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Research Article

Teachers' Diagnostic Competences and Levels Pertaining to Students' Mathematical Thinking: The Case of Three Math Teachers in Turkey*

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Abstract

The main purpose of this research is to determine mathematics teachers' diagnostic competency levels. Diagnostic competence, which is described as the ability to understand and analyze student thinking, has been examined in four levels through the components of teachers' general knowledge on learning processes and their skills at considering, scrutinizing, and interpreting student thinking. The research model is based on a case study, and the participants consist of three elementary mathematics teachers. Teachers' lessons were observed for 17 weeks, and data has been collected through (a) observation notes, (b) video recordings, (c) written documents, and (d) interviews with teachers. According to the findings, the teachers have Level-2 diagnostic competences due to their limited general knowledge on learning processes and scrutiny skills. Additional findings are as follows: (a) While teachers have been defined as having Level-2 diagnostic competences in the classroom, lower and higher levels seem able to emerge in environments that used prompts. (b) The limitations of teachers' general knowledge on learning processes can also weaken other diagnostic competency skills. (c) Teachers care more about students' errors and mistakes when diagnosing and do not feel the need to diagnose ideas that provide correct results.

Keywords

Diagnostic competence • Consideration • Scrutiny • Teacher knowledge • Interpretation

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There is a big difference between good teaching and effective teaching. Good teaching is about what a teacher does while teaching, whereas effective teaching relates to what learners can learn from this teaching (Airasian, 2001). This is also true for teaching mathematics; teachers are expected to have more math skills than math knowledge (Cooney, 1999). For example, skills can be mentioned such as using one's understanding of mathematical knowledge when teaching and seeing one's students both as individuals and as math learners (National Council of Teachers of Mathematics [NCTM], 2000; Shulman, 1987).

To effectively teach math, teachers need to be able to analyze their pupils' and own actions towards teaching and consider how these actions can affect student learning (NCTM, 2000). For this necessary consideration, teachers are expected to be able to interpret students' perceptions as well as their learning outcomes (Aufschnaiter et al., 2011; Cooney, 1999; Zembat, 2013). This interpretation requires being able to analyze students' perceptions and error sources (Ball, Hill, & Bass, 2005) and to understand student thinking in the context of valuing pupils' ideas (Dewey, 1902; Duschl, Schweingruber, & Shouse, 2007; Graeber 1999). Teachers' abilities to analyze and understand student thinking have been introduced into the literature through the concept of *diagnostic competence*.

Diagnostic competence can be associated with skills for accurately evaluating individuals (Brünken, 2009; Helmke, Hosenfeld, & Schrader, 2004), with the analytical skills required for performing this assessment (Edelenbolos & Kubanek-German, 2004; Prediger, 2010), or with both analytical and evaluation skills (e.g. National Science Foundation [NSF], 2007). Prediger (2010) mentioned that this competence is used for teachers' skills in understanding and analyzing pupils' thinking process –without any concern for grading them. Therefore, diagnostic competence is necessary for making assessments in teaching (Edelenbolos & Kubanek-German, 2004) and involves more skills than just judging student achievement (Schwarz, Wissmach, & Kaiser, 2008). Similarly, this study associates diagnostic competence with understanding and analyzing the nature of student thinking (Prediger, 2010), and its theoretical framework is structured around this meaning.

What is Diagnostic Competence?

The first thing that comes to mind with the concept of diagnosis is the health sector (such as doctors' diagnoses); the word diagnosis is even oft seen used in the field of medicine (Edelenbolos & Kubanek-German, 2004; Hoth et al., 2016). Studies carried out in the field of education on this concept have usually been implemented by German scholars. According to Schrader (cited in 2001, Schwarz et al., 2008), who is one of the most frequently mentioned names, diagnostic competence is the ability of an evaluator to analyze the performance of individuals according to predefined categories, terms or concepts, or readiness in this regard. From another aspect, diagnostic competence is both knowledge and skills in understanding and assimilating students' learning processes as well as their current learning difficulties in order to assess (Deutsches PISA Konsortium, 2001, p.132, as cited in Richter, 2010). This competence can be used not only for student assessment or analysis, but also for adapting to student-centered instruction (Helmke et al., 2004). Therefore, the purpose for which and how diagnostic competence is used is important. This issue becomes clear under the following headings.

What is the Role of Diagnostic Competence in Teacher Education?

The role of diagnostic competence in teacher education becomes a greater issue with the idea that it is necessary for successful and effective teaching (Helmke & Schrader, 1987; Klug, 2011). This role can be explained by the fact that diagnostic competence reflects the education and training aspects in terms of diagnosis and the profession aspect in terms of competence. In this respect, diagnostic competence establishes a bridge between the teaching profession's requirements and education (Buch, 2008; Klug, 2011; Schrader, 2006). In addition, this competence includes skills that can help a teacher become more qualified (Edelenbolos & Kubanek-German, 2004) and is used in the formation of judgments that constitute the basis for decisions concerning students (Brüenken, 2009).

Diagnostic competence can be used for designing classroom environments to meet the needs of students (Klug, Bruder, Kelava, Spiel, & Schmitz, 2013; Opdenakker & Van Damme, 2006) and in adapting activities that can make learning more effective according to students' needs (Hoge & Colodarci, 1989; Klug, Bruder, & Schmitz, 2016). If one wants to move teaching ahead, one must be able to determine students' weaknesses and strengths, where they start, where they currently stand, and what they are capable of (Klug et al., 2016). Thus, this competence is important in terms of the necessities teachers may need when teaching; a teacher's specialization in diagnosis positively contributes to teaching and interactions with students.

According to Prediger (2010), a need exists for diagnostic competence in two cases in particular. The first one is for determining student thinking, which Prediger defined as the "individual starting point of the learning process" (p. 76); the second is for maintaining the sustainability of the learning process, That is, adequate diagnosis is important for both being able to identify student thinking and taking their ideas into consideration when designing and implementing teaching. In the literature, studies in which teachers' diagnostic competence were used in the process of designing or implementing teaching have gained in intensity (e.g., Cohen, 2004; Kunter et al., 2013; Prediger, 2010). Along with these studies, the role of diagnostic competence in the sustainability of the learning process has become more prominent. As Prediger (2010) points out, however, diagnostic competence is not just necessary for the sustainability of the learning process. This study differs from those in the literature in that diagnostic competence has been researched only in order to identify student thinking.

How Can One Analyze Teachers' Diagnostic Competences?

Answering this question is important in the approach to diagnostic competence. Studies that associate diagnostic competence with accuracy in judgment (e.g., Colodarci, 1986; Demaray, & Elliot, 1998; Helmke & Schrader, 1987; Martinez, Stecher, & Borko, 2009) perceive this concept as consistency of assessment. These studies have compared students' actual performances with teachers' predictions about their performances. The coefficient of concordance between teacher judgment and student performance has been described as an indicator of a teacher's diagnostic competence. Thus, studies on diagnostic competence that reconcile student performance with student success concentrate their basis on diagnosing student achievement (Kaiser, Retelsdorf, Südkamp, & Möller, 2013; Klug et al., 2013; Klug et al., 2016). Nevertheless, these studies have left a question mark in mind as to what kind of consequences may result from accurately diagnosing student achievement (Klug, et al., 2013). According to Klug et al. (2013), such a diagnosis cannot provide information on how teachers should shape their teaching or what kind of support they can give students.

Although correlating diagnostic competence with accuracy of student achievement is still an important point of view, it is no longer the only solution (Hoth et al., 2016; Klug, 2011; Klug et al., 2013). Studies also exist that regard this competency as a process and deal with acquiring specific skills and knowledge use. For example, Klug (2011) developed a process model and examined teachers' diagnostic competences in terms of learning behaviors. On the other hand, Prediger (2010) examined this competence through its components, but her explanations about these components lack clarity on what qualifications a teacher should have to be thought of as sufficient at diagnosing.

As noted above, studies on diagnostic competence often focus on student success. As so many factors affect achievement, diagnosing student achievement seems to have a structure that can turn into an unclear and complex situation. Approaching diagnostic competence through a clear and deep concept instead of through success is thought to be more meaningful. Prediger (2010) chose to base diagnostic competence on student thinking. With this approach, Prediger (2010) focused on the meanings of mathematical concepts and inspired us with the idea of mathematical thinking. In this study, we aim to determine diagnostic competency towards diagnosing situations unique to mathematics that underlay student thinking.

According to Dewey (1910), thinking implies what is in the mind, and one can only think about something they have not seen, heard, smelled, or tasted directly. While Dewey is assumed to have aimed at pointing out the process of acquiring very conscious knowledge of things not received directly by the senses (as opposed to an automatic process of information acquisition), thinking being defined as conscious action occurring in the mind and resulting within the production of ideas seems appropriate. Various definitions of mathematical thinking are found in the literature (e.g., Tall, 2002; Stacey, Burton, & Mason, 1985; Stenberg & Ben-Zeev, 1996; Yıldırım, 2011). In this study, mathematical thinking will be used to mean conscious actions resulting with a mathematical idea. So, in which way and how can these conscious mental actions of students be diagnosed? What role do teachers play in this process? The question that teachers and teacher educators wonder is whether individuals who teach are able to diagnose their students' mathematical thinking and, if so, to what extent. If teachers' diagnostic competence is considered as a journey, determining this competence is very important in terms of illuminating the starting point, direction, and distance to go.

The Components of Diagnostic Competence

According to Prediger (2010), diagnostic competence has various components. These components have been found appropriate for the framework of our research, adapted to the conditions of Turkey, and clarified in the following framework.

Diagnostic competence has four different components. Understanding skills includes consideration and scrutiny; analytical skills include knowing/implementation, and interpretation. Now we will explain these components and what meanings are assigned to these components within the scope of this study.

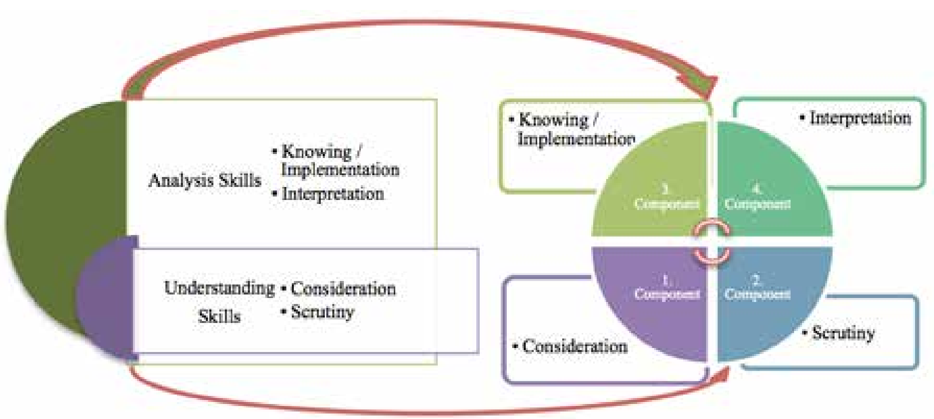

Figure 1. The components of diagnostic competence.

Consideration. The literature explains that teachers should focus on their students' thinking when teaching (e.g., Ball, 2001; Franke & Kazemi, 2001; Levin, Hammer, & Coffey, 2009; NCTM, 2000; Schifter, 2001; Schoenfeld, 2000). This focus can be achieved by showing concern for students' thoughts and being involved in the way they think (Graeber, 1999; Levin et al., 2009). Prediger (2010) has linked this component, which she named *interest in student thinking*, to teachers' curiosity towards students' ideas. She mentioned that teachers may not show the necessary care towards students' thoughts without this curiosity.

Scrutiny. Dewey (1902) assumed concepts to be the product of searching, inquiring into, and being curious about the truth, using the term *psychologizing* to refer to the situation of looking from the learner's perspective. According to him, the subject area is not just its formal form but also the form in which it was learned. A part of skillfully teaching mathematics requires teachers to show a percept of students' ideas (Ball & Cohen, 1999) and to look at their teaching from students' perspective rather than their own (Ball, 2013; Ball et al., 2005; Selter, 2001). The important thing in this component, to which Prediger (2010) refers as the *tendency of teachers to influence student thinking,* is that the perspective should change from immediate judgments that can lead to error-oriented decisions (What is wrong with the student's response?) towards understanding the inner rationality of student thought (In which circumstances does the student's thinking become meaningful?).

Knowing/Implementation. This is a complex process where teachers' shift themselves from a self-centered perspective to seeing concepts through students' eyes (Ball, 2000; Ball, 2001; Ball, & Cohen, 1999; Schifter, 2001). It includes the ability to hear and comment on what the learner says, as well as to expertly scrutinize when students' ideas lack clarity (Hill & Ball, 2009). Therefore, a teacher who considers and anticipates students' ideas also uses some knowledge, dealing not only with content knowledge but also with merging ideas together and arranging for more thinking than knowing (Ball, Lubienski, & Mewborn, 2001).

Prediger (2010) talks about this component's theoretical structures, referring to them as *teachers' general knowledge of learning processes*, presenting Tall and Vinner's (1981) definition and image of the concept as well as Shulman's (1986) pedagogical content knowledge as examples of theories that might be useful. However, no explanation is found as to how, where, or which theory may be useful. At this stage, we asked which subcomponents are necessary for teachers to analyze and understand their students and agreed it would be appropriate to classify them as teachers' (a) subject matter knowledge, (b) knowledge of students' understanding, (c) knowledge of the curriculum, (d) knowledge of instructional strategies, and (e) knowledge of assessment.

Interpretation. The purpose of this component, which Prediger (2010) referred to as *content-specific mathematical knowledge for teaching and analyzing with focus on meaning*, is to analyze the difference between a given mathematical concept and the state of this concept in a particular student. The difference between the ideas in the students' minds and what they mention usually can be characterized through the meanings students assign to mathematical concepts (Prediger, 2010). Additionally, a concept's meaning strictly connects to its definition (Kilpatrick, Hoyles, & Skovmose, 2005), and determining the meaning a student has is possible by referring to their definition of that concept (Adler, 2005). Thus, the skill of interpretation can be examined in terms of students' conceptual definition and individually placed meanings (Argün, Arıkan, Bulut, & Halıcıoğlu, 2014).

When taking into account this meaning of teachers' diagnostic competence, a hierarchical structure is seen to exist among the skills of consideration, scrutiny, and interpretation, which are components of this competence. Therefore, a teacher's ability to first consider, then scrutinize, and finally interpret students' mathematical thinking has become accepted for being able to diagnose student thinking. Teachers' knowledge about learning processes and how they use this knowledge when diagnosing is a component that must be investigated within other components. This resulting hierarchical structure gave us the idea that we can discuss this competence through levels. In this context, the study discusses *levels of diagnostic competence* for the first time in the literature. This study researches diagnostic competence in terms of four different levels (described under the heading of Method). Therefore, the research purpose is to examine elementary mathematics teachers' levels of diagnostic competence. In accordance with this aim, the answer to what level elementary school mathematics teachers have regarding diagnostic competence is researched, and sub-questions are identified as follows:

(a) How are elementary mathematics teachers' skills in understanding student thinking? (a1) How are elementary mathematics teachers' skills in considering student thinking? (a2) How are elementary mathematics teachers' skills in scrutinizing student thinking? (b) How are elementary mathematics teachers' skills in analyzing student thinking? (b1) How is elementary mathematics teachers' general knowledge of learning processes? (b2) How are elementary mathematics teachers' skills in interpreting student thinking?

Method

This study aims to determine elementary mathematics teachers' levels of diagnostic competence without any generalized purpose using an interpretive approach. As such, the qualitative approach has been adopted with a focus on the nature of the research question, "How are elementary mathematics teachers' diagnostic levels?" This competence has been studied through a process using its contexts and components. Therefore, the study design constitutes a case study model with a search for how and why (Merriam, 1998; Yin, 2003), presenting all the factors related to the situation holistically (Yıldırım & Şimşek, 2011).

Case studies have a long-term design with undefined boundaries between the studied case and its context; they aim for the contextual conditions that might explain a phenomenon (Yin, 2003). In this study, the phenomenon of diagnostic competency is defined as a condition with levels and researched in a context that can be understood in its own environment (Gillham, 2000), teachers in this case, through the support of various data sources. This is because the levels of diagnostic competency gain meaning with the teachers and can manifest alongside teachers' contextual conditions such as their knowledge and skills. In addition, an in-depth examination on the subject has been carried out during the research and, as Merriam (1998) stated for case studies, the focus is on the process rather than the results, on the environment rather than a specific variable, and on exploration rather than proof.

Participants and Data Collection Tools

The participants of the research constitute three volunteer elementary mathematics teachers who were identified using the convenience sampling method (Patton, 2002). The first participant, coded T_Y, stated having had 8 years of experience. The second participant, coded T_G, stated having had 18 years of experience. The last participant, coded T_H, stated having had 20 years of experience. After identifying the participants, the teachers' lessons were video-recorded and in-class observations were made. After the observations, interviews were conducted, and various written documents such as photocopy papers (including problems solved during the course) were collected from the teachers. Therefore, the data collection tools used in this study are (a) observations, (b) video recordings, (c) interviews (conversation-style interviews [Yıldırım, & Şimşek, 2011] and semi-structured interviews), and (d) written documents.

Observations have been carried out through the role of non-participant observer (Creswell, 2003; Fraenkel, Wallen, & Hyun, 2012), as teachers' diagnostic competencies should be observed in their natural environment. The primary author watched each teacher for an average of four hours per week for 17 weeks and conducted conversation-style interviews. Video recordings from these classes and observation notes were shared with the second author at specific time intervals and discussed in the context of diagnostic competencies. While watching the video recordings, diagnostic processes were determined and classified below; interview questions were also prepared. Semi-structured interviews were held with each teacher for an average of one hour.

The diagnostic processes observed while analyzing the video recordings have been determined to parallel the subjects students have difficulty understanding in the literature (for more information, see Bingölbali & Özmantar, 2009; Özmantar, Bingölbali, & Akkoç, 2008). Type-1 diagnostic processes mainly relate to recordings thought to be of pedagogical origin, such as perceiving relatively prime numbers' greatest common denominator (GCD) to be zero, wherein students' mistakes are clear. Type 2 relates to recordings thought to be of epistemological origin, such as exponential numbers and algebraic expressions, wherein students' mistakes are not clear. The recordings, which the teachers were shown during their interviews, have been classified according to this distinction first, then subjected to a secondary classification. The second classification is based on whether the recordings in the first classification are special cases encountered from one teacher or general situations encountered from more than one teacher. After these classifications, seven different class sessions, which is thought to provide maximum diversity, have been shown during the semi-structured interviews to each teacher, at least one of which is their own class session recording.

Data Analysis

When analyzing the data, observations and video recordings have been taken as the primary basis. After examining these records, diagnostic processes were revealed and transcribed. The paths for each component have been clarified, and meaningful data units have been identified and coded through diagnostic processes. Afterwards, one-on-one interviews were conducted with the teachers, and the interview records were compared with the previously obtained data from the transcriptions. Data not included in the transcripts, such as non-verbal responses like teachers' shaking their head at a student's answer, must not be overlooked. That is why transcripts and video recordings have been used together in the data analysis. The level indicators are worth mentioning in order to be able to answer what elementary school mathematics teachers' levels of diagnostic competences are in this process, and we have elaborated deeply in the following paragraphs. Taking into account the skills of consideration, scrutiny, and interpretation (the components of teachers' diagnostic competence), we have formed the levels of diagnostic competence as follows:

(a) A teacher who only considers a student's mathematical thinking indicates Level-1 diagnostic competence.

(b) A teacher who considers a student's mathematical thinking and scrutinizes directly with a solution indicates Level-2 diagnostic competence.

(c) A teacher who considers a student's mathematical thinking and scrutinizes by clarifying indicates Level-3 diagnostic adequacy.

(d) A teacher who considers a student's mathematical thinking, scrutinizes by clarifying, and interprets indicates Level-4 diagnostic adequacy.

Because the literature lacked provisional support while developing these levels, the doctor-patient relationship was believed to be a good model in consideration of the insight it provides into the relationship between teacher and student (Çelikten, 2006; Saban, 2004). For this reason, a specialist was interviewed about what to pay attention to when making a diagnosis. In this interview, the specialist stated to (a) listen to the problems of the patient, (b) ask questions about the problem (e.g., How long has it been going on? Has pain increased or decreased?) and case history (e.g., Have you ever had a similar illness? What genetic diseases run in your family?), (c) make a general physical examination (e.g., Listening to patient's breathing) and a special physical examination (peculiar to the area of specialization) for the problem, and (4) request various tests (e.g., blood test). In addition, the specialist stated that diagnoses and alternative diagnoses come to mind in the first three stages approximately 60-70% of the time, and a clearer diagnosis is made by comparing the results of the tests and the data gathered in the fourth stage.

To establish an analogy between a doctor's diagnosis and a teacher's diagnosis, the ability of a teacher to diagnose student thinking can be correlated in the following stages.

Consideration. This resembles the stage where the doctor listens to the patient. This skill is based on the assumption that, because a doctor cannot diagnose patients without listening to them (Bowen, 2006; Özkan, 2008), teachers cannot make a diagnosis without considering their students' thinking. When analyzing data related to this component, the data that emerged during diagnostic processes in the classroom constitute the basis; data from the interviews have also been used to support the underlying data. Below is the route for the component of consideration.

Table 1

Route Map for the Component of Consideration

Question asked for the analysis of consideration data	What needs to be known?	How can it be known?	
		Data sources	Major criteria
How can a teacher show interest in student thinking?	In what ways can student thinking be observed?	Video recordings Observations	-Students' verbal explanations -Students' written explanations - Students' hesitations
	In what ways can a teacher consider student thinking?	Video recordings Observations	-Considering students' explanations / hesitations -Not considering students' explanations / hesitations

When students' thinking emerges, a teacher can adopt two different ways: (a) the teacher can consider this thinking or (b) not consider this thinking. In order for a teacher to be able to talk about a diagnostic competence, situation (a) is expected to occur; this is called Level-1 diagnostic competence.

Scrutiny. It is similar to the stage in which a doctor asks a patient about the problem and case history (Bowen, 2006; Özkan, 2008). This skill is based on the assumption that the teacher must find a way to get into the students' thinking to clarify it. There are three different forms of scrutiny in the literature, and by questioning what path is taken, we have named them as: (a) direct scrutiny (a way of clarifying using direct questions), (b) indirect scrutiny (a way of clarifying using indirect questions; Ball & Forzani, 2009), and (c) not showing an error-oriented approach (this contains steps to clarify thinking by allowing students to discuss among themselves). The route of the data analysis for this component is shown in Table 2.

Table 2
Route Map for the Scrutinizing Component

Question asked for analyzing data on scrutiny	What needs to be known?	How can it be known?	
		Data sources	**Major criteria**
How can a teacher get involved in student thinking?	What kind of tendencies can teachers show when considering student thinking?	Video recordings Observations Interviews	- Shows an error-oriented tendency towards student thinking; Starts direct treatment -Shows a tendency to understand the in-trinsic flux of student thinking, clarification (direct/indirect scrutiny, doesn't show an error-oriented approach)

A teacher's consideration of student thinking has been mentioned for being able to talk about Level-1 diagnostic competence. From the moment, a teacher considers student thinking, two different ways can be adopted: (a) starting direct treatment according to the possible diagnosis one has in mind, or (b) clarifying student thinking more. Situation (a) also has a diagnosis, of course, but this diagnosis is only as sound as the diagnosis a doctor makes listening to the patient. Thus, a teacher who starts direct treatment is accepted as having Level-2 diagnostic competence, and clarifying is accepted as having Level-3 diagnostic competence.

Knowing/Practicing. This stage has similarities with the stage where doctors examine the patient and use their medical knowledge (Bowen, 2006; Clancey, 2014). Teachers must use the necessary information when diagnosing their students. Other people who are involved may have this information, but the part that should be in this component is information usage. This skill is based on the assumption that as the doctor can make a diagnosis using the necessary information (Clancey, 2014) a teacher can only make a healthy diagnosis by using general knowledge of learning processes. Moreover, this component is not considered to be a factor in determining levels but as a necessary component of the levels.

Fennema and Franke (1992) mentioned that teacher knowledge is meaningful alongside the environment where teachers teach and that this knowledge should be studied within the teaching environment. Acting on this thought, the information given in Table 3 is considered to be meaningful in the classroom (the natural environment) while determining teachers' level of diagnostic competence. Therefore when analyzing data related to this component, the data generated in class have been taken as the basis, and the data obtained from the interviews have been used to support the underlying data. Unlike the other components, the diagnostic processes occurring within the classroom have not been sufficiently considered, and information from teachers on all the observed processes has also been analyzed.

Table 3
Route Map for Knowing/Implementation Component

Question asked for the analysis of knowing/practicing data	What needs to be known?	How can it be known?	
		Data sources	**Major criteria**
How can teachers use their general knowledge of learning processes while diagnosing student thinking?	How a teacher's knowledge on the following areas can be observed: Subject matter Students' understanding Curriculum Instructional strategies Assessment	Video recordings Observations Interviews Written documents	Performance Indicators (Categories like mathematical method, student pre-knowledge, learning outcome knowledge; mostly adapted from Turkey's document on mathematical competencies)

Interpretation. This has similarities with the stage where the doctor compares all on-hand data with the results from the tests. This skill is based on the assumption that the teacher must make a comparison between the data obtained up to now and the structure of mathematical ideas and concepts. In the context of concept definition and the meaning that individuals place on concepts, the following is expected:

a) The teacher should distinguish whether a mathematical idea belonging to the student is formal knowledge; a concept image, experience, or perception of the student; or a relative combination of these two.

b) The teacher should be able to compare students' mathematical ideas with their created forms, analyze them, and determine how well they match with each other, where they match, and where they don't.

The route for the criteria that will guide to determining the participant elementary mathematics teachers' interpretation skills is shown in Table 4.

Table 4
Route Map for Interpreting Component

Question asked for analyzing data on interpretation	What needs to be known?	How can it be known?	
		Data sources	**Major criteria**
How can a teacher interpret student thinking?	When a teacher recognizes/considers student thinking and shows a tendency towards this thinking's internal meaning, what ways is the teacher able to interpret this thinking?	Video recordings Observations Interviews	-Distinguish the structure of student thinking (formal, intuitive, or conceptual imagining) -Comparing student thinking with formal structures of the mathematical idea

Skipping the stage where the doctor makes use of the test results can result in an incorrect diagnosis and, therefore, an incomplete/incorrect treatment. The same is true for a teacher's diagnostic competence. Misdiagnosis and, consequently, incomplete treatments can be made as long as interpretation goes unused. Talking about that similarity between the doctor and the teacher from a different standpoint is also useful. If what determines the quality of the doctor's diagnosis is the proper progression of all these steps, one can say the same applies to teachers as well. In other words, the strength and quality of a teacher's diagnosis is determined by the level of diagnostic competence.

Validity and Reliability

For the validity and reliability of this study, the criteria proposed by Lincoln and Guba (1985) (credibility, transferability, dependability, and confirmability) are discussed. For example, long-term interaction has been established for the credibility of the research, data triangulation has been carried out, and an expert study has been conducted. The purposeful sampling method and detailed descriptions have been used for transferability. For dependability control, appropriate criteria have been defined for each component, and explanations have been given for the terms that can lead to confusion. For confirmability, direct quotations have been used, and data have been analyzed multiple times at different points.

Findings[2]

The purpose of this study is to examine mathematic teachers' level of diagnostic competence regarding student thinking. In the findings presented for this purpose, quotations have been presented marked with the initial of the teacher speaking (Y, G, and H for teachers; A for the researcher; or S for the student). In the non-dialogue findings, teachers have been referred to as T_Y, T_G, or T_H for teachers Y, G, and H, respectively.

The findings have been presented by first examining T_Y, then T_G, and finally T_H through their diagnosis skills. Teachers' diagnostic competence and levels are recognized by teachers' ability to understand and analyze student thinking and by their consideration, scrutiny, knowledge/practice, and interpretation skills. Teacher knowledge, a component of diagnostic competence, has been accepted as a necessary preliminary component for competence level. Therefore, starting with teachers' knowledge through to the presentation of the findings is thought to make for an easier read.

2 For detailed information on all the findings mentioned under this heading see Kaplan, 2015.

T_Y's General Knowledge of Learning Processes

T_Y generally composed associations among mathematical ideas and provided environments that promote mathematical reasoning. For example, when teaching the learning outcome for students to be able to divide using fractions, T_Y focused on division's meaning of finding the total number of one type of unit contained in another unit and started teaching using two fractions with the same denominators. Primarily, T_Y preferred fractions with numerators that can be fully divided with no remainder (e.g., 3/4 ÷ 1/4) and then used numerators that left a remainder (e.g., 7/9 ÷ 2/9), asking questions about how to find solutions using strip modeling. After finding solutions, T_Y presented an environment related to the abstraction that numerators are divided into each other when dividing two fractions with the same denominator; this time T_Y asked how two fractions with different denominators can be divided. T_Y, in whose classes we could frequently observe this and similar kinds of processes, has the principle of teaching class from basic to complicated (from known to unknown), in addition to using several and effective strategies for teaching mathematical thinking. However, T_Y occasionally experienced problems sustaining students' active participation; this can complicate students' application of the reasoning and associations being taught.

Based on T_Y's work experience, T_Y claimed that having problems related to GCD and least common multiples (LCM), which includes the concept of volume in the 6th graders' book, to be inappropriate. Because calculating the volume of three-dimensional objects is placed as a future outcome and, considering that the concepts of GCD and LCM are already challenging enough for the students, relating them to volume is not yet feasible. This finding, supportable by this and other similar kinds of data, is related to T_Y's knowledge about the distribution/sequencing of outcomes in the teaching program and scrutiny of the teaching materials. Additionally, another finding is that T_Y knows about students' previous knowledge, both in terms of learning outcomes and in terms of student understanding.

T_Y usually pays attention to individual differences, constructing other ideas based on what students already know. For example, when teaching division by negative whole numbers, T_Y first reminded students of division by natural numbers and its meanings (allocation, finding out how many of one whole another contains). Afterwards, by using division's allocation meaning, T_Y implied -12 : 3 = -4, and by using the meaning of finding how many of one whole in another, implied -12 : -3 = 4. Additionally, T_Y cares about developing students' problem-solving and problem-posing skills, presenting environments useful for interpreting a problem. T_Y's practices are devoted to not only developing practical skills but also to providing conceptual understanding.

T_Y's ability to understand students' thinking.

Consideration. On the occasions when student thinking emerged, T_Y has been observed to consider their thinking each and every time. This consideration has been observed through T_Y's body language (i.e., shaking the head to mean yes or no), explanations, and changes in tone of voice (see Table 5).

Table 5

Percentage Table for T_Y's Understanding of Student Thinking

T_Y's	Categories and Percentages
Diagnostic processes not considered	0%
Ways of consideration	Using body language (14%) Providing comments about thinking while explaining the solution (80%) Changing tone of voice (6%)
Environment in which a considered thought has shown up	Student's oral explanations (88%) Student's written explanations (10%) Student's hesitations (2%)
Tendencies toward student thinking	Starting direct treatment/Inclined toward errors (98%) Inclination toward understanding inner perspective (2%)
Factors determining the types of scrutiny	Whether or not the error is obvious in student thinking
Types of scrutiny	If the error is obvious; providing guidance (46%) and making corrections (48%) If the error is not obvious; reinforcement of content (6%)

Scrutiny. As can be seen from Table 5, T_Y usually starts with direct treatment when student thinking shows up. Therefore, T_Y can be said to be mostly error-oriented toward student thinking. This approach appeared in three different ways: If a mistake is evident, T_Y either corrects or directs student responses; if an error is not apparent, T_Y reinforces the content.

T_Y's ability to analyze students' thinking.

Interpretation. In the interviews, T_Y mentioned concept images that students might have when expressing their ideas attributed to the possible meanings of simple fractions and notations for them (i.e., it should be a single digit in the whole number section). For example, T_Y commented on a student's answer of 7,520 answer to the question "What is the largest/smallest numbers that can be made from the digits 7, 5, 2, and 0" and said the students as well as those on the video had similar misconceptions (limiting the number of digits). According to T_Y, the students usually think there must be at most two or three digits in total in the whole part, and the students on the video thought there should have been one digit. In addition, T_Y stated that the students on the video had not placed a zero in the integer part because they did not consider zero as an integer. After this phase, T_Y added the following about the possible reasons behind students' thoughts regarding not placing zero in the integer's ones column:

It could be about this: When we mention mixed fractions, we say 2 ¼, but we never say 0 ¼. I mean, maybe the child takes inspiration from that. Maybe they think that zero never comes at the beginning because it is not written as an integer. All of this is possible.

When expressing simple ratios as fractions, T_Y stated that the integer part is not written (e.g., integer part of 1/3 is zero but does not need to be written in this representation). This means that the student might expect a similar situation when using decimal notation. In the teacher's opinion, students have trouble positioning zero when they don't see it in a representation.

T_Y's level of diagnostic competence. T_Y has Level-2 diagnostic competence. During the observations, the observer rarely encountered processes that could be characterized as Level-3 (see Table 5). However, discussions involving repeated questions that end after a certain stage were noticed, as opposed to being able to clarify student thinking.

Through the performed interviews, T_Y focused scrutiny on the underlying causes of students' thinking and even provided clues to the interpretative ability. For example, T_Y depended on the GCD of relatively prime numbers being perceived as zero on the assumption that the students could not see 1 in the algorithm for finding the GCD.

So the kids don't see anything. I make them take notes while teaching GCD and LCM. I say, "Put the common ones in the circle or mark them with a star or heart shape. Do whatever you want in order to notice them. "There were two and three as common divisors. I marked them. Then we multiplied it to six. Then, when we don't mark anything, the student says there are no common divisors. Then I realize that it was not written down. The children perceive what they see. If it is written, it is apparent. If not, it doesn't exist. One is not the prime number there. We cannot write it there.

The inadequacy in observing such scrutiny in T_Y's lessons and the inconsistencies between the expressions in the interviews and practices seem to prevent T_Y's level of diagnostic competence from advancing one step further. Nevertheless, T_Y is also thought to have the potential to look at things from the student's perspective and can advance the level of diagnostic adequacy beyond where we have qualified the teacher.

T_G's General Knowledge of Learning Processes

When introducing the concepts of GCD and LCM, T_G first emphasized the terms divisors and multiples, then common divisors/multiples, and lastly greatest common divisor and least common multiple. T_G wrote the divisors 18 and 24 on the board, made students circle common divisors, and mentioned the greatest divisor in common between the two numbers is known as the GCD. Even when introducing the LCM, students themselves created the idea that common multiples could grow forever. T_G's performance of processes similar to these when teaching certain concepts such as equations and absolute value aside from GCD and LCM were observed to be open to reason. Thus, one can argue that T_G constitutes associations among mathematical ideas in the context of certain learning outcomes and offers an environment that encourages mathematical reasoning.

T_G presented the following data regarding knowledge of students' thinking. T_G pointed out that one of the biggest mistakes students had made when dividing by fractions during a lecture was to remain on the division process instead of the multiplication in reverse and the multiplication algorithm. Another mistake made during the written examinations when T_G talked about students' was extended repetition of the decimal's transferor part, even when the transfer line was drawn. One can argue that, for T_G, predictions about the ideas that students might have difficulty with and the errors or misconceptions students have developed on them can be said to have been concentrated on concepts' operational and terminological dimensions. T_G was also observed to know the distribution/ranking of the outcomes in the curriculum and to occasionally scrutinize the content and limitations of learning outcomes.

T_G stated that even when describing how to calculate the area of a triangle with the help of rectangles in class, the students would ask, "Where did this '2' come from?" The few who had *mathematical intelligence* would understand the logic of where it came from. T_G also mentioned that other students would learn the area of a triangle is half the base times the height by memorizing this as a formula. According to T_G, memorization is a learning method that should even be applied in elementary school and sixth grade. For example, students make more mistakes in upper-level mathematics when they do not memorize multiplication tables. Thus, the importance given to intelligence and memorization in T_G's teaching can be clearly seen.

Apart from the lessons where T_G teaches learning outcomes about posing problems, T_G expects students to form questions/problems. Problem-posing and problem-solving activities can be seen to constitute an important part of T_G's student-comprehension knowledge through T_G's idea of "Because you can solve the problem you have created yourself, you understand the subject."

T_G has been observed occasionally to evaluate students using different strategies. In the interviews, T_G made self-evaluations of his/her teaching, saying, "It is partly our mistake… Our failure," and pointed out that teaching strategies may be a reason underlying students' thinking.

T_G's ability to understand student thinking.

Consideration. T_G mostly considered students' thinking. Consideration of thoughts was observed through T_G's body language and explanations. Additionally, while the point where student thinking became apparent the most during T_G's lessons was in students' verbal explanations, the frequency observed in written explanations relative to other teachers is a notable finding (see Table 6).

Table 6
Percentage Table for T_G's Understanding of Student Thinking

T_G's	Categories and Percentages
Diagnostic process not considered	1%
Ways of consideration	Using body language (6%) Providing comments about thinking while explaining the solution (94%)
Environment in which a considered thought has shown up	Students' oral explanations (84%) Students' written explanations (16%)
Tendencies toward student thinking	Starting direct treatment/Inclined toward errors (100%)
Factors determining the types of scrutiny	Whether the error is obvious in student thinking or not
Types of scrutiny	If the error is obvious, providing guidance (56%) and making corrections (28%) If the error is not obvious; reinforcement of content (13%) and giving short answers (3%)

Scrutiny. T_G started direct treatment in all diagnostic processes. According to the findings, T_G can thus be argued to have displayed an error-oriented approach toward student thinking. This approach also emerged in four different ways. If the error is obvious, T_G either provides guidance or corrects students' mistakes. If the error is not obvious, T_G either reinforces the content or prefers giving short answers like "okay" (see Table 6).

T_G's ability to analyze student thinking.

Interpretation. T_G's ability to interpret student thinking was observed to be a constant repetition of similar expressions: "In order to notice some things, one should have a certain level of intelligence. Those who cannot notice, learn by memorizing. Abstract thinking can take place later." Nevertheless, at least data could be obtained where T_G linked students' thoughts to sensory resources (i.e., eyesight, hearing). According to T_G, perceiving the GCD of relatively prime numbers to be zero depends on things not being seen (not seeing 1), and to say 3/5 is between 3 and 5 depends on things being seen. In a dialogue about a student falling into error who thought $0.631 \cong 0.62$, T_G referred to hearing by using a rhyme analogy.

A: Have you encountered it?

G: Many times. One rounds up or rounds down when it needs to be written a certain way. I have come across it many times, not in exams but in the process of teaching like this. What could be the reason for this?

A: Why might one think like that?

G: I think he has developed a rule of his own. If the digit being rounded is five or greater, we round up. If it is less, we round down.

A: Do you think it is logical for a student to think like that?

G: It makes more sense than erasing it directly. It's his rule, like a rhyme. If it's five or greater go higher, if less go smaller. It sounds more musical and is easier to learn. The rule he developed is easier. He didn't listen to me well. But this student is a very intelligent child, a very intelligent student, but did he misunderstand what he heard? Either he didn't pay attention, or it sounded more musical like that. Learning this is easier. That might be the reason. It is easier saying if it's big, I round up; if it is small, I round down.

All these findings can signal that T_G thinks the students refer to their own intuition using their senses. In this context, T_G offers little clue as to T_G's ability to interpret student thinking.

T_G's level of diagnostic competence. T_G also has Level-2 diagnostic competence: T_G considers student thinking but scrutinizes toward errors. In the interviews with T_G, although T_G indicated thinking the students were right in some expressions, this situation is not considered satisfactory for the phenomenon of seeing things through students' eyes. Because T_G indicated being unable to scrutinize the understanding of inner perspective, making generalizations like, "In order to notice some things, one should have a certain level of intelligence. Those who cannot notice, learn by memorizing. Abstract thinking can take place later."

T_H's General Knowledge of Learning Processes

One way T_H teaches sixth-grade students about calculating GCDs relates to choosing bases in which the exponent is the smallest, writing the numbers as exponential numbers. For example, after writing the equations $18 = 3^2.2^1$ and $24 = 2^3.3^1$, T_H said, "The numbers with the smallest exponents among the common multipliers are chosen and multiplied" when finding the GCD of these numbers. In other words, the smallest common prime factors for 18 and 24 are 2^1 and 3^1, and they get multiplied. Presenting this kind of calculation without reasoning and depending only on memorization does not qualify as an efficient method for teaching sixth graders who are just recently getting acquainted with GCDs. The frequency of such data supports the finding that T_H uses inappropriate/ineffective mathematical methods in his/her lessons.

T_H is also seen to have limited curriculum knowledge. For instance, by asking students to calculate $x^3(-x^2 + 3)$, for the learning outcome of students' being able to multiply two algebraic expressions, expecting them to find the answer provides us with an idea about T_H's knowledge regarding outcome limitations. This is because here, the limitation is the "variables' exponents be two at most by the end of operations using algebraic expressions." Similarly, another example appeared in a lesson where T_H dealt with finding the general term in the number pattern. An example of a general term presented to the students, who had yet to confront the next outcome (exponential numbers) was 2^n, while the other one was $3n^2$. In the limitations of these outcomes, the choice of algebraic expressions involving a single operation such as $n + 1$, $n - 2$, or $3n$ is emphasized. This choice of patterns can surely be used for different purposes. However the support of other findings from T_H allows this data to be evaluated within the scope of limited instructional knowledge.

The characteristic features of T_H's lessons limit the ideas that students can offer and has them repeating T_H's sentences. For example, T_H had six students in class repeat "If a point is symmetrical with respect to the *x*-axis, then *y* changes; whereas if the point has symmetry with respect to the *y*-axis, then *x* changes. If it is taken according to the origin, both *x* and *y* change" and had 12 students repeat the following word for word: "Corresponding angles have one interior and one exterior angle; alternate exterior angles are both exterior and reverse angles, and alternate interior angles are both interior but reverse angles."

T_H's difference from other teachers is subject-matter knowledge, and therefore T_H's other knowledge is limited; this also unfavorably affects T_H's approach to student thinking. T_H's knowledge of learning processes can be said to be limited not only for diagnosing but also for correcting/directing errors. Our experience of the following process supports all the findings here. T_H is calculating $\frac{2}{5}.(-1\frac{5}{6})$ on the board. T_H finds the solution by simplifying without converting the second fraction into an improper fraction (T_H has the 5, the denominator of the first fraction, cancel out the 5 that is the incomplete numerator of the second fraction). The interesting part is that T_H is unaware of this mistake. Afterwards, T_H has the following dialogue with a student:

S: Can I please ask something?

H: Yes.

S: If we had started with an improper fraction, it would not have simplified (referring to $-1\frac{5}{6}$).

H: It would have. Let's try. Is the answer positive or negative?

S: Negative.

H: Let's put it in a box. Mert don't sit yet. Özlem says that if we transform it into an improper fraction, it would have simplified. We get the same result, Özlem. Let's try once your way. Now start by first transforming. Make a line in between so that we don't get confused.

S: Should I write again?

H: Write the problem $\frac{2}{5}.(-1\frac{5}{6})$. Yes, let's transform it. Multiply by 2/5 multiply. Yes… -11/5, sorry 11/6 [$-\frac{11}{6}$]. Six and two are gone. Let's mark it with purple.

S: The result is different.

H: Why is that different?

S: Then it is wrong.

H: Ok, we did it the right way.

One cannot consider T_H's mistake to be a simple miscalculation or a result of recklessness because once the student realized the mistake, T_H insisted on claiming the results would be the same and actually self-questioned, "Why is that different?"

T_H's ability to understand student thinking.

Consideration. T_H's categorization of diagnostic processes has been observed to be much less than those of other teachers. In these processes, T_H often considers students' thinking and shows this using body language and explanations (see Table 7). Also, while the most common point where thinking occurs during T_H's lessons is with students' verbal explanations, the diagnostic processes often come out with students' questions/discourses.

Table 7

Percentage Table for T_H's Understanding of Student Thinking

T_H's	Categories and Percentages
Diagnostic processes unconsidered	6%
Ways of consideration	Using body language (19%) Providing comments about thinking while explaining the solution (81%)
Environment in which a considered thought has shown up	Students' oral explanations (97%) Students' written explanations (3%)
Tendencies toward student thinking	Starting direct treatment/Inclined toward errors (100%)
Factors determining the types of scrutiny	Whether or not the error is obvious in student thinking Having limited/sufficient content knowledge
Types of scrutiny	If the error is obvious, providing guidance (27.5%) and making corrections (36%). If the error is not obvious and T_H has sufficient content knowledge, making explanations (27.5%) If the error is not obvious and T_H has limited content knowledge; dogmatizing the answer (9%)

Scrutiny. T_H began direct treatment in all diagnostic processes where student thinking appeared. Therefore, T_H can be said to have an error-oriented approach toward student thinking. If the error is obvious, T_H either provides guidance or makes correction, and if the error is not obvious, another factor shows up: whether T_H has limited or sufficient content knowledge. T_H was observed to prefer to making explanations when his/her content knowledge is sufficient and dogmatizing students' statements when it is limited. For example, dialogue between T_H and a student scrutinizing the multiplication of decimals and the algorithmic process of addition follows:

S: When multiplying 1.5 with 0.5; the result contains two digits in the decimal place because of the sum of the decimal digits of the multipliers. Why is this case not the same in addition?

H: Because that is addition.

S: Why is that so?

H: I wouldn't know, Gosh! [The teacher looks at the camera] One decimal point has to be under the other when adding.

T_H's stating, "One decimal point has to be under the other" and "It is the rule of division" when responding to the student asking why the quotient starts with zero when calculating the decimal notation for $\frac{4}{6}$ in division can be shown as examples of cases where T_H dogmatizes student thinking. T_H's limited knowledge of learning processes is perhaps seen as the most important factor in determining T_H's attitudes toward scrutiny.

T_H's ability to analyze student thinking.

Interpretation. Concerning observations when questioning T_H's ability to interpret student thinking, the subject constantly was changed and the interview questions could not be understood. For example, the following dialogue with T_H ensued within the interview on perceptions of relatively prime numbers' GCD as zero:

A: Here the answer given by the student is important, not the teacher's. The student mentioned zero. Had you ever encountered this situation? It is very interesting.

H: Zero. I don't remember it. Where is this?...

A: Why might the student have thought that way?

H: The student thinks zero nullifies, that it is non-existent.

A: But why did the student say zero here?

H: One student said that there. They are not like that overall. That student is meddlesome.

The interview findings are able to support skills other than T_H's interpretation. We briefly summarize these findings: (a) T_H thinks that general errors arise from students' mistakes; specific errors arise from students' meddlesomeness, and (b) the underlying reason for T_H's error-oriented approach is the belief that students make mistakes.

T_H's level of diagnostic competence. T_H appears to have Level-2 diagnostic competence, but T_H's limited general knowledge on learning processes manifests itself at every stage. This limitation, which can be more clearly understood in the above explanations, permits T_H's diagnostic competence to be labeled between Levels 1 and 2.

Results and Discussion

This study, which aims to examine elementary mathematics teachers' diagnostic competence and level, has researched diagnostic competence in terms of the components of consideration, scrutiny, and interpretation. Based on the assumption that teachers' knowledge of learning processes is a preliminary component, participants can be classified under three different categories: (a) T_Y is a teacher whose general knowledge on learning processes seems adequate and provides general in-class usage of knowledge, (b) T_G is a teacher who uses general knowledge within certain contexts (e.g., teaching some specific mathematical ideas) even if T_G's knowledge of learning processes seems adequate, and (c) T_H is a teacher who appears to have limited general knowledge of learning processes. Findings from the consideration component suggest that teachers often consider student thinking and do this in similar ways. In addition, times when teachers did not consider student thinking were rarely observed. For example, a student asked, "Teacher, can I ask something?" to which T_G answered "No," during a lesson. Answering no, as happened to this student, destroys the environment in which thinking can arise. This leads to the conclusion that not considering student thinking is related to preventing/inhibiting student thinking.

A relationship has been seen to exist between the environments where teachers consider student thinking and the way they teach. This association has also been observed by Sherin, Linsenmeier, and Van Es (2009). This relationship can be expressed by the way that categories from which student thinking arises (such as verbal explanations and written explanations) depend on a teacher's teaching style and type of communication with students. The emergence of student thinking being mostly through their verbal explanations can be explained in the context of the frequency of verbal communication established within the classroom. T_G, who had a higher level of written explanations, can be observed to have called students to the board more frequently, and these students used their notebooks more.

Teachers were observed to tend to correct student mistakes more than scrutinize their thinking during the lesson. A pattern was found among the treatment they applied in their error-oriented approach: Teachers with sufficient knowledge about the content (T_Y and T_G) were observed to have different methods such as asking questions, giving clues, and using body language before correcting student mistakes. If they did not notice any mistakes in students' thinking, they supported it using necessary explanations. These actions by the teachers can be likened to a medical doctor following a cure for suppressing only the visible symptoms of the disease without finding the actual cause of the disease. As long as the underlying cause is not found, the risk of symptoms recurring at different times will be confronted, not improvement.

One teacher with limited content knowledge (T_H) has been seen to deal with student mistakes in ways that differ from other teachers. This different approach is thought to be caused by his/her limited knowledge. Hoth et al. (2016) noted that teachers with limited knowledge frequently overlook the learning/teaching aspects of the learning process and are more focused on student behavior. Similarly, the findings indicate that the teacher with limited knowledge scrutinized by dogmatizing students' statements or issuing decrees. With these forms of scrutiny, T_H preferred to pass the ball to the students instead of using his/her own knowledge.

Teachers' scrutiny has become the most important factor determining the ability of teachers to understand student thinking. All the teachers demonstrated an error-oriented approach in their lessons and behaved by correcting students' responses. That is, they acted in a way totally unrelated to the phenomenon of seeing through students' eyes. The attention given to student thinking should not only be assessed by their mistakes or by right/wrong attitudes, but also by creating the perception of seeing things through students' eyes (Levin et al., 2009; Prediger, 2010). Teachers have also been observed in the interviews to be able to understand students' perspectives, even when they had used an error-oriented approach in practice. However, teachers have put a question mark in our minds as to how well they can apply this interpretation in the classroom environment. For example, T_G often considered students' responses in the interviews as "logical" but did nothing to scrutinize student thinking in his/her lessons. T_G's labeling students as correct from their own point of view seems inadequate in the context of diagnostic competence because, according to Hill and Ball (2009), the act of seeing through someone else's eyes involves scrutinizing the points where students have closed ideas. In the context of diagnostic competence, this is why differences exist between acknowledging students to be right or logical and seeing through their eyes; acknowledging students to be right is not accepted as sufficient for diagnostic competence.

As mentioned earlier, teacher knowledge is not considered as a determinant in level of diagnostic competence but as a necessary preliminary component of the level. As such, we were confident that this information would affect diagnostic competence but were curious to see how. Findings from the research show that teachers' knowledge of learning processes affects their diagnostic competences, both directly and indirectly. Teachers' knowledge of learning processes directly affects their consideration, scrutiny, and especially interpretation of student thinking. As much as this result resembles the relationship of teacher knowledge to their predictive skill found in König et al.'s (2014) research, it in fact differs in that no relationship is found with the ability to perceive. We can only explain this difference in the context of the limitation of teachers' knowledge of learning processes. As long as this information is limited, teachers' ability to consider is affected negatively. We have no mention of any finding on the effect of having sufficient knowledge of learning processes on teachers' consideration. This result, especially evident in T_H, can be interpreted in terms of the higher frequency with which T_H did not take student thinking into account (see Table 7). Looking at the findings obtained from this teacher holistically, T_H having inhibitory environments with regard to student thinking seems to be related to T_H's limited knowledge because this also determines T_H's teaching and has prevented the emergence of diagnostic ability processes.

Teachers' knowledge of learning processes is important and effective for diagnostic competence (Aufschnaiter et al., 2011; Busch, Barzel, & Leuders, 2015; Klug, 2011;

Klug et al., 2016; König et al., 2014), and this study has shown this knowledge to be the most important factor affecting analysis skills. In addition, this knowledge type has also been observed to be affected through direct use as well as through the way it is used. In other words, the fact that teachers have satisfactory knowledge levels of learning processes is not enough for diagnosing; this knowledge also needs to be used when diagnosing. The indirect effects of teachers' learning-processes knowledge can be mentioned in regard to diagnostic competence. For example, as long as students' thinking isn't diagnosed, teachers can find classes to have similar ways of thinking at various stages of a lesson. Limitations in this knowledge, or a lack of using it at this stage, can provide the continuance of similar thoughts being formed.

As a result, while limited knowledge of learning processes affects teachers' levels of diagnostic competence the most, having error-oriented scrutiny skills is also a determinant. We would like to refrain from saying that teachers should never have error-oriented tendencies. What we want to explain is that having error-oriented tendencies is reasonable at a certain time and level. However, the percentage encountered here is not considered reasonable for diagnostic competency because wrong/incorrect answers are usually not caused by a lack of concern or intelligence (Graeber, 1999). Certain logic can exist in these errors and mistakes (Fischbein, 1987), or students may not be able to express their ideas the same as adults can (Ball, 1993, 2001; Schifter, 2001). In addition, students produce output that cannot simply be considered right or wrong in most educational environments (Sadler, 1989). For example, although a student may seem not to understand a concept, they may have a certain logic to their thinking (Ball, 1988; 2001; Schifter, 2001). Or vice versa, even though they seem to understand, there may be unreasonable aspects to their ideas (Ball, 1993; Doyle, 1988). In this study, given that students were able to calculate 79.8 ÷ 6 without being taught (even if they answered wrong) the damage that error-oriented approaches can cause did become more noticeable. As long as teachers are error-oriented when diagnosing their students, they are not just unable to understand them, but are also unable to see that the source of error does not always originate from the students. Teachers can overlook the problems that may exist in the ideas of students who present correct output (Ball, 2001). Therefore, the need to change teachers' perspectives is very important (Jacobs, Lamb, & Philipp, 2010; Steinberg, Empson, & Carpenter, 2004), and this study once again emphasizes this fact.

While seeking answer to questions about elementary mathematics teachers' level of diagnostic competence, data have been collected using different methods, particularly observations and interviews. According to findings from the observations and video recordings, teachers' appear to be at Level 2 in terms of their practices and lessons. When considering the improvability of diagnostic competency skills, the emergence of this level is an expected result for teachers who have not received training on this subject (Sherin, 2001). This is because while a teacher can easily recognize when students perform well, being able to define exactly what to look for or expect becomes difficult for the teacher (Mason, 1998; Moscardini, 2014; Sadler, 1989) and performing these skills is not as easy as it seems (Ball, 1993, 2001; Chamberlin, 2005; Schifter, 2001). Differences in the interview findings are due to teachers having Level-2 skills in their lessons, yet they show indications that their level can change when prompted (questions like "Why do students think like this?"). For example, in the case of perceiving the GCD of relatively prime numbers as zero, while T_Y and TG evaluated students who stated the GCD to be zero to be logical and stated their opinions on this subject, T_H continued using an error-oriented approach. This situation, which stems from T_H's limited knowledge, negatively affects scrutiny and, naturally, the level of diagnostic competence. Therefore, the diagnostic competence levels of teachers who had been identified at the same level in class showed differences in the interviews.

When examining these obtained results, teachers clearly diagnose under two different contexts: their in-class diagnostic competence (i.e., their practice) and their diagnostic competences while being interviewed. Although their levels of diagnostic competence appear similar in class, their show of different competence levels in the interviews leads to the emergence of the second context. For example, T_H, who showed limited knowledge of learning processes, seems to be at a lower level. On the other hand, T_Y, who has sufficient knowledge of learning processes and uses this knowledge generally, can carry his/her level potentially higher. This context, which we call the potential of diagnostic competence, seems important in terms of the emergence of skills that cannot be observed in class. Although we have questions about whether or not potential skills can be put into practice, the discovery of these two contexts allows diagnostic competence to be characterized at different levels under different contexts. The answer to this study's query, the question of how are elementary mathematics teachers' levels of diagnostic competence, is as follows: While teachers demonstrate Level-2 diagnostic competences, they probably can perform at higher or lower levels depending on if prompts are given.

Suggestions

Given that diagnostic competence covers learnable skills, and based on the distinction between potential and practice as obtained from this study, these skills appear able to be supported. Therefore, various seminars, workshops, or summer schools can be arranged for developing teachers, and in-service/pre-service training can be given. Explanations (e.g., Busch et al., 2015; Schifter, 2001) emphasizing that teachers' perspectives may change in such trainings support our suggestion. For example, a video-based teaching application could be designed. The reason for our video-based suggestion is that observations of the effects of video recordings on teachers and similar effects have been mentioned in the literature (e.g., Bruckmaier, Krauss, Blum, & Leiss, 2016; Sherin et al., 2009; Seidel, Stürmer, Blomburg, Koberg, & Schwindt, 2011; Tripp & Rich, 2012). This application should include skills on how to analyze or interpret student thoughts, as well as how to listen to students' ideas and think like them.

Finally, we have suggestions worth mentioning for guiding researchers. The teachers' different levels of diagnostic competence in class and in the interviews can likely lead to a variety of research. For example, the levels of diagnostic competence can be designed to include both actual and potential diagnostic competence, or transform potential into practice. Additionally, shaping the levels of diagnostic validity can utilize the result of teacher knowledge being a cornerstone, which this research has obtained. The research data on teacher knowledge have been analyzed on the basis of performance indicators adapted from documents on mathematical competences in Turkey. Different studies, wherein teacher knowledge is analyzed through various means, can aim to bring the role of teacher knowledge in diagnosing adequately to a more descriptive and comprehensive state by using other tools.

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