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Research Article

An Analysis of Middle School Mathematics Textbooks from the Perspective of Fostering Algebraic Thinking through Generalization*

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Abstract

The efficient use of generalization in textbooks, which are regarded as a supplementary teaching material and constitute a substantial place in the processes of learning and teaching, is important for effective mathematics learning and teaching, as well as for developing mathematical and algebraic thinking. This research seeks to understand how and to what extent middle school mathematics textbooks include both components of generalization in terms of developing algebraic thinking. This study uses the document analysis approach, a qualitative research method, for collecting data. The tasks and exercises in the textbooks have been analyzed using a coding matrix based on the two basic dimensions of the research's analytical framework. As a result of the research, the tasks and exercises in the textbooks are seen to sufficiently support the component of Generalizing Arithmetic and Quantitative Reasoning, while partially ignoring the component of Patterns and Functional Relationships/Variables. Moreover, the tasks and exercises that are expected to allow students to explore and express generalizations are seen to have been mostly ignored. In addition, generalizations are observed to have been generally associated with everyday life, in parallel with the curriculum.

Keywords

Mathematics education • Mathematical thinking • Algebraic thinking • Generalization • Mathematics textbooks

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Although the concept of algebraic thinking is considered related to algebra, it is seen to have a wider meaning than algebra and also to have many definitions emphasizing its different aspects. Algebraic thinking can be said to be a subset of mathematical thinking and to use many basic skills such as reasoning, representation, functional thinking, and generalization (Bednarz, Kieran, & Lee, 1996; Driscoll & Moyer, 2001; Kaput, 2000; Mason, 1996). Of these, generalization is prominent in terms of its central role in mathematics.

While Mason (1996) defined generalization as the heart of mathematics, Kaput (2008) defined the focus of algebraic thinking as a complex process of symbolization that serves the aim of generalization. Polya (1957) referred to generalization as the center of mathematical activities and as a basis for developing mathematical knowledge. Many studies have emphasized the importance of generalization in developing students' thinking processes (Blanton, 2008; Common Core State Standards for Mathematics [CCSSM], 2010; Mason, Johnston-Wilder, & Graham, 2005; National Council of Teachers of Mathematics [NCTM], 2000; Van de Walle, Karp, & Bay-Williams, 2012).

CCSSM, which is regarded as the backbone of educational reform in the United States, and NCTM, the leading organization in mathematics education, advocate that mathematics exercises that include generalizations improve mathematics education. Concordantly, Turkey's mathematics curriculum, which was revamped in 2006 and 2013, has emphasized the importance of providing students with generalization skills. Also when students are trying to improve their problem-solving skills, generalizations are provided as a necessary solution. In addition, an indicator that should be taken into consideration for providing students with reasoning skills is stated in the curriculum as something that "makes logical generalizations and inferences" (Milli Eğitim Bakanlığı [MEB], 2006, 2013).

In short, generalization is a fundamental cognitive function in the thinking process (Dumitrașcu, 2015) and thus has a critical role in teaching and learning (Kaput, Carraher, & Blanton, 2008). In the process of mathematics teaching and learning, students need high-level thinking that emphasizes generalization; otherwise learning difficulties are inevitable. Indeed, many studies on students' ability to generalize in every grade in mathematics confirm this view (Akkan & Çakıroğlu, 2012; Bishop, 1997; Çayır & Akyüz, 2015; Gray, Loud, & Sokolowski, 2005; Haldar, 2014; Kaput & Blanton, 2000; Lee & Lee, 2015; MacGregor & Stacey, 1997; Özdemir, Dikici, & Kültür, 2015). For instance, some studies have determined students to have difficulty with: formulating and writing mathematical thinking when generalizing (Kaput & Blanton, 2000), expressing simple relationships using algebraic notations (Bishop, 1997), generalizing arithmetic (Haldar, 2014), and generalizing patterns (Akkan & Çakıroğlu, 2012; Çayır & Akyüz, 2015; Lee & Lee, 2015; Özdemir et al., 2015).

For all these reasons, generalizations have been identified as the critical point of this research focused on algebraic thinking.

Textbooks, by guiding teachers and providing resources for learning, are a rather important teaching material. The quality of a textbook largely guides teaching activities and contributes to how students learn subject (Demirel & Kıroğlu, 2005; Güzel & Adıbelli, 2011). Textbooks are also tools for implementing curricula (Duman, Karakaya, Çakmak, Eray, & Özkan, 2001). While a textbook does not fully reflect what happens in the classroom, it does however show the instructional objectives that can influence students' mathematical knowledge (Dumitraşcu, 2015). All these significant points have led studies to analyze textbooks under various subject areas in the mathematics education literature (Ashcraft & Christy, 1995; Bakılan-Mutu, 2008; Freeman & Porter, 1989; Jitendra, Deatline-Buchman, & Sczesniak, 2005; Kerpiç & Bozkurt, 2011; Tanışlı & Köse, 2011; Taşdemir, 2011; Yeniterzi & Işıksal-Bostan, 2015).

Studies on mathematics textbooks have analyzed different dimensions, including how the textbook relates to the class and curriculum; how it corresponds to the objectives and standards, teaching, assessment and evaluation, reflection of skills, concepts, and examples from daily life; and how it influences some of students' mathematical difficulties. Some other studies have also been based on teachers' opinions about textbooks (Bakılan-Mutu, 2008), where in these teachers have stated textbooks to contain some mistakes and to have inadequate content and evaluations. On the other hand, no study has examined middle school mathematics textbooks from the perspective of generalization. These situations constitute the second important point of this study. Certain previous studies indicated that students, under proper instruction, have made significant progress with generalizations (Olkun, Şahin, Akkurt, Dikkartın, & Gülbağcı, 2010) and that students can be inclined toward algebraic thinking as a result of proper teaching (Blanton, 2008). Taking into consideration that textbooks are important materials for leading teaching activities, having textbooks support generalizations is important for effective mathematics teaching and learning as well as for developing mathematical and algebraic thinking. Therefore, the crucial issue of research here is seen as examining whether or not textbooks include activities for gaining the skill of generalization as proposed in the curriculum, and if so, how the textbooks do this. Based on this issue, the research seeks to understand how and to what extent middle school mathematics textbooks include generalizing arithmetic and quantitative reasoning as well as patterns and functional relationships in terms of generalizations when developing algebraic thinking.

Analytical Framework

Algebra and algebraic thinking are indispensable parts of mathematics literacy in terms of educational aims and expectations (Ersoy & Erbaş, 2002). Sutherland and

Rojano (1993) defined algebra as a mathematical language used to describe ideas in mathematics and other disciplines. Algebra, as a large and important field of mathematics, opens the doors of abstract thinking and logical reasoning to students (MacGregor & Stacey, 1997). Algebraic thinking includes essential skills for mathematics such as reasoning, using representations, understanding variables, explaining the meaning of symbolic representations, working with models to develop mathematical ideas, and transforming among representations (Kaf, 2007). Additionally, algebraic thinking as a reflection of mental activities represents the bond established between algebraic relations by assigning meaning to symbols; revealing thoughts through different multiple representations; and describing concrete, semiabstract, and abstract concepts in algebraic relations, resulting in the ability to reason (Kaya & Keşan, 2014).

Algebraic thinking consists of three basic skills: using symbols and algebraic relations, utilizing multiple representations (symbols, graphs, tables, etc.), and formulating generalizations (Çelik, 2007). According to Kaput (2008), the core of algebraic thought contains two essential elements: one is using symbols and problem solving to represent mathematical ideas and the other is generalizing. Here, generalization is defined as a process that can come from or cause special situations (Davydov, 1990; Krutetskii, 1976, Polya, 1957) or as a way of transferring information in order to create a statement that is valid for all objects' properties (Dörfler, 1991). Generalization has also been referred to as a process requiring high-level thinking that improves reflective abstraction (Piaget, 1970). Additionally, generalization is an important indicator in the development of algebraic thinking, as well as a preparation process for subsequent algebraic learning and teaching (Cooper & Warren, 2011). Considering that algebraic thinking is an important step in mathematical thinking and not just limited to algebra, generalization should be considered as a process involving mathematical situations and patterns in all learning areas.

Many researchers have focused on analyzing the nature and content of algebraic thinking, as well as the development of students' symbol usage and generalization skills in their work. One of the most impactful studies on conceptualizing algebraic concepts as an all-purpose activity in past years is Kaput's (2008) theoretical model.

According to Kaput, algebraic thinking and mathematics share two aspects: firstly, mathematics is concerned with generalizations and expressing generalizations; secondly, mathematics is concerned with using customized symbol systems in order to reason using generalizations. Kaput transformed these two core aspects into a theoretical framework for a content analysis of the algebra. This framework consists of two core aspects:

Core Aspect A (Expressing generalizations). In Core Aspect A, algebra is the generalization of regularities and constraints and expresses these generalizations using increasingly systematic and conventional symbol systems.

Core Aspect B (Using representations to express generalizations). In Core Aspect B, algebra is syntactically guided reasoning and action based on generalizations that are expressed using conventional symbol systems.

In Core Aspect A, generalizations are produced, justified, and expressed in various forms. In Core Aspect B, context is associated with symbols, and symbols are considered independent of meaning. Both processes have observable symbolism. However, Core Aspect B has a fully-algebraic symbolization that uses traditional symbol systems, whereas Core Aspect A uses semi-algebraic symbolization for the same generalizations and reasoning but less conventionally. More clearly, semi-algebraic symbolization can be supported by the concept of quasi-variable (Fuji, 2003). Using symbols is not necessary when reasoning using quasi-variables. An example is the equation of $78 - 49 + 49 = 78$, where both 78 and 49 can be considered to act as quasi-variables; this indicates the relationship that a number (e.g., 78) remains unchanged if something (e.g., 49) is subtracted and then added to it. The observed intention is to not introduce children to expressions like $a - b + b = a$, but rather to get them to understand that this equation belongs to a numerical equation that holds true no matter what number is subtracted and added back. At this point, the observational level can be "looking at" or "looking through" depending on the level of focus on students' actions (Kaput, Blanton, & Moreno, 2008). For example, when a student "looking at the representations used in the activities, they can comment on the characteristics of the representations, generalize regularities and constraints, and express these generalizations in increasingly systematic, conventional symbol systems. However, when examining the representations in depth, students can syntactically guide their reasoning and actions on generalizations and use the properties they analyze to help them recognize the properties of conceptual processes or objects.

In order to better analyze the core aspects, a problem exemplifying Core Aspects A and B is presented in Table 1.

Options a) and b) of the problem in Table 1 ask students to find the result of the problem using two different calculation strategies. The aim here is to allow students to observe mathematical relations by having them calculate it themselves rather than introduce the associative property of addition algebraically in the form of $(a + b) + c = a + (b + c)$. In this way, students can generalize regularities and constraints (i.e., even if the addends change in addition, the result will be the same whether the operation is performed from right to left or from left to right) in these mathematical relations through analysis and express these generalizations systematically using conventional symbol systems. Therefore, options a) and b) can be said to support the process of expressing generalizations, specifically Core Aspect A. However, although generalizations about the associative property of addition based on only one

Table 1
Example Problem Related to Core Aspect A and Core Aspect B

Example Problem	Murat bought 250 g of assorted nuts, 2.5 kg of apple, and 3 kg of fish from a supermarket. Find the total weight using the following operations: a) (250 + 2500) + 3000 b) 250 + (2500 + 3000) c) Compare the results you find in options a) and b). Explain.
Core Aspect A Expressing generalizations	Express the total weight in grams. 2.5 kg = 2,500 g, 3 kg = 3,000 g a) (250 + 2500) + 3000 → 2750 + 3000 = 5,750 g = 5.75 kg b) 250 + (2500 + 3000) → 250 + 5500 = 5,750 g = 5.75 kg
Core Aspect B Using representations to express generalizations	c) The results I found for both options are the same. The results do not change when processing from right to left or from left to right in an addition. *a*, *b*, and *c* are three natural numbers; $a + (b + c) = (a + b) + c$ This is the associative property of addition.

example are not expected of students in options a) and b), the environment needed for students' awareness of the mathematical relationship has been established using an activity prior to this problem.

Option c) asks students to compare their findings. At this point, students are expected to associate the contents of the problem with symbols and to consider these symbols independent of their meaning in the problem. Thus, they can transform the numbers (i.e., 250, 2500, 3000) previously used as quasi-variables into algebraic symbols (a, b, c) and can reason using symbolic expressions. Put another way, they can use syntax to direct reason in the generalizations they had expressed using traditional symbol systems. For this reason, option c) can be said to lead students to use representations for expressing generalizations, thus supporting Core Aspect B.

The core aspects in Kaput's (2008) theoretical model correspond to algebraic reasoning and generalizing. Algebraic thinking and generalization also have several principles and characteristics in common. For this reason, the core aspects of algebra have been used for the generalizations in this study. In addition, Kaput (2008) identified three strands of these two core aspects, which this study uses as the components of generalization. Furthermore, the sub-components have been identified by synthesizing various studies on algebraic thinking and generalization (Blanton, 2008; Mason et al., 2005; Van de Walle et al., 2012). These strands of algebraic thinking constitute the components of the analytic framework, the sub-components of which are explained below.

Table 2
Generalization of Arithmetic and Quantitative Reasoning, and Sub-Components

Structuring the number system using explicit and abstract counting	Properties of number systems	Addition and Subtraction Identity Property: $a + 0 = a$, $a - 0 = a$ Inverse Property: $a - a = 0$ Commutative Property: $a + b = b + a$ Associative Property: $a + (b + c) = (a + b) + c$
		Multiplication and Division Identity Property: $a \times 1 = a$, $a \div 1 = a$ Inverse Property: $a \div a = 1$, $a \neq 0$ Zero Property: $a \times 0 = 0$ Commutative Property: $a \times b = b \times a$ Associative Property: $a \times (b \times c) = (a \times b) \times c$
		Assumptions Derived from Basic Properties $a + b - b = a$ $a \times b \div b = a$, $b \neq 0$ $a \times (b + c) = (a \times b) + (a \times c)$ $a \times (b - c) = (a \times b) - (a \times c)$
	Odd *(O)* and even *(E)* relationships	$E + E = E$ $E - E = E$ $E \times E = E$ $O + E = O$ $O - E = O$ $O \times E = E$ $O + O = E$ $O - O = E$ $O \times O = O$
	Generalization using connections between operational properties	Sum of five consecutive integers: $n + (n + 1) + (n + 2) + (n + 3) + (n + 4)$ Sum of three consecutive even integers: $n + (n + 2) + (n + 4)$ Sum of four consecutive odd integers: $n + (n + 2) + (n + 4) + (n + 6)$
	Derive shortcuts using properties	$1 + 2 + 3 + \ldots + 98 + 99 + 100$ Grouping the totals that equal 101; $\left.\begin{array}{l} 100 + 1 \\ 99 + 2 \\ 98 + 3 \\ 97 + 4 \ldots \end{array}\right\} \frac{100 \times 101}{2}$
The meaning of the equals sign and relational thinking	Conceptualizing the equal sign as a balance	Write 23 + 14 instead of 37 $23 + 14 = 10 + 27$ $23 + 14 = 19 + 18$ $23 + 14 = 14 + 23$
	True/false and open sentences	$73 + 56 = 71 + 58$ (true) $73 + 56 = 70 + 58$ (false) $73 + 56 = 71 + \square$ (open)
	Relational thinking	$8 + 4 = \_\_ + 5$ Because the difference between 4 and 5 is 1, the blank should be 7. $8 + 4 = (7 + 1) + 4 = 7 + (1 + 4)$ $8 + 4 = 7 + 5$
Quantitative Reasoning	Interpreting quantitative information and drawing conclusions	Eren, Alper, and Cem are three close friends and often talk on the phone. In the event any one of them has less phone credits, the other two friends send credits to that person. The amount of credits these three friends got from each other over a period of time is as follows: - Eren got 12 credits from Alper and 18 credits from Cem. - Alper got 24 credits from Eren and 16 credits from Cem. - Cem got 5 credits from Alper and 8 credits from Eren. According to this information, compare the amount of Alper's initial credit balance to the amount of his final credit balance. *The solution based on quantitative reasoning:* I think of myself as Alper. The money my friends sent me is greater than what I sent them, which means I sent them less than they sent me. My friends sent me 24 and 16 credits. I sent 12 and 5 credits to my friends. The credits they sent me is greater than what I sent them. So now I need to have more credits than I had in the beginning.

Strand 1: Generalization of arithmetic and quantitative reasoning. This strand has structures and systems abstracted from computations and relations arising in arithmetic and quantitative reasoning:

- Structuring the number system using explicit and abstract counting.
- The meaning of the equals sign and relational thinking.
- Quantitative reasoning.

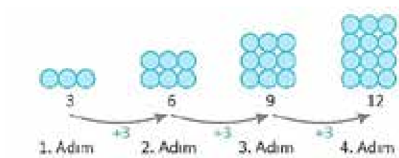
Generalization situations that exemplify the subcomponents of Strand 1 are presented in Table 2.

Strand 2: Patterns and functional relationships/variables. Generalization of patterns, functional relationships, and combined variables:

- Patterns and functional relationships.
- The meaning of variables.

Generalization situations that exemplify the sub-components of Strand 2 are presented in Table 3.

Table 3
Patterns and Functional Relationships/Variables, and Sub-Components

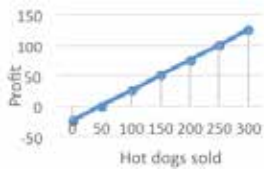
Patterns and Functional Relationships	Generalization of patterns	Repeating patterns	■ ■ ■ ■ ■ ■ ■ ■ ■ ■... △ △ △ △ △ △ △ △ △... A B B A B B A B B A B B...
		Generalization of number patterns	2, 4, 6, 8, … 1, 4, 7, 10, 13, … 2, 6, 12, 20, 30, …
		Generalizations of shape patterns	
	Graphs and exchange ratio (Slope)		
The meaning of variables	Variables used as unknown values	$x + 5 = 7$ $7 = x + 5$ $7 = 5 + x$	
	Variables used as quantities that vary	$y = 3x - 5$ $2x - 3y = 4z$ $a - 2b = 1$	
	Variables used as parameter	$mx + n = y$	

Strand 3: Modeling. This strand applies a cluster of modeling languages both in and out of mathematics:

- Multiple representations

Generalization situations that exemplify the sub-components of Strand 3 are presented in Table 4.

Table 4
Modeling, and Sub-Components

<table>
<tr><td rowspan="5">Modeling</td><td rowspan="5">Multiple representations</td><td>Context</td><td>Brian is trying to make money to help pay for college by selling hot dogs from a hot dog cart at the coliseum during major performances and ball games. He pays the cart owner $35 per night for the use of the cart. He sells hot dogs for $1.25 each. His costs for the hot dogs, condiments, napkins, and other paper products are about 60 cents per hot dog on average. The profit from a single hot dog is, therefore, 65 cents.</td></tr>
<tr><td>Table</td><td>Number of Hot Dogs Sold (Independent Variable) and the Profit (Dependent Variable)
<table>
<tr><th>Hot Dogs Sold</th><th>Profit</th></tr>
<tr><td>0</td><td>-35.00</td></tr>
<tr><td>50</td><td>-2.50</td></tr>
<tr><td>100</td><td>30.00</td></tr>
<tr><td>150</td><td>62.50</td></tr>
</table></td></tr>
<tr><td>Verbal Description</td><td>The profit depends on (is a function of) hot dog sales.
You multiply each hot dog sold by $0.65; then you subtract the $35 for the cart.</td></tr>
<tr><td>Symbols</td><td>Hot Dogs Sold: S
Profit: P
P = (0.65 x S) – 35</td></tr>
<tr><td>Graphs</td><td>
</td></tr>
</table>

Effective mathematics teaching should directly relate to everyday life, and the knowledge and skills students gain need to be useful in daily life. In recent years, resources for teaching mathematics have supported the use of everyday-life problems that improve students' algebraic skills, as opposed to simple algorithmic problems that lead to memorization (Kabael & Tanışlı, 2010, p. 218). Thus, the generalizations examined in the research have been examined in two separate categories: daily-life situations and mathematical situations. Lastly, only the first two strands have been included in the analysis because including the modeling strand (Strand 3) is thought to cause the investigation to be overly extended.

The Purpose of the Study

The main purpose of the study is to determine how textbooks give importance to generalizing arithmetic and quantitative reasoning, patterns, and functional relationships (i.e., the components of algebraic thinking) in the context of generalizations that foster algebraic thinking, as well as the extent to which the textbooks do this. As part of this general purpose, answers have been sought for the following questions:

1) How and to what extent do middle-school mathematics textbooks include activities related to the component of generalizing arithmetic and quantitative reasoning?

2) How and to what extent do middle-school mathematics textbooks include activities related to the component of patterns, functional relationships, and variables?

When taking into account the importance of helping students develop the ability to generalize in developing algebraic thinking and high-level thinking skills, having textbooks include activities aimed at developing the ability to generalize becomes quite important. This study presents how problem situations are handled when textbooks cover them and provides suggestions on how textbooks that don't cover these situations can fill this gap, helping guide them to be prepared or re-edited in the future. Therefore, this research can be said to be important for Turkey. The analytical framework adapted in the research is also believed able to contribute to the literature on mathematics teaching and the findings and results obtained at the end of the research to be able to contribute to future field-training studies for teaching the concept of generalization.

Method

Research Design

The document analysis approach, a qualitative research method, has been used to collect the research data. Document analysis, which can also be used as a sole data-collection method in qualitative research, includes the analysis of written materials containing information about what is planned for research (Yıldırım & Şimşek, 2003).

Participants

Criterion sampling, a purposive sampling method, has been adopted in the research. The main intent of criterion sampling is to study all the conditions that meet a previously determined criterion's range. The criterion (or criteria) mentioned here can be formed by the researcher (Yıldırım & Şimşek, 2003). In this sense, grade, class, and publisher have been determined as the three basic criteria. In accordance with these criteria, textbooks prepared according to the middle-school mathematics curriculum of Turkey, which was revised in 2013 and taught from 2013 to 2016, have been evaluated. The Head Council of Education and Morality chose two textbooks belonging to private publishing houses and one belonging to the Turkish Ministry of Education (MEB [Milli Eğitim Bakanlığı]) to be taught per grade for fifth through eighth grades from 2013 to 2016. In order to compare and contrast books belonging to MEB publications and the private publishing houses, the research has included one book from each publishing house in the sample, giving priority to the most recent editions in this process. Therefore, the sampling of the research consists of eight textbooks, two from each grade. These publishing houses are referred to throughout the study. With concern for ethical principles and the principle of confidentiality, the pseudonyms of Çember Publications and Daire Publications have been used.

Data Analysis

In the data analysis, the coding matrix seen in Table 5 was formed by the researchers by taking the first two stages of the analytical framework of the research (Kaput, 2008) as a basis and then analyzing the data using this matrix. This coding matrix has also been used to determine the frequency of generalization situations. In the table, columns marked with Çx represent books published by Çember Publications, columns marked with Dx represent books published by Daire Publications; the numbers that replace the sub-symbol *x* represent the grade level. The frequency of generalization situations in textbooks has been calculated as percentages and grey-scaled. The grey scales and percentage bands representing frequency are shown in Figure 1.

Table 5
Coding Matrix

Strands / Core Aspects		(1)						(2)			
	Components / Situations	**S**		E		N		**Ö**		D	
		$Ç_x$	D_x	$Ç_x$	D_x	$Ç_x$	D_x	$Ç_x$	D_x	$Ç_x$	D_x
A	Gh										
	Md										
B	Gh										
	Md										

20+%	**19.9% – 16%**	**15.9% – 12%**	**11.9% – 8%**	**7.9% – 4%**	**%3.9 – 0.1%**	**0.0%**

Figure 1. Color and percentage bands representing frequency.

Abbreviations used in the coding matrix for interpreting the findings are explained below.

1*: Generalization of Arithmetic and Quantitative Reasoning (Strand 1)*

S: Structuring the number system using explicit and abstract counting

E: The meaning of the equals sign and relational thinking

N: Quantitative reasoning

2*: Patterns and Functional Relationships / Variables (Strand 2)*

Ö: Patterns and functional relationships

D: The meaning of variables

A: Expressing generalizations (Core Aspect A)

B: Using representations to express generalizations (Core Aspect B)

Gh: Daily-life situations

Md: Mathematical Situations

Shown below is an example of the coding matrix used for analyzing situational generalizations in the textbooks.

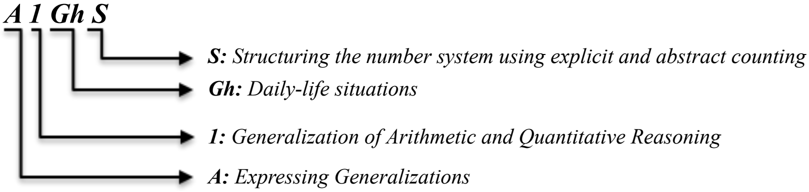
A problem situation from a 5th-grade mathematics textbook. There are 50 words on each page of the book next to you. If the book contains 20 pages, how many words are in the book?

Number of Words on One Page Number of Pages = Total Number of Words in the Book

50 x 20 = 1,000

For a story book with 1,000 words per page: If I read 1 page, I will have read 1,000 words. If I read 10 pages, I will have read 10,000 words. If I read 100 pages, I will have read 100,000 words. If I read 1,000 pages, I will have read 1,000,000 words.

<u>Code</u>: <u>A 1 Gh S</u>



Justification. The learning outcome intended for students in this problem is to be able to read and write nine-digit numbers at most. In this context, students are presented with a daily life (Gh) problem that requires four operations, and the objective is to have students generalize the results of the problem as a nine-digit number. In line with this objective, the number of pages read is systematically increased and consequently the number of words read reaches a nine-digit number. Because the steps in the problem-solving process foster students' process of expressing generalizations, they belong to Core Aspect A (expressing generalizations). In this process, the student is expected to make sense of arithmetical operations and make a quantitative connection between the number of pages and number of words read. For this reason, the problem in the book includes the component of generalization of arithmetic and quantitative reasoning (Strand 1, a component of generalization), and the component of structuring the number system using explicit and abstract counting (S, a sub-component of generalizations).

Continuation of the above problem situation. Even if we read 80 words per minute, we need to read non-stop for 9 days to finish 1,000,000 words. Can you imagine how difficult this is? If so, "one million" becomes a large number.

***Code*: B 1 Gh S**

S: *Structuring the number system using explicit and abstract counting*

Gh: *Daily-life situations*

1: *Generalization of Arithmetic and Quantitative Reasoning*

B: *Using Representations to Express Generalizations*

Justification: In the previous problem situation, the student who has worked with specific situations is expected to express the obtained results using different representations. Therefore, because one million being a large number was expressed through verbal representation in the text of the previous problem situation, it has been coded as Core Aspect B (using representations to express generalizations). The other three labels in the code have been thoroughly explained in the previous example's justification.

Reliability of the Coding System and the Role of the Researchers

Before moving onto data analysis, two students obtaining their masters in mathematics teaching and having had taken a class related to algebraic thinking were included in the first stage of the study in addition to the researchers to test the reliability of the coding system. The purpose here was to determine whether consensus could be reached when examining the coding system prepared by the researchers from a different perspective. During this process, the coding system was introduced and one of the researchers gave a step-by-step tutorial to the students. The components of the coding system were discussed and a few samples were studied. Afterwards, a specific topic was chosen from the textbooks, and the students were asked to code problem situations on generalization in this context. A week later a meeting was held to compare the codes determined by the students with the researchers' coding. After the comparison, some clear differences were determined between the students' and researchers' codes. These differences resulted from the students using a single code when the problem situation contained more than one code. The discrepancy between the codes was resolved upon re-clarifying the codes, and consensus was reached.

In the second stage, the researchers used the coding system independently to study the generalization situations in the textbooks. Afterwards, the two researchers compared their analyses and identified items with unanimous and conflicting opinions. The reliability formula suggested by Miles and Huberman (1994) was used to calculate the reliability of the coding at 90%.

Results

The research results obtained after document analysis of the two middle-school mathematics textbooks from two different publishing houses and taught for all grade levels (fifth through eighth) from 2013-2016 are included in this section. All data obtained from this analysis of textbooks from Çember and Daire Publications are presented in Table 6. This allows for comparing and contrasting the data by grade level as well as by publisher for the same grades.

Table 6
Incidence Percentages of Generalization Situations in Textbooks According to Analytical Framework

Strands / Core Aspects	Components / Situations	(1)						(2)			
		S		E		N		Ö		D	
		Ç$_5$	D$_5$	Ç$_5$	D$_5$	Ç$_5$	D$_5$	Ç$_5$	D$_5$	Ç$_5$	D$_5$
A	Gh	25.4%	13.6%	2.5%	3.4%	2.5%	0.6%	1.4%	1.0%	0.0%	0.0%
	Md	25.4%	26.6%	13.2%	24.9%	0.0%	1.3%	3.2%	1.0%	11.0%	8.1%
B	Gh	2.5%	4.7%	0.0%	0.0%	0.0%	0.0%	1.4%	1.0%	0.0%	0.0%
	Md	5.4%	6.1%	2.1%	6.1%	0.0%	0.0%	3.2%	1.0%	0.0%	0.0%
		Ç$_6$	D$_6$	Ç$_6$	D$_6$	Ç$_6$	D$_6$	Ç$_6$	D$_6$	Ç$_6$	D$_6$
A	Gh	17.3%	12.8%	15.9%	7.7%	17.5%	11.4%	3.4%	3.1%	4.6%	4.5%
	Md	17.4%	21.4%	19.2%	18.7%	0.0%	0.0%	5.7%	3.3%	17.9%	13.1%
B	Gh	3.6%	1.2%	0.0%	0.0%	0.0%	0.0%	1.6%	1.3%	2.3%	1.9%
	Md	16.3%	11.2%	0.9%	0.7%	0.0%	0.0%	3.5%	2.1%	3.7%	5.4%
		Ç$_7$	D$_7$	Ç$_7$	D$_7$	Ç$_7$	D$_7$	Ç$_7$	D$_7$	Ç$_7$	D$_7$
A	Gh	10.6%	6.6%	3.3%	5.5%	6.3%	2.5%	1.6%	0.6%	2.0%	2.6%
	Md	14.5%	24.0%	7.9%	9.5%	0.0%	0.0%	6.1%	2.8%	20.2%	16.0%
B	Gh	2.6%	4.4%	0.0%	0.0%	0.0%	0.0%	0.5%	1.1%	0.6%	0.7%
	Md	7.7%	8.3%	1.5%	0.9%	0.0%	0.0%	3.8%	2.3%	10.2%	11.6%
		Ç$_8$	D$_8$	Ç$_8$	D$_8$	Ç$_8$	D$_8$	Ç$_8$	D$_8$	Ç$_8$	D$_8$
A	Gh	6.0%	4.3%	3.7%	3.9%	2.8%	3.2%	4.0%	3.7%	0.7%	0.9%
	Md	11.3%	16.8%	13.4%	19.3%	0.0%	0.0%	10.9%	15.3%	17.2%	17.4%
B	Gh	0.3%	0.2%	0.0%	0.0%	0.0%	0.0%	3.2%	0.4%	4.3%	0.2%
	Md	5.7%	6.1%	1.4%	1.2%	0.0%	0.0%	6.0%	3.4%	8.3%	3.1%

20+%	19.9% – 16%	15.9% – 12%	11.9% – 8%	7.9% – 4%	%3.9 – 0.1%	0.0%

When examining Table 6, situational problems related to generalizing arithmetic and quantitative reasoning (Strand 1) are clearly seen to be included in 5th- and 6th-grade textbooks more than problem situations related to generalizing patterns and functional relationships (Strand 2), while 7th- and 8th-grade textbooks have equal presence. Generalizations regarding the components of structuring the number system using explicit and abstract counting and the meaning of the equals sign and relational thinking stand out in Strand 1. One of the reasons for this is that while numbers and operations are included in all grade levels of the learning domain, the grade in which these components are included the most is fifth grade. While natural numbers, fractions, decimal notations, and percentages make up a great part of fifth-grade learning outcomes, sixth-grade learning outcomes (as a continuation of this) include learning outcomes related to the order of operations in natural numbers, factors and multiples, integers, and operations with fractions and decimals. The numbers-and-operations learning domain, which starts with multiplication and division of integers in seventh grade, continues with integers, rational numbers, ratios, proportions, and percentages. In eighth grade, factors, multiples, exponential numbers, and square roots are studied. Thus the fact that problem situations related to number systems and the meaning of the equals sign are seen more frequently in 5th- and 6th-grade textbooks corresponds to the aims and objectives of the curriculum. The component of generalizing arithmetic and quantitative reasoning, seen more frequently in fifth and sixth grades, is replaced by other components related to the algebra-learning domain being given more importance in the 7th- and 8th-grade learning outcomes. Daire Publications apparently includes the component of structuring the number system using explicit and abstract counting in all grades more than Çember Publications. Meanwhile, Çember Publications includes problem situations that foster the component of the meaning of the equals sign and relational thinking more than Daire Publications only for their 6th-grade textbooks; Daire Publications is more dominant for all other grades. The quantitative reasoning component is evidently neglected for all grades by both publishers, and this component is seen most frequently in Çember Publications' 6th-grade textbooks.

Table 7
Problem Situations Related to Generalization of Arithmetic and Quantitative Reasoning in Textbooks

Publisher	Code	Situations Supporting Generalization	Explanation
$Ç_5$	A1GhS A1MdE	There are 31 walnuts in a basket.18 walnuts were added to basket. Find out mentally how many walnuts are in the basket without using a paper and pen. **Solution:** We separate the numbers to easily add them. **31 = 30 + 1** **18 = 10 + 8** 31 + 18 = (30 + 10) + (1 + 8) = 40 + 9 = 49	A1GhS In this problem related to daily life, the aim is for the student to express the generalization of the problem situation using the properties of the number system.
D_5	A1GhS A1MdE	Detective Ahmet Açıkgöz read 27 books last year and his partner 24 books. Can you calculate mentally how many books Detective Ahmet Açıkgöz and his partner read last year? **Solution:** **27 = 20 + 7** **24 = 20 + 4** **27+24 = 40 + 11 = 51**	A1MdE The aim is to conceptualize the equals sign used in mathematical sentences as a balance to support students' relational thinking.

In Tables 7 and 8, problem situations with sample generalizations of arithmetic and quantitative reasoning are chosen from among different publications and compared to each other.

In this situational problem from Çember Publications (coded: A1GhS & A1MdE; see Table 7), students are expected to make sense of the equals sign by using the properties of the number system, writing the appropriate mathematics equation for the problem, and interpreting the relationships for the given daily-life problem. The equals sign is the main way to point out these relations, and when these relations are generalized and expressed symbolically, the symbols can be evaluated independent of meaning and become usable for study with other numbers. With these kinds of generalizable relations, the problems in textbooks from Çember Publications are vital in developing students' abilities to use these generalizations, as well as the different representations in algebraic terms they will encounter in subsequent grades.

The problem situation (coded A1GhS and A1MdE) in Daire Publications' textbook has the same objectives as similar problems in Çember Publications'. In solving the problem, first tens and then ones are put together, then the results are added together and their digit values combined going from larger to smaller. However, with the question "Can you also reach a conclusion by using another strategy?", which was directed at students as a continuation of the problem, the aim is to get students to determine their own strategies; thus they are given the opportunity to generalize operational properties using the properties of number systems to express these generalizations.

Such problem situations enable students to systematically express these gener-

Table 8
Problem situations Related to Generalizing Arithmetic and Quantitative Reasoning in the Textbooks

Publisher	Code	Situations Supporting Generalization	Explanation
$Ç_7$	A1MdS B1MdS A1MdÖ	Find the value of the expressions 2^0 and 10^0. Reduce the exponent with division by starting from an exponent of 2 and 10 and create a pattern until the zero exponent is found. $2^5 = 32$, $2^4 = 16$, $2^3 = 8$, $2^2 = 4$, $2^1 = 2$, $2^0 = 1$ (: 2) $10^4 = 10\ 000$, $10^3 = 1000$, $10^2 = 100$, $10^1 = 10$, $10^0 = 1$ (: 10) For the non-zero integer n, $n^0 = 1$ When we examine the relations in the patterns, we see that	A1MdS Students are expected to logically reason using their knowledge and experience of exponential numbers to find the value of numbers to the 0-power in arithmetic problem.
D_7	A1MdS B1MdS	Calculate the 0, 1st, 2nd, 3rd, and 4th powers of 2. $2^0 = 1$ (The 0-power of all non-zero integers is 1.) $2^1 = 2$ (The first power of all integers is equal to itself.) $2^2 = 2 \times 2 = 4$ $2^3 = 2 \times 2 \times 2 = 8$ $2^4 = 2 \times 2 \times 2 \times 2 = 16$	

alizations using conventional symbol systems by generalizing the regularities and constraints, then the problem syntactically guides their reasoning and actions over the generalizations they'd expressed using conventional symbol systems. Problem situations that guide students through these objectives are very important for developing their algebraic thinking.

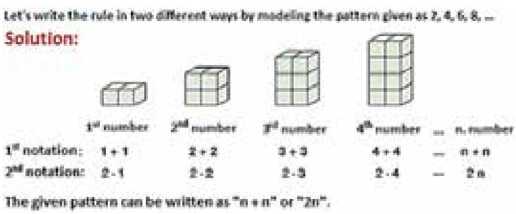
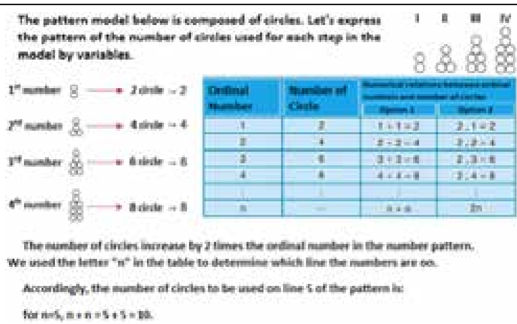
These problem situations (coded: A1MDS, B1MDS, & A1MDS; see Table 8) have students find the values of exponential expressions using the properties of the number system. In this process, students are supposed to use logical reasoning through their knowledge and experiences related to exponential expressions to reach a generalization about exponential expressions to the 0-power. In order to provide students with understanding of the effect on the result of increasing the power of an exponential expression, Çember Publications' textbooks formed a numerical pattern starting with the example of 2^5 and reducing the power one by one. In this pattern, the students are expected to notice that when an exponential expression's power is reduced by one, the result is actually divided by the base, creating a geometric sequence. Therefore, when 2^1 is divided by 2, 1 is obtained. In order to reinforce this result, the same solution process is also applied to exponents for base-10, and the value of the exponential expression, 10^0, is resultantly found to be 1. After these two specific examples, the aim is for students to make sense of the relationship in the patterns and to express that numbers to the 0-power are equal to 1. Using a variable, the resultant generalization is expressed as $n^0 = 1$ where n is a real, non-zero number.

In the textbook from Daire Publications, no pattern was found for numbers to the 0-power, with this situation being first approached with the expression $2^0 = 1$. The generalization is expressed as "the 0-power of all non-zero numbers is 1" right after the example. In this case, no guidance is given to the students for reaching their own generalizations, and the process of expressing the generalization is ignored. One can say that when evaluating this process in terms of students' algebraic-thinking development, specifically in terms of developing their generalization skills, the process passes directly to Core Aspect B without any mention of Core Aspect A. In other words, students are led to memorize the rule without being included in the generalization process.

When examining the data for the patterns and functional relationships/variables stage in Table 6, the meaning of variables is seen to be the most frequently included component for all grades. Both the components of variables' meanings and the patterns and functional relationships are included in all grade levels, with the least in fifth grade and the most in eighth grade. When looking at the curriculum, this situation can be said to be directly related to the distribution of objectives in the algebra-learning domain. Objectives in the algebra-learning domain are first found in sixth grade. The aim is that students in this grade find the intended term in arithmetic sequences, understand algebraic expressions, and perform addition and subtraction operations in algebraic expressions. When looking at seventh grade, two sub-learning domains can be seen: equality/equating and linear equations. In the eighth grade, the algebra-learning domain is given much wider coverage. At this level, the objectives relate to algebraic expressions and identities, linear equations, and inequalities. Therefore, more coverage being given to problem situations fostering generalizations in the strand of patterns and functional relationships/variables in 8th-grade textbooks is due to the extent of the curriculum.

Çember Publications are seen to cover the components of patterns/functional relationships and the meaning of variables, which are subcomponents of Strand 2 (patterns and functional relationships/variables), in all grade levels more frequently than Daire Publications are seen. However, just because Çember Publications textbooks' include these components more does not mean that it supports the processes of using representations to express generalizations or expressing generalizations in the context of algebraic thinking. Problem situations illustrating this comparison are examined in Table 9.

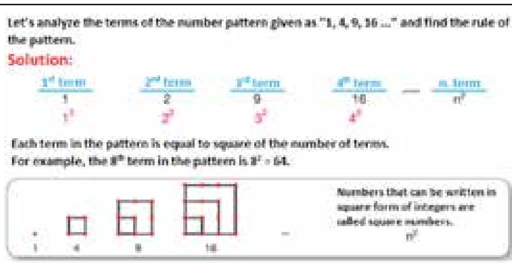
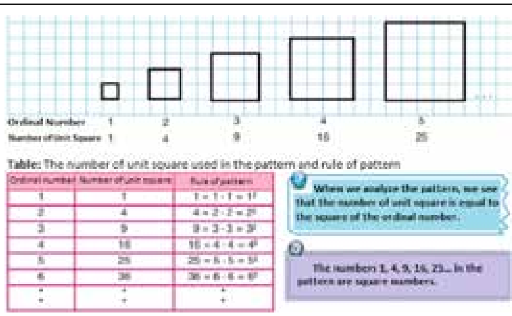
Table 9
Problem Situations Related to Patterns and Functional Relationships/Variables in Textbooks

<table>
<tr><th>Publisher</th><th>Code</th><th>Situations Supporting Generalization</th><th>Explanation</th></tr>
<tr><td>$Ç_6$</td><td>A2MdÖ
B2MdÖ
A2MdD
B2MdD</td><td>
</td><td rowspan="2">A2MdÖ / B2MdÖ
A2MdD / B2MdD
The student is expected to find the relationship between the terms and number of terms and to express the rule of the pattern without using a variable.</td></tr>
<tr><td>D_6</td><td>A2MdÖ
B2MdÖ
A2MdD
B2MdD</td><td>
</td></tr>
</table>

In the problem situation (coded: A2MdÖ, B2MdÖ, A2MdD, and B2MdD) from Çember Publications (see Table 9), the students are asked to model the given number pattern as a figure pattern, thus aiming to have them switch between representations. They are then expected to observe functional relationships in the pattern, obtain a generalization about these relations, and then express this generalization through conventional symbol systems. In this process, first the functional relationship between the ordinal number and the term are shown two different ways and then expressed using *n* as a variable. At the same time, the pattern is also seen generalized first by its additive and then its multiplicative relation, depending on the structure of the shape. On the other hand, the arrived-at generalization is not expressed through verbal representation nor are there any examples about finding a term in the pattern or finding the ordinal number from the term (inverse operation). Therefore, while creating a pattern, recognition, and extension (which are very important for developing algebraic thinking) takes place effectively in practice, generalization using different representations and testing the generalization are not included. Thus one can say the pattern problem in Çember Publications does not support students in guiding their reason and actions about generalizations expressed in conventional symbol systems.

The pattern given in Daire Publications' textbook is given as a figure pattern, which is different from Çember Publications', and the students are expected to find the number pattern from the figure pattern. In this process, without analyzing the pattern structure of the given shape (i.e., by moving away from the shape pattern), the recursive relationship between consecutive terms of the number pattern and the close terms of the pattern are shown in two different ways using tabular representation. Independent of the shape pattern, a number-focused rule is seen to be reached by making use of both the additive and multiplicative relations between the terms. However, analyzing the structure of the shape and reaching a generalization related to this structure are quite

Table 10
Problem Situations Related to Patterns and Functional Relationships/Variables in the Textbooks

Publisher	Code	Situations Supporting Generalization	Explanation
$Ç_8$	<u>A2MdÖ</u> <u>B2MdÖ</u> <u>B2MdD</u>		<u>A2MdÖ</u> / <u>B2MdÖ</u> / <u>B2MdD</u> Students are expected to find the relationship between the number of terms and terms in the given number pattern, either by reaching the number pattern from the given shape pattern or to express the rule of the pattern with or without using a variable.
D_8	<u>A2MdÖ</u> <u>B2MdÖ</u>		

important in algebraic thinking, particularly with shape patterns. Therefore, this situation can be said to have been neglected in Daire Publications' textbooks.

In Table 10, the problem situation coded as A2MdÖ, B2MdÖ and B2MdD from Çember Publications expects the students to observe the mathematical relations in the given number patterns and generalize on these relations using a rule containing a symbol. First in solving the problem, the table representation of where the first four terms are the square of the ordinal numbers (even though the table representation does not exactly meet this) is represented as n^2 using *n* as the variable. The rule for the pattern is given at the beginning of the solution process, which doesn't give the students the chance to reach their own generalizations. The number pattern that is initially provided is turned into a shape pattern after giving the rule for the pattern. On the other hand, perfect square numbers are defined without associating the terms (points) with the ordinal numbers in the given shape pattern. However, analyzing the structure of the shape and reaching a generalization related to this structure is quite important in algebraic thinking, particularly in shape patterns. Therefore, this situation can be said to have been neglected in the textbook.

The pattern in Daire Publications' textbook, unlike Çember Publications', is given as a shape pattern, and the students are expected to reach the number pattern by making use of the shape pattern. The structure of the shape pattern is seen analyzed and expressed as a multiplicative relation to reach a general rule. However, this generalization is only expressed verbally, and the use of different representations is neglected. Therefore, while creating a pattern, recognition, and extension (which are crucial to the development of algebraic thinking) are effectively involved in practice, generalizing using different representations and testing generalizations have not been included.

After reviewing the generalization situations in the textbooks according to grade, the common and differing characteristics for all grade levels are determined by comparing them; those characteristics are given in Table 11.

When examining the common and differing characteristics of the generalization situations in the textbooks in Table 11, even though most of these generalization situations support the process of expressing the generalizations, no problem situations are seen that support using representations to express generalizations. In other words, the problem situations in textbooks can be said to support the process of generalizing the regularities and constraints and expressing these generalizations in increasingly systematic, conventional symbol systems. However, the process of syntactically guided reasoning in these generalizations and the process of using relations and symbols to recognize conceptual processes cannot be said to be supported. Generalization situations involving daily-life situations are found included in almost the same amount in textbooks from both publishing houses, but Çember Publications' textbooks have

Table 11
Comparing Textbooks' Problem Situations for All Grades in Terms of Generalization

	Common Characteristics	Different Characteristics
Structuring the Number System Using Explicit and Abstract Counting	• This component is seen in Core-Aspect-A processes more frequently than in Core Aspect B. • In both core aspects, problem situations containing mathematical situations are included more frequently than problem situations containing daily-life situations. • For the 6th and 7th grades, both publishers include the same number of problems that foster Core-Aspect-A processes in problem situations containing mathematical situations.	• Çember Publications has more problems that foster Core-Aspect-A processes in problem situations containing daily-life situations. • For 5th and 8th grades, Çember Publications has more problems that foster Core-Aspect-A processes in problem situations containing mathematical situations.
The Meaning of the Equals Sign and Relational Thinking	• This component is seen more in Core-Aspect-A processes than Core Aspect-B processes. • In both core aspects, problem situations containing mathematical situations are included more frequently than problem situations containing daily-life situations. • For 7th and 8th grades, both publishers have the same number of problems fostering Core Aspect-A process in problem situations containing daily-life situations. • For 5th, 6th and 7th grades, both publications include the same number of problems that foster Core-Aspect-A processes in problem situations containing mathematical situations.	• For 5th and 6th grades, Çember Publications has more problems that foster Core-Aspect-A processes in problem situations containing daily-life situations. • For 8th grade, Daire Publications have more problems that foster Core-Aspect-A processes in problem situations containing mathematical situations.
Quantitative Reasoning	• This component is not included in Core-Aspect-B processes. • For 8th grade, both publishers have the same number of problems that foster Core-Aspect-A processes in problem situations containing daily-life situations. • Both publishers have the same number of problems that foster Core-Aspect-A processes in problem situations containing mathematical situations for the 5th grade, but have none for 6th, 7th and 8th grades.	• For 5th, 6th, and 7th grades, Çember Publications has more problems that foster Core-Aspect-A processes in problem situations containing daily-life situations.
Patterns and Functional Relationships	• This component is seen in Core-Aspect-A processes more frequently than Core Aspect B. • In Core-Aspect-A processes, problem situations containing mathematical situations are included more frequently than problem situations containing daily-life situations. • Both publishers include the same number of problems that foster expressing generalizations in problem situations containing daily-life situations for all grade levels.	• For 5th, 6th, and 7th grades, Çember Publications has more problems that foster Core-Aspect-A processes in problem situations containing mathematical situations while Daire Publications has more of these for the 8th grade.
The Meaning of Variables	• This component is seen in Core-Aspect-A processes more frequently than Core Aspect B. • In both core aspects, problem situations containing mathematical situations are included more frequently than problem situations containing daily-life situations. • For 6th, 7th, and 8th grades, the same number of problems that foster Core-Aspect-A processes in problem situations containing daily-life situations are included, but not for 5th grade.	• For all grades, Çember Publications has more problems that foster Core-Aspect-A processes in problem situations containing mathematical situations. • For 8th grade, Çember Publications has more problems that foster using representations in problem situations containing daily-life situations.

been determined to include a contribution to the development of algebraic thinking. While the component of quantitative reasoning is seen, albeit rarely, in the process of expressing generalizations, it is not seen in the process of using representations to express generalizations. Smith and Thompson (2007) argued that focusing on quantitative reasoning can improve students' ability to develop conceptualization, reasoning, quantities and quantitative interrelationships and that quantitative-reasoning-based algebra education improves students' chances of success in algebra and makes arithmetical and algebraic knowledge more meaningful and productive. At the same time, placing importance on quantitative reasoning during primary and middle school enables students to develop mathematical ideas about general situations and, through these mathematical ideas, make sense of the expressions in algebraic representations. In other words, quantitative reasoning skills provide conceptual content for powerful forms of algebraic representation and orientation. Because developing quantitative reasoning skills supports the development of algebraic thinking, the fact that textbooks do not include the component of quantitative reasoning poses a negative situation for students in terms of developing algebraic thinking. Additionally, many problem situations in textbooks aim to help students acquire operational abilities only using arithmetic operations with no associative basis. Only 6th-grade textbooks were determined to give as much importance to problems involving daily-life situations as to problems involving mathematical situations. In the light of all these evaluations, the textbooks examined according to the components of the analytical framework created within the scope of the study cannot be said to have been designed to support the perspective of generalizations in the context of algebraic thinking.

Discussion and Conclusion

Mathematics textbooks are seen as one of the primary sources of supplementary teaching materials teachers use for their lessons (Altun, Arslan, & Yazgan, 2004; Demirel & Kıroğlu, 2005; Güzel & Adıbelli, 2011; Tutak & Güder, 2012). For this reason, having textbooks be organized in a way that will help students develop their mathematical and algebraic thinking skills and use mathematical concepts and associations in everyday life and other disciplines is certainly important. In addition, these books should be prepared in accordance with the Middle School Mathematics Curriculum published by MEB by taking the general aims of the program, the learning-teaching approach, and the basic skills envisaged for the students to gain into consideration. At the same time, as emphasized in the curriculum's main objectives for mathematics, these books should help students acquire the ability to solve real problems in everyday life (MEB, 2013). When looking at studies in this field, middle school students are seen to sufficiently use operations when solving problems, but most of them fail to solve daily-life problems (Akkuş, 2008; Doruk & Umay, 2011; Erdem, Gürbüz, & Duran, 2011; Guberman, 2004; Inoue, 2008; Karataş & Güven, 2010). As Inoue (2008) points out, if students can face situations relatable to the real world and visualize them in the school environment, their ability to relate current situations to reality should improve. When taking these points of views into account, all the examined textbooks could be seen to have tried associating generalization situations with daily life, parallel with the curriculum and the examined studies. When looked at it in terms of publishing houses, Çember Publications' textbooks' more comprehensive covering of generalization situations involving daily life compared to Daire Publications' is noteworthy.

Studies by NCTM (2000), a leading institution in the United States that dominates mathematics education world-wide, and CCSSM (2010), the backbone of new educational reform in the United States, and research on the development of algebraic thinking (Akkan & Çakıroğlu, 2012; Bishop, 1997; Blanton, 2008; Çayır & Akyüz, 2015; Haldar, 2014; Kaput, 2008; Lee & Lee, 2015; Mason et al., 2005; Özdemir et al., 2015; Radford, 2011; Rivera & Becker, 2011) emphasized that, in terms of algebraic thinking, generalization situations are important that allow the development of different strategies for expressing generalizations, establish different computational strategies, establish associations with other concepts similar to the generalizations reached, and perform inverse operations to discover number properties. Patterns in curriculum are shown as tools for teaching the properties of operations, having students be able to make sense of these characteristics, teaching functions, and visually representing recursive patterns (MEB, 2013). Therefore, the abundance of pattern problems encouraging students to generalize undoubtedly makes them an active participant in the algebraic-thinking process. According to Radford (2008), while a characteristic related to terms can be generalized in the process of pattern generalization, a rule that can be used when calculating any term is not obtainable. This process of course has a generalization situation, but this is only part of Core Aspect A. In this context, all the publishers' textbooks are seen to frequently include problem situations that support the development of generalizations at the level of Core Aspect A and that require arithmetic-processing skills (finding a number's multiples, increasing or decreasing number orders according to a certain rule, determining the pattern in operational properties, etc.). Such problem situations can be said to have a very important place in algebraic thinking because they have the potential to improve generalizability at the level of Core Aspect B and at the same time allow students to form different pattern types (patterns in number-multiples, patterns in common multiples of different numbers, etc. (Akkan & Çakıroğlu, 2012; Çayır & Akyüz, 2015; Lee & Lee, 2015; Özdemir et al., 2015). However, the problem situations in textbooks are also seen to sometimes support only Core Aspect A generalization development while neglecting Core Aspect B. When evaluating textbooks separately according to publisher, Çember Publications' textbooks, compared to Daire Publications', are seen to include more pattern problems at the Core Aspect B level, which require the effective use of analysis, synthesis and abstraction (Mason et al., 2005), providing students with an opportunity to generalize. Also, pattern problems presented in the form of problem situations where students can establish functional relationships helps students take their functional thinking skills to the next level. In the examined textbooks, however, no generalizations were found for this purpose.

The process of expressing generalizations (Core Aspect A) also includes problem situations that require operational properties to be expressed numerically or verbally. Such problems are also considered a sub-component of Strand 1 as they include operational properties that are adopted as useful strategies in solving arithmetic problems. In the textbooks, the generalizations belonging to this level are sometimes seen expressed using conventional symbols and sometimes using visual representations (e.g., figures, pictures, graphs), and the transition from symbolic to visual representation is seen covered more frequently in Çember Publications' textbooks than Daire Publications'. In solving arithmetic problems at the level of Core Aspect B, first analyzing numerical expressions and equations then synthesizing the common features resulting from these analyses are performed; lastly commonly identified properties are expressed in general representations (Krutetskii, 1976), thus successfully constructing the general representation used for the generalized situation. According to Kaput (2008), activities where these processes can be clearly expressed and generalization situations examined can be described as algebraic. However, both publishing houses' textbooks intuitively regarded these properties and generalizations as correct then used them only for arithmetic operations. In addition even though insufficient, Çember Publications textbooks have generalization situations that allow students to construct general representations; this is practically non-existent in Daire Publications' textbooks.

Bastable and Schifter (2008) stated that the targeted generalization must be reached step-by-step in a problem situation, and that in this context the hierarchy between number sets must be monitored in order to prove whether or not the operational properties are valid. In other words, generalization situations must first apply to natural numbers, then to integers, rational numbers, and finally to all sets of real numbers. However, examples in the textbooks about operational properties are limited to integers. For instance, examples are given only from the integers set with respect to the commutative property of multiplication **(**3 x 7 = 7 x 3; +7 x -5 = -5 x +7**)**, and examples from other number sets are not included $\left(\frac{3}{5}\,x\,\frac{15}{12}=\frac{15}{12}\,x\,\frac{3}{5}\right)$, $(\sqrt{2}\,\mathrm{x}\,\sqrt{3}=\sqrt{3}\,\mathrm{x}\,\sqrt{3})$. Therefore, problem situations supportıng Strand 1 in the textbooks can be said to be limited to the process of expressing generalizations (Core Aspect A). However, this limitation can be removed by changing the numbers used in arithmetic problems with variables (*a x b* = *b x a*) and the relations between variables.

Throughout this study, Kaput's (2008) theory of algebraic thinking has been used to identify problem situations with the potential that can have students be a part of the algebraic-thinking process. Kaput has set two main objectives in his theory: to move the students' algebraic thinking levels from Core Aspect A to Core Aspect B and to give students a new perspective to facilitate their transition from Strand 1 to Strand 2. However, the results of the study conclude that, irrespective of publisher, textbooks in Turkey do not fully purpose moving students' levels of algebraic thinking from Core Aspect A level (expressing generalizations) to Core Aspect B (using representations to express generalizations) and that Strand 2 has been considerably neglected. Therefore, textbooks improper handling of Strand 2 is an important factor limiting students' development of algebraic thinking. Moreover, Çember Publications textbooks clearly support generalizations in many respects more effectively in the context of algebraic thinking, and that Daire Publications textbooks are clearly inadequate compared to Çember Publications'. However, the inclusion of activities that support generalizations in textbooks used by students and teachers as the primary learning source for diversification of learning domains has a critical importance in the development of algebraic thinking.

Suggestions

1. Based on the results of the study, the following suggestions can be given for similar future research:

2. Deficiencies in problems related to generalizations that the curriculum includes but the textbooks do not adequately include should be corrected by editing existing textbooks or writing new ones.

3. Intuitively accepting generalizations in textbooks as correct and the using them to solve arithmetic problems may prevent students from constructing their own generalizations. Therefore generalization situations should be organized in a way that allows students to create their own generalizations and encourage them to explore.

4. Effective mathematics teaching should directly relate to daily life and be arranged in such a way that enables students to use in daily-life the knowledge and skills they have gained. Therefore, more importance should be given to daily life problems in textbooks' problem situations.

5. Among the textbooks published so far in Turkey, textbooks that support generalization from a wider perspective are not seen as books that have been selected for teaching. For this reason, different policies that allow teachers to select the textbook that best meets their objectives may be developed.

6. Textbooks from other publishing houses that have been recently accepted by the Head Council of Education and Morality for printing textbooks to be taught can be examined in another study.

7. The question of how and to what extent the component of modeling (Strand 3 of the analytic framework formed within the scope of this study) supports generalizations can be examined in another study.

Finally, this study, by having created awareness in the context of generalizations, can guide those who develop curriculum to design quality problems that support the development of algebraic.

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