaching.

According to the preservice teachers' instructional plans after the first microteaching and self-evaluation, they were found to have made progress by also emphasizing contradictions in terms. The following explanations from the preservice teachers belong to the interviews after the second microteaching, which shows the progress in their opinions on the concepts of: difficulty, misconception, error, and their correlations.

> [I] realized I had used the concept of error instead of all these other concepts. Many of my friends noted when criticizing my first instruction that while I was lecturing on the student's error it was actually a misconception. Accordingly, I investigated these concepts and reread them... Difficulty is a more general term involving error and misconception. Error is something done once, that doesn't feature faulty thinking. It's akin to a one-time error of operation. But if a student explains it with a different interpretation when we ask why the error was made, we should address it as a misconception. (PT43)

[F]or example, the majority of students can have difficulty perceiving the height of a triangle only as the perpendicular drawn base. When deciding the height, if a student says, "Assuming perpendiculars are drawn from all sides to their opposite vertices, only the perpendicular line drawn down to the base is the height," and gives the explanation that "only the perpendicular drawn down to the base from the opposite vertex is the height, we can't take the other perpendiculars drawn to other sides," this is a student misconception. But if the student says, "I didn't notice the other perpendiculars. Of course the perpendiculars drawn to other sides are heights, to," this means it had escaped the student's notice and is an error. (PT50)

When considering preservice teachers' first and second microteachings, they were seen unable to use computer software efficiently for overcoming student difficulties. Even though the preservice teachers had managed to identify computer software convenient for overcoming student difficulties, they were unable to put forth strategies clear enough to eliminate these difficulties. In other words, the computer software packages that had been preferred could not be utilized efficiently for overcoming the difficulty.

The difficulty PT5 addressed in the second microteaching on how to use the software and the recommendations made for strategies that could be followed in this instruction are given in Figure 3.
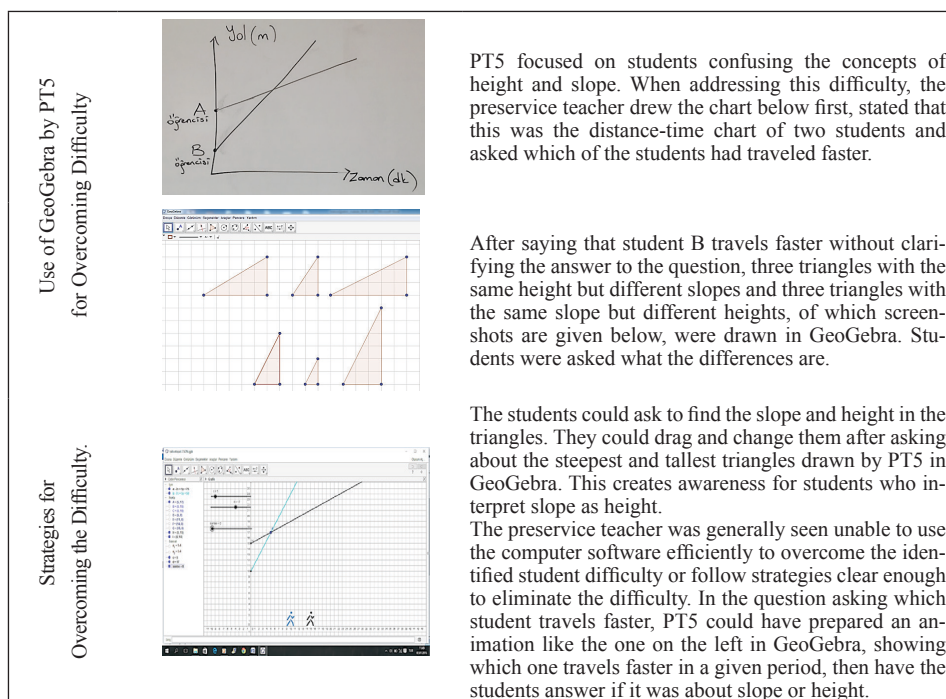


*Figure 3.* PT5's use of GeoGebra and recommended strategies for overcoming difficulties.

According to the computer software and difficulties addressed in the second microteaching, the fact that most of those who used Excel focused on algebraic

difficulties and the majority of those who used Graphical Analysis focused on graph difficulties proves that difficulties addressed in regard to software type might have been limited. While some preservice teachers supported this limitation with explanations in the interviews, others argued the reason only one was preferred and not all difficulties were addressed was for focusing on the difficulty better and creating a more realistic plan for the amount of time.

Another intriguing finding on this topic is that the preservice teachers did not consider the potential problems students might face in terms of technology use. The preservice teachers (especially those using Cabri3D in their instructional plans) stated thinking students may have trouble with software packages because Turkish is not supported. As a precaution, they offered to clearly explain to the students the basic functions and buttons when the software is in English. Most preservice teachers argued that students would not have any problems because almost all of them are accustomed to technology; if they were to face a problem, they could solve it by asking their peers or teacher for help. The preservice teachers were seen to have not addressed other foreseeable problems, such as not knowing or understanding a program's commands or basic functions, not having diverse levels of technological knowledge or different preparedness levels of technology use, and being unaware of student prejudices against technology use. Therefore, precautions were not mentioned for solving these issues.

**Findings Regarding the Methods and Strategies for the Instruction of the Concept**

When examining the methods and strategies preservice teachers preferred in their instructions, no general change was seen between their first and second microteaching. The findings regarding their preferred methods and strategies in microteaching are given in Table 4.

Table 4

*Frequencies of Preservice Teachers' Methods and Strategies in Microteaching*

| Method and Strategy | First Microteaching Frequency (*f*) | Second Microteaching Frequency (*f*) |
|---|---|---|
| Discovery Learning | 28 | 33 |
| Direct Instruction | 43 | 38 |
| Discussion | 36 | 43 |
| Research | 3 | 3 |
| Question & Answer | 41 | 49 |
| Teaching with Scenario | 2 | 2 |
| Brainstorming | 37 | 31 |
| Learning in Group Work | 8 | 11 |
| Learning by Problem Solving | 13 | 16 |
| Teaching with Games | 1 | 1 |
| Demonstration | 4 | 5 |

As seen in Table 4, discovery learning, direct instruction, discussion, question and answer, and brainstorming are the most preferred methods and strategies in preservice teachers' first and second microteaching.

According to the levels of technology use in their method and strategies for the specified goals, an increase was observed in the number of Level-3 activities in the second microteaching, which was performed after examining the peer and self-evaluations from the first microteaching. Table 5 shows preservice teachers' average percentages by level in their first and second microteaching. According to these values, the average percentages for Level 1 and Level 2 decreased while the average percentage increased for Level 3.

Table 5
*Pre-service Teachers' Level of Technology Use in Microteaching*

| Levels | First Microteaching Percentage (%) | Second Microteaching Percentage (%) |
|---|---|---|
| Level 1 | 41 | 24 |
| Level 2 | 32 | 19 |
| Level 3 | 23 | 48 |

Exemplification based on the calculation of percentages for each level in the microteachings is presented in Figure 4 in detail through PT19's second microteaching, which addressed the goal of "Student creates the area relation of the triangle and solves related problems."

Table 6
*PT19's Technology Use Levels in Microteaching*

| Levels | Total duration | Percentage (%) |
|---|---|---|
| Level 0 | 5 minutes 40 seconds | 24 |
| Level 1 | 1 minutes 05 seconds | 5 |
| Level 2 | 2 minutes 15 seconds | 10 |
| Level 3 | 14 minutes 10 seconds | 61 |
| Total | 23 minutes 10 seconds | 100 |

PT19's technology-use levels in microteaching are given in Table 6. According to the percentage of detailed level-use for microteaching in Table 6, Level-3 activities are seen to be dominant.

**Findings Regarding the Component of Technology and Conceptual Assessment-Evaluation**

The instructional plans and assessment-evaluation tools prepared and used by the preservice teachers for their first microteaching, their reason for using them, and the frequency and percentage of use are shown in Table 7.

[Level 0: 2 minutes 10 seconds]
PT19 reminded students of the area of rectangular by drawing it on the whiteboard before going on to the area of the triangular.
[Level 1: 1 minutes 05 seconds]
PT19 changed the environment by drawing a rectangle in GeoGebra.
[Level 2: 2 minutes 15 seconds]
PT19 asked students to find the areas of the rectangles with the help of unit squares by drawing different rectangular areas on the grid in GeoGebra (the preservice teacher used GeoGebra software to run area operations quicker.)

[Level 3: 6 minutes 55 seconds]
PT19 presented the activity prepared using the grid function of GeoGebra before class. The presentation reminded that the area of the rectangular area is the total of the unit squares. Next, two triangles were obtained by dividing this rectangular area in half from any of its diagonals. This showed, using the rotation function of GeoGebra, that the triangles were matches. Consequently, students were led to discover that the area of the triangle was half the area of the rectangle.

[Level 3: 7 minutes 15 seconds]
In this stage, PT19 reminded students that the area of a parallelogram is the width times the height of the side in an activity that had been prepared again prior to class using the grid function of GeoGebra. Next, two triangles were obtained by dividing this parallelogram into two from one of its diagonals. Using the rotation function of GeoGebra, the triangles were shown to match. Consequently, students were shown that the area of the triangle was half the area of the parallelogram, and they were led to discover that the area is half of one side multiplied by the height of that side.
[Level 0: 3 minutes 30 seconds]
The preservice teacher asked students to answer the questions on the distributed worksheets about the area of triangles.

*Figure 4.* Sections regarding the levels and durations of technology use in PT19's second micro-instruction.

Table 7

*Assessment-Evaluation Tools Used by the Preservice Teachers in Their First Microteaching*

| Assessment-Evaluation Tool | Purpose of Usage | | Frequency (*f*) | Percentage (%)* |
|---|---|---|---|---|
| | Summative | Formative | | |
| Worksheet | 28 | 8 | 36 | 69 |
| Asking Questions | 41 | 35 | 76 | 146 |
| Activity | 12 | 7 | 19 | 37 |
| Sample question | 46 | 32 | 78 | 150 |
| Homework | 50 | 4 | 54 | 104 |
| Computer-Assisted Worksheet | 1 | 3 | 4 | 8 |
| Computer-Assisted Homework | 2 | - | 2 | 4 |

**\*** Percentage values may exceed 100% because some preservice teachers used multiple tools for different purposes.

According to Table 7, the preservice teachers used different assessment-evaluation tools for different purposes in their first microteaching. The use of technology in the assessment-evaluation process by only six pre-service teachers draws attention.

With the answers given in the semi-structured interviews on their self-evaluations after having examined the peer evaluations, the participants proved that they were knowledgeable about technological assessment-evaluation tools (e.g., computer-assisted worksheet, computer-assisted activities, computer-assisted homework, e-portfolios, concept map software.) Most of the preservice teachers were also seen to have integrated technology with formative assessment-evaluations in their second microteaching. The instructional plans and assessment-evaluation tools the preservice teachers prepared and used for the second microteaching, their purpose for using them, and the frequency and percentage of use are shown in Table 8.

Table 8

*Assessment-Evaluation Tools Used by the Preservice Teachers in their Second Microteaching*

| Assessment-Evaluation Tool | Purpose of Usage | | Frequency (*f*) | Percentage (%)* |
|---|---|---|---|---|
| | Summative | Formative | | |
| Asking Questions | 25 | 38 | 63 | 121 |
| Sample question | 16 | 24 | 40 | 77 |
| Homework | 2 | 3 | 5 | 10 |
| Computer-Assisted Worksheet | 26 | 35 | 61 | 117 |
| Computer-Assisted Homework | 19 | 23 | 42 | 81 |
| Computer-Assisted Research | - | 3 | 3 | 6 |
| Computer-Assisted Activity | 7 | 46 | 53 | 102 |

**\*** Percentage values may exceed 100% because some preservice teachers used multiple tools for different purposes.

Some of the preservice teachers were observed to construct computer-assisted worksheets in particular for formative purposes in consideration of the age range they would teach. For example, some preservice teachers presented the previously developed dynamic activity (they reported that providing the previously developed version of the activities for the concepts they thought would be hard to construct due to the abilities of students in this age range to use software) and asked accompanying questions, while others also provided

the tips from the preparation stages of the dynamic activity and aimed both at improving students' ability to use the software as well as at shaping their learning. Moreover, the preservice teachers used computer-assisted homework for summative purposes.

The following is PT8's dynamic activity and the accompanying questions preservice teachers had asked for the purpose of shaping learning. PT8 reported providing the previously developed activity in particular because the ability of 11-14 year olds to use Cabri3D software is not suited to the construction of this activity.
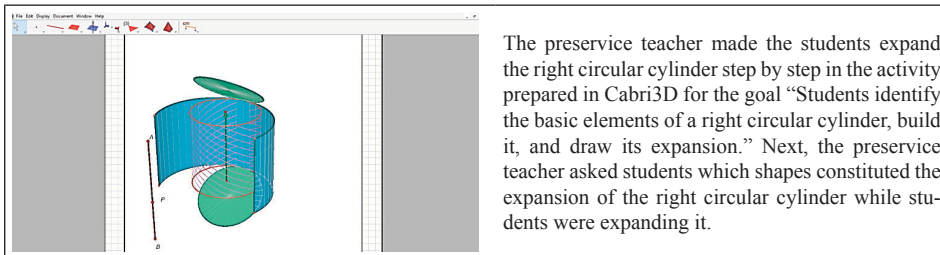


The preservice teacher made the students expand the right circular cylinder step by step in the activity prepared in Cabri3D for the goal "Students identify the basic elements of a right circular cylinder, build it, and draw its expansion." Next, the preservice teacher asked students which shapes constituted the expansion of the right circular cylinder while students were expanding it.

*Figure 5.* A portion of PT8's second micro-instruction.

Figure 6 additionally presents a computer-assisted homework used by the preservice teachers for summative purposes. PT38 gave the students activities similar to those in the microteaching for homework. To this end, the preservice teacher asked the students to do the file ödev.xlsx (*homework.xlsx*) on their computer desktops.



*Figure 6.* Screenshot of the homework assigned by PT38 in the second microteaching.

In summary, when examining the assessment-evaluation tools preservice teachers used and their purposes, they are seen to have benefitted from technology and mainly to have used formative methods in their assessment-evaluation processes for the second microteaching compared to the first.

**Findings Regarding the Component of Teaching Concepts in the Curriculum Using Technology**

From the preservice teachers' first microteaching and instructional plans, most of them are understood to prefer goals involving the statement "information and communication technologies can be benefited from" in the curriculum so that they can achieve technological integration. The preservice teachers were discovered to not have presented the objectives they'd set for the computer software used for teaching the concept they had reflected on nor for multiple representations of the concept and misconceptions as attainments.

In light of the goals in the instructional plans the participants' had prepared for the first microteaching, eight preservice teachers were seen to have performed activities above the level of related grade/age targeted by the curriculum. In the interviews performed with the participants about their self-evaluations, these eight were found to not know that they had conducted activities above the level of grade/age and that the other preservice teachers had not even criticized the level of their activities in their observations. The instructional process of one preservice teacher who had performed an above-level activity is in Figure 7.
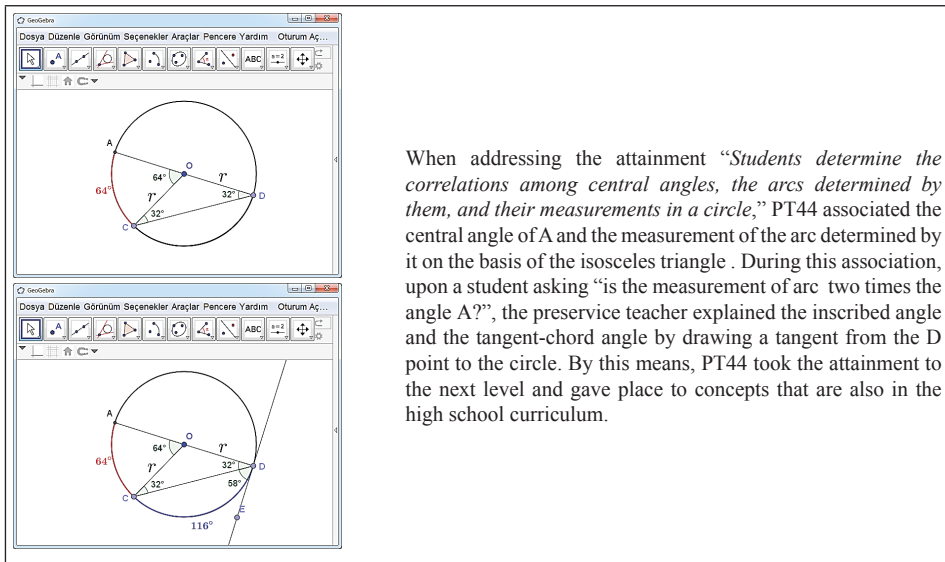


When addressing the attainment "*Students determine the correlations among central angles, the arcs determined by them, and their measurements in a circle*," PT44 associated the central angle of A and the measurement of the arc determined by it on the basis of the isosceles triangle . During this association, upon a student asking "is the measurement of arc two times the angle A?", the preservice teacher explained the inscribed angle and the tangent-chord angle by drawing a tangent from the D point to the circle. By this means, PT44 took the attainment to the next level and gave place to concepts that are also in the high school curriculum.

*Figure 7*. A portion of PT44's first micro-instruction performed above the level targeted by the program.

In the interviews on the self-evaluations, some preservice teachers emphasized that no activities were present regarding the use of technology for supporting instruction, either within the program or in the course books. They indicated that they had not managed to make their microteaching efficient because they couldn't evaluate how to address the attainments with the help of technology on various sources when planning their instructions. The following are their opinions on the subject.

[I] went to the schools and talked to the teachers there in the first place to make preparations. They couldn't help me because they hadn't used technology in this way, so I borrowed books from them for help. I couldn't find anything of use in the books. It was difficult to prepare when there was nothing to look up. (PT43)

We had always been told that technology use is important in the program and the course books. But I can't say they helped when I reviewed them all. There is nothing in the course books that was of any help as they weren't for teaching my goals. (PT17)

## Discussion, Conclusion, and Recommendations

This section involves the discussion of the findings specific to the components of TPCK. Recommendations have also been presented both specific to the TPCK components and its framework.

### Technology and Multiple Representations of Concept

When examining preservice teachers' instructional plans from the first microteaching, even though many participants used different representations, the preservice teachers were seen to not have inter-related the representations, which have an importance place in understanding the subjects conceptually. Previous studies show that the expected benefit can often not be obtained from using different representations in concept instruction (Ainsworth, Bibby, & Wood, 1998), and the most important reason for this is considered to be teachers leaving the representations' correlation to students (Mallet, 2007). Considering the age range (11-14 year olds) that the preservice teachers will teach, leaving the task of correlating representations to these students increases the odds that they'll have great difficulty doing this, not be able to notice the importance and necessity of this task, and/or concentrate on only one representation (Berthold et al., 2009; De Jong et al., 1998). After examining their peers' and self-evaluations, the effort participants put forth was revealed within the framework of interrelating representations in the second microteaching. The preservice teachers benefited from technology and the means offered by technology for different concept representations during their practices. Yet the fact that most preservice teachers did not benefit from technology for interrelating representations but instead tried to have the students discover these correlations through verbal discussions indicates the lack of a very important component of the technological integration process (Alagic & Palenz, 2006; Juersivich et al., 2009). The studies in the literature regard teacher incompetence in technology use, their lack of knowledge on how to present course content though technology, and inability to see the contribution of technology in concept presentations as the biggest obstacles facing the integration process (Hew & Brush, 2007; Pelgrum, 2001). However, none of these

obstacles mentioned in the literature justifies the fact that preservice teachers did not benefit from technology in correlating representations. Undoubtedly, one of the most important components for technological integration in mathematics teaching is teaching concepts by correlating representations through the benefits offered by technology (Juersivich, Garofalo, & Fraser, 2009). This point here is that the preservice teachers did not benefit from technology for interrelating representations even though they utilized the means offered by technology for the different concept representations. What one should actually consider here is how to benefit from technology for correlating representations.

To review the content of the application process within the scope of multiple representations, what they are, the importance of the role correlating them plays in concept teaching, and the representations that can be offered by different software packages have been addressed. The emphasis in this study is on how the different representations of mathematical concepts can be interrelated, the role played by technology in clarifying the correlations for the student, the importance of correlating representations in making the concept understandable with the help of technology, and the means and limitations of specific software. In light of the related component results in this study, one can say the preservice teachers are not competent in benefiting from technology, especially in correlating representations. Therefore, supporting the benefits and necessities that can be provided by correlating representations through different preservice activities is obviously important in being able to create awareness so that these shortcomings can be overcome.

### Technology and Student Difficulties Regarding Concepts and Misconceptions

When examining the instructional plans preservice teachers prepared in the first microteaching, they were seen unable to precisely clarify the concepts in the attainment examples in terms of difficulties, misconceptions, errors, and their correlations. In addition, the fact that most of the participants did not benefit from computer software when planning the methods of addressing these concepts is intriguing. Preservice teachers' knowledge of these concepts is important; only then will they be able to develop productive and useful strategies for readdressing students' understandings (Bingölbali & Özmantar, 2009; Shulman, 1986). Following their self-evaluations and after examining their peers' evaluations, the participants showed progress in terms of understanding difficulties, misconceptions, errors, and their correlations. Findings show that most preservice teachers understood these concepts and their relationships. Yet the preservice teachers were also found to have not made sufficient progress in using computer software efficiently for overcoming student difficulties. Even though they had identified suitable computer software suitable for overcoming student difficulties, they could produce no strategies clear enough to eliminate these difficulties. When considering TPCK as the

relationship between technology and content, and certain software being addressed as an example in terms of its content regarding a given mathematical concept (Mishra & Koehler, 2006), because the preservice teachers preferred appropriate software for specific student difficulties, progress can be said to have occurred in their TPCK. On the other hand, preservice teachers were unable to show sufficient progress in planning by benefiting from technological tools for overcoming difficulties regarding a given concept, which is the focal point of this study. Nevertheless, the limited way difficulties were addressed by the computer software packages in the instructional plans (i.e., difficulties in algebra used Excel, difficulty with graphs used Graphical Analysis) grabs one's attention. Other than this limitation, the reason all the difficulties for attainment were not addressed can be blamed on focusing on the difficulty more and creating a more realistic plan for better use of time. Another point that was neglected is that students' problems with the software were ignored. The preservice teachers only considered the issue of software language being in English. They stated that the basic functions and navigation buttons of the related software need to be explained clearly so that this problem can be eliminated. Most preservice teachers argued that students would not encounter any problems because almost all of them are accustomed to technology; if they were to face a problem, they could handle it by asking their peers or teachers for help.

The preservice teachers were unable to make sufficient progress in using the computer software efficiently for overcoming students' difficulties, even though the findings generally show that they had awareness and knowledge of the potential problems students might face in regard to the concepts from their plans and microteaching. In accordance with this result, the need to provide preservice teachers with environments in which they can select the appropriate software for overcoming difficulties regarding the concept(s), for making efficient plans to eliminate the difficulties by benefiting from technology, and for applying these plans efficiently so that they can show progress in related topics has now become obvious. Thus one should dwell on student difficulties regarding technology use much more and provide different preservice activities that can create top-level awareness of the potential problem sources and solutions.

### Technology and the Methods and Strategies for Instructing Concepts

According to the instructional methods and strategies preservice teachers' preferred in their applications, no general change occurred between their first and second microteachings. In these instructions, discovery, direct instruction, discussion, question & answer, and brainstorming were the most preferred methods and strategies for preservice teachers' first and second microteachings.

The preservice teachers can be said to have made progress in technological integration for the methods and strategies of the specified attainments. They not only used technology but achieved effective technological integration in their microteaching.

The participants showed this progress especially through the increase in the number of Level-3 activities they performed in their second microteachings after examining their peers' and self-evaluations after the first microteaching. According to Hughes (2005), achieving transformation and quality learning through technology is possible when technology is used at the third level. In parallel with the appropriate methods and strategies the preservice teachers used, how they started to use technology more efficiently is a significant indicator of progress regarding this component.

**Technology and Concept Assessment-Evaluation**

The analyses conducted for the assessment-evaluation component of TPCK revealed that the preservice teachers had made progress in being informed about the different technology-assisted tools of assessment-evaluation and using them in technology-assisted environments for formative and/or summative purposes. According to the preservice teachers' instructional plans for the first microteaching, their use of several assessment-evaluation tools in accordance with their different purposes yet only six using technology during the assessment-evaluation process is remarkable. In the interviews performed following their self-evaluations after they had examined their peers' evaluations, the participants showed knowledge of technological assessment-evaluation tools (e.g., computer-assisted worksheets, computer-assisted homework, e-portfolios, and concept-mapping software). Kissane et al. (1994) stated that the aspect of assessment-evaluation has been ignored in research studies on technology. Providing preservice teachers with a wide repertoire of technological tools is important so they can integrate technology with the assessment-evaluation approach in parallel with the learning process. Moreover, technology should obviously not be ignored in the assessment-evaluation process of technology-assisted instruction. However, preservice teachers, who are at the center of this study, being able to prepare and use these tools in the assessment-evaluation process is also important, as well as providing them with information about these tools in their instructional practices. When examining preservice teachers' second microteaching, most integrated technology with formative assessment-evaluation, which has positive impacts on learning (Black & Wiliam, 1998). The preservice teachers were observed to particularly have constructed computer-assisted worksheets for formative purposes in consideration of the age range they would be teaching. In addition, the preservice teachers used computer-assisted homework for summative purposes. As a result, the preservice teachers made progress in planning and using different technological assessment-evaluation tools for both summative and formative purposes of assessment-evaluation. Technologically integrated assessment-evaluation, which teachers and preservice teachers have been emphasized for not taking into consideration (Kissane et al., 1994), was used efficiently by the preservice teachers in this study.

**Teaching the Concepts in the Curriculum Using Technology**

The preservice teachers were seen to prefer attainments in the curriculum involving the statement "information and communication technologies can be benefited from" for achieving technological integration. They were discovered to not present the objectives they had set for the computer software used in instructing the concept they had reflected on, nor for the multiple representations of concepts and misconceptions as attainments. Thus, in a general sense, the preservice teachers identified some new attainments while addressing the attainments in the curriculum. A similar finding was interpreted by Demir (2011) as an indispensable part of efficient technological integration, and the differentiation in attainments were stated to result from performing several actions with technology that would otherwise not be possible during instruction. Yet based on this perspective, differentiation in performing several actions through technology and achieving attainments may cause activities to be conducted way above the level targeted by the curriculum in some cases (Harris et al., 2009). In these cases, students subjected to concept instruction above the curriculum-specified level may think that using technology makes it harder rather than easier to learn. Some of the preservice teachers were seen to perform activities above the curriculum-targeted level in the microteaching. In the interviews conducted with the participants on their self-evaluations, they were seen to ignore the distribution by year for the information that students need to learn throughout their schooling, as well as information on subjects and attainments specific to this end. The lack of criticism for preservice teachers' performance of activities above students' levels by the other participants in the evaluations is also intriguing. This indicates that the preservice teachers had not considered the information on the subjects and concepts that students of a certain grade should learn in a year. This was addressed by Magnusson et al. (1999) as horizontal information in information content regarding the curriculum.

When reviewing the content in the study's application process, the importance of investigating and discussing how attainments on the axis of vertical articulation are taught by age to increase the awareness of vertical articulation becomes obvious when addressing the component of teaching concepts in the curriculum using technology. Another point the findings shed light on is the lack of materials that can be used in technology-assisted mathematics teaching that are prepared according to the curriculum. With the inclusion of technology into the process, teaching and learning are fundamentally altered (Kieran & Drijvers, 2006). This leads to the need for teachers to use resources. This need was also put forth by the participants. The preservice teachers emphasized the lack of activities regarding technology use for supporting instruction, either within the program or in the course books. As a consequence, they stated being unable to make their microteaching efficient because they could not evaluate from any resources how their attainments were being addressed with the help of technology when planning their instructions.

Self-Efficacy Scale in relation to Computer-Based Education and the Computer-Assisted Mathematics Instruction Questionnaire were applied before and after the microteaching applications in consideration of the key role of preservice teachers' self-efficacy perceptions and opinions on computer-based education. Preservice teachers' perceptions of self-efficacy regarding computer-based education and their opinions on computer-assisted mathematics teaching were examined, and no statistically significant difference was found between their pretest and posttest mean scores. Even though the quantitative results can make one think that no reinforcement of the qualitative results occurred, one reason why no significant difference was found may be how the pretest and posttest were applied upon a certain level of awareness, as the preservice teachers had already completed their infrastructure on TPCK in previous courses (i.e., Mathematics Curriculum, Applications of Technology in Education, Instructional Technologies and Material Design). If the pretests from the data collection instruments had been applied before the students had taken those courses, more significant and detailed results could possibly have been achieved.

Consequently, the preservice teachers used technological tools to achieve the goals they specified in their instructional plans. When considered within this study's TPCK framework, the findings show the preservice teachers to have made noteworthy progress along the axis of TPCK components. Efficient microteaching practice plays a significant role in this progress. Allowing preservice teachers to monitor their in-class performance (Wakwinji, 2011), allowing them to gain experience on what to do or not do when instructing in real classroom settings (Gürses, Bayrak, Yalçın, Açıkyıldız, & Doğar, 2005; Marulcu & Dedetürk, 2014), and allowing other participants to monitor and evaluate, as well as providing instructional videos to enable self-evaluation (Fernandez, 2005; Kpanja, 2001; Peker, 2009), are of great importance in the effective practice of this method. This indicates the shortcomings and proposed solutions presented in the discussion, as well as the effectiveness of microteaching programs based on the TPCK conceptual framework. Furthermore, developing instructions to ensure that this microteaching method can be applied in real classroom settings and also be applied in undergraduate lessons (including school experience and teaching practices) are recommended.

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