Research Article

Effects of a Schema Approach for the Achievement of the Verbal Mathematics Problem-Solving Skills in Individuals with Autism Spectrum Disorders[*]

Caner Kasap[1]
Anadolu University

Yasemin Ergenekon[2]
Anadolu University

Abstract

Teaching verbal mathematics problem-solving skills to individuals with developmental disorders is important for these individuals to understand cause and effect relations. The present study aimed to determine the effectiveness of a schema approach for individuals with Autism Spectrum Disorders (ASD) acquiring and sustaining verbal mathematics problem-solving skills. The study also aims to investigate the participants' verbal mathematics problem-solving skills, their retention levels after the termination of the application, and generalization of this skill to different types of mathematics problems. Three individuals with ASD aged 9, 11, and 14 participated in the study. A single-subject, multiple-probe design with probe conditions across participant research method was used. The findings demonstrated that instruction with the schema approach increased the participants' verbal mathematics problem-solving performance. This increase was retained 1, 3, and 5 weeks after the instruction was completed. Furthermore, two participants were able to generalize the verbal mathematics problem-solving skills for comparison-type problems with unknown results in comparison-type problems with unknown difference amounts. Social validity data collected from the participants' mothers and classroom teachers showed that both groups had positive views on the use of the schema approach in the instruction of verbal mathematics problem-solving skills.

Keywords
Verbal mathematics problem-solving • Comparison-type problems • Schema approach •
Direct instruction • Individuals with ASD

Individuals with development disorders experience inefficacies in cognitive processes, such as memory, generalization, metacognition, language, and academic skills (Friend, 2013, p. 8), and problems in retaining and generalizing newly acquired knowledge and skills (Heward, 2009, pp. 132–135). As a result, they lag behind their peers exhibiting typical development (Heward, 2009, pp. 132–135; Thomas, 1996, p. 135). Therefore, it is necessary for individuals with developmental disorders to acquire various information, skills, and behavior to prepare for social life, gain their independence, and lead their lives with minimum dependency on others around them (Çiftci-Tekinarslan, 2012, pp. 157–158; Ergenekon, 2012, p. 158). Individuals with developmental disorders need special assistance to acquire the skills that individuals with typical development learn through daily experiences (Sucuoğlu, 2009, p. 67).

In addition to communication, socialization, and behavioral disorders, individuals with Autism Spectrum Disorder (ASD), considered a development disorder, often have disabilities in cognitive skill areas such as reading, writing, language, and mathematics (Minshew, Goldstein, Tylor, & Siegel, 1994), which are also considered cognitive skills (Shapiro, 2011, p. 212).

Mathematical skills are important for the decisions made in daily life (Reyna & Brainerd, 2007). Individuals resort to mathematical skills every day without even recognizing it (Prater, 2007, p. 358). The use of mathematical skills increases an individual's independence and autonomous decision making at home and in professional and social lives (Browder & Snell, 2000, p. 497). Numbers, geometry, measurement, and data are the areas of learning in mathematics (Gürsel, 2013, pp. 448–449). Within their context, process skills that students should acquire in mathematics classes include (a) problem solving, (b) reasoning, (c) communication, (d) association, and (e) projection skills (Gürsel, 2013, pp. 447–448; National Council of Teachers of Mathematics [NCTM], 2010). To be able to use the abovementioned skills effectively, individuals need to develop visual skills as well (Donaldson & Koffler, 2010).

The three learning levels in mathematical skills instruction are "concrete, semi-concrete, and abstract", and verbal mathematics problems are related to the semi-concrete skill level (Bender, 2009, p. 65). McCoy and Gehrke (2009, pp. 498–499) defined verbal mathematics problems as mathematical exercises that include real or imaginary situations expressed in words. Verbal mathematics problems can be in one of three types: "change," "grouping," or "comparison" (Jitendra, 2002; Marshall, 1991, p. 72). The following are some examples of these problem types:

a. Change-type problems: Fatma had 4 pencils. Her mother gave her 2 pencils. How many pencils does Fatma have now? Change-type verbal mathematics problems consist of a beginning (i.e., 4 pencils), change (i.e., 2 pencils), and ending set (unknown set).

b. Grouping-type problems: Caner has 15 red and clutch pencils. 5 of these pencils are red. How many clutch pencils does Caner have? Grouping-type verbal mathematics problems consist of two small sets (i.e., 5 red pencils and an unknown set) and a larger set (i.e., 15 red and clutch pencils).

c. Comparison-type problems: "Ali has 2 books. Mehmet has 3 more books than Ali. How many books does Mehmet have?" Comparison-type verbal mathematics problems consist of a compared set (i.e., 2 books), difference set (i.e., 3 more books), and referent set (unknown set) (Jitendra, 2002). The problem schemas of these problem types are shown in Figure 1.
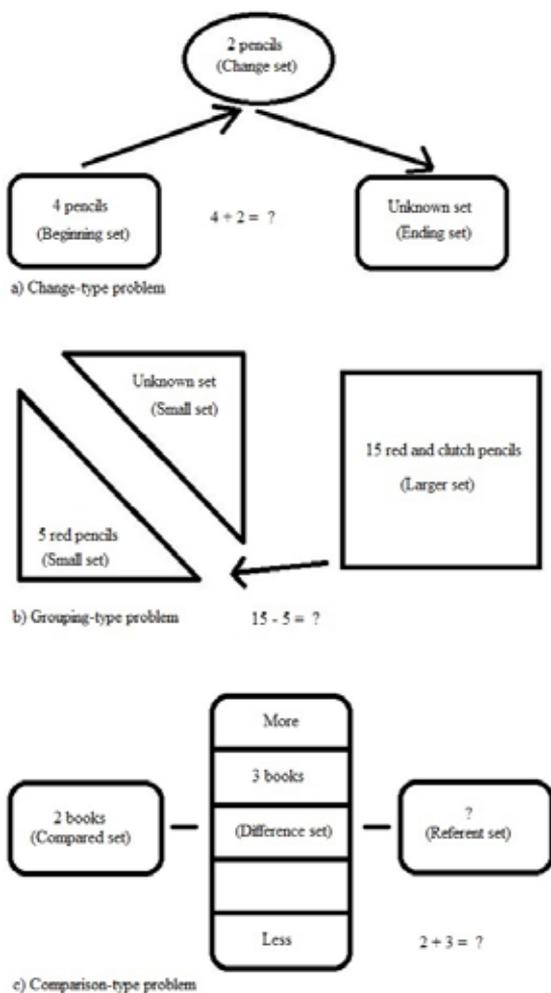

Figure 1. Change-, grouping-, and comparison-type problem schemas.

According to Gooding (2009), difficulties encountered in solving verbal mathematics problems occur due to (a) individuals being unable to comprehend or analyze the words used in the problems when solving verbal mathematics problems or a lack of self-esteem when reading the problems, (b) individuals experiencing difficulties in transforming the words in verbal mathematics problems into numbers, (c) the context of the verbal mathematics problem and magnitude of the numbers on affecting individuals' calculation strategies related to the tendencies and individuals' choices of calculation strategies, or (d) errors that occur as a result of neglecting real-life variables when solving verbal mathematics problems. To alleviate these difficulties, several instructional approaches have been developed in the literature on solving verbal mathematics problems, for example, process approach, keyword approach, and schema approach (Bender, 2009, p. 111; Bottge, 2001; Gurganus, 2007, pp. 150–162; Jitendra & Hoff, 1996).

In the process approach, a verbal problem is solved step by step, and the solution to the presented problem to the student is taught (De Kock & Harskamp, 2014; Star, 2005). A student who learns the process and correctly solves the problem related to this process during the process approach instruction could experience difficulties when encountering the same verbal problem with an unknown variance, instead of an unknown result (Engelbrecht, Bergsten, & Kagesten, 2009). The schema approach was developed to compensate for the disadvantages of the process approach (Bender, 2009, p. 111; Jitendra et al., 2007). Schemas used in the solution of verbal mathematics problems are achieved through the direct instruction method (Rockwell, 2012), which involves four steps: (a) creating a need, (b) setting an example, (c) guided applications, and (d) independent application steps (Aykut, 2012, pp. 204–209; Dağseven-Emecen, 2008). When the problem type is discovered by the individual using the schema approach, the individual needs to find the set in which the unknown amount is located. Then, the schema adequate for the problem is drawn, and the unknown set is marked on the schema. The required operation is determined based on known and unknown items (Jitendra, 2002). Jitendra et al. (2009) offered a four-step strategy to solve verbal mathematics problems using the schema approach. These steps are (a) find the problem type, (b) organize the information in the problem using a diagram, (c) plan to solve the problem, and (d) solve the problem.

Studies in the existing literature report that the schema approach is effective in teaching verbal mathematics problems to typically developing individuals (Griffin & Jitendra, 2009; Jitendra et al., 2009) as well as those with learning disabilities (Griffin & Jitendra, 2009; Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra & Hoff, 1996; Jitendra, Hoff, & Beck,1999; Jitendra et al., 2007; Na, 2009; Xin, 2008; Xin, Jitendra, & Deatline-Buchman, 2005), emotional and behavioral disorders (Jitendra, George, Sood, & Price, 2010), visual impairment (Tuncer, 2009), and intellectual

disorders (Baki, 2014; Jitendra et al., 1998; Karabulut, Yıkmış, Özak, & Karabulut, 2013; Kot, 2014). However, only a limited number of studies have been conducted with individuals with ASD (Rockwell, 2012; Rockwell, Griffin, & Jones, 2011). The present study was conducted to fill this gap in the literature as well as contribute to and expand it.

Furthermore, since problem-solving skills are the prerequisite for decision making and independent action skills, they need to be taught to individuals with disabilities (Lombardi & Savage, 1994). Solving verbal mathematics problems contribute to the independence of individuals with ASD in social life. Ergenekon (2004) reported that teachers utilized real objects when teaching mathematics courses to students with developmental disabilities and included the withdrawal of clues in this process. It was reported in the literature that while teaching academic skills to individuals with developmental disorders, instructional methods and strategies enabling the individuals to gain more experience, render academic skills functional, and combine them with daily life should be prioritized (Pedrotty-Rivera & Deutsch-Smith, 1997, p. 242). Swanson, Orosco, and Lussier (2014) reported that individuals who experience difficulties in acquiring mathematical skills could develop these skills with a combination of visual and verbal instructions. Although they experience difficulties in verbal communication, individuals with ASD are defined as strong visual students in the literature (Brill, 2011; Tissot & Evans, 2003). Thus, utilizing visualization when instructing individuals with ASD could contribute significantly to their learning. Therefore, it is necessary to teach alternative instructional approaches, methods, and strategies that include visualization, such as the schema approach, to pre-service special education teachers during their undergraduate courses so that they can teach their students higher level mathematical skills, such as solving mathematics problems; this fact forms the basis for the present study.

Based on the abovementioned requirements, the objective of the present study is to determine the effects of the schema approach on comparison-type verbal mathematics problem-solving skills of individuals with ASD that require addition and subtraction operations. The following research questions were tested within the context of that general objective:

1. Is the schema approach effective in the instruction of comparison-type verbal mathematics problem-solving skills with unknown results for individuals with ASD?

2. Is the schema approach effective in the generalization of comparison-type verbal mathematics problem-solving skills with unknown results to solving skills of verbal mathematics problems with unknown difference amounts by individuals with ASD?

3. If verbal mathematics problem-solving skills are acquired through the schema approach, will participants retain the acquired skills 1, 3, and 5 weeks after the instruction?

4. What are the views of the mothers and teachers of individuals with ASD on the instruction of verbal mathematics problem-solving skills using the schema approach, for individuals with ASD?

Methodology

Participants

Three primary-school-age individuals who were diagnosed with ASD and could perform addition and subtraction operations with single-digit numbers but could not solve verbal mathematics problems, the researchers, and observers participated in the present study. The prerequisite criteria for the participants were (a) reading-writing skills, (b) reading comprehension skills, (c) understanding and following instructions that included five or more words, (d) visual perception skills, (e) performing single-digit addition and subtraction operations, (f) directing attention to an activity for at least 15 min, and (g) knowing the meanings of keywords (e.g., more, less). The first author's tests determined that the three potential participants met these criteria and possessed these prerequisite skills. The participants' characteristics are summarized in the Table 1, followed by the detailed explanations.

Table 1
Participant characteristics

Participants	Diagnosis	Gender	Age	Class Type
Ekin	ASD	Female	14	Special education class
Zehra	ASD	Female	11	Special education class
Dilek	ASD	Female	9	Inclusion class

Ekin, Zehra, and Dilek were 14-, 11-, and 9-year-old female students diagnosed with ASD based on hospital reports. They had no systematic education history on verbal mathematics problem-solving skills with the schema approach. All three participants could understand and complete five-word instructions given to them. They could count by ones, twos, threes, fours, and fives to 100; add and subtract single-digit numbers using their fingers; identify numbers from 1 to 100 shown to them on numbered cards; and read text up to 20 sentences and answer the five WH questions about texts they read with the help of verbal cues. They could also concentrate on desk-bound activities for at least 15 min and had fine and gross motor skills.

All applications of the research were conducted by the first author, who was a graduate student and research assistant in the field of special education. The present study was the first application experience by the first author in providing instruction in verbal mathematics problems to individuals with ASD.

Effectiveness, follow-up, and generalization data were collected by viewing the video recordings of completed sessions. Reliability data for the study were collected by two research assistants who were experienced in working with children with development disabilities, had undergraduate degrees in special education, and were continuing their graduate education in the same field. The first author provided detailed information to the observers on the application process and how to collect data.

Setting

Beginning level, application, probe, generalization, and follow-up sessions were conducted in quiet rooms in the participants' homes in one-on-one instructional settings. Participants sat on the left side of the table, and the researcher sat on the right side of the table. This was done to prevent the obstruction of the participant's field of vision while the researcher was setting an example. The video camera was mounted in front of the window in these quite rooms of the participants' homes and on the left side of the participant to more clearly record her reactions.

Materials

In teaching the target behavior, the following materials were used: (a) worksheets with comparison problems and spaces below to solve them, (b) pencil, (c) eraser, (d) pencil sharpener, (e) video camera, (f) tripod, (g) collection forms for beginning level, application, probe, generalization, and follow-up sessions data, (h) table, and (i) two chairs.

Reinforcers

Reinforcers used in the study were determined by consulting the participants' mothers. Appropriate participant behaviors were reinforced with verbal, social, object (e.g., surprise eggs), and activity reinforcers (e.g., playing with the phone, listening to music). In the beginning level and intermittent probe sessions, correct responses were reinforced with verbal reinforcers. In the instructional sessions, correct responses were reinforced verbally on a continuous reinforcement schedule. At the end of each session, an object or activity reinforcer was presented based on the participant's selection. In the generalization sessions, participants were given verbal reinforcement upon generalization of the target behavior. In follow-up sessions, a variable rate of reinforcement (VR3) schedule was used. After the follow-up sessions were completed, the participants were presented with certificates of achievement for participating in the study.

Research Model

In this study, to determine the effectiveness of the schema approach for verbal mathematics problem-solving skills instruction in individuals with ASD, a single-subject, multiple-probe design with probe conditions across participants research method was used. In this design, empirical control is established by the occurrence of variation in the skill level of the application recipient only when the independent variable is applied and by the lack of a significant change when the independent variable is not applied as well as by a repetition of the abovementioned effect subsequently in other skills (Gast & Ledford, 2014, pp. 251–296; Tekin-İftar, 2012a, pp. 219–254).

Dependent and Independent Variables

The dependent variable was the accurate written response rate that participants gave to comparison-type mathematics problems with unknown results requiring the addition or subtraction of two single-digit numbers. In an instructional session, 18 problems were presented to the participants. Of these 18 problems, 9 required addition and the rest 9 required subtraction operations. Within each direct instruction modeling stage, guided application stage, and independent application stage, 3 addition and 3 subtraction verbal mathematics problems were posed. Thus, a total of 6 problems, 3 addition and 3 subtraction, were used in each stage of the study's direct instruction. The schema approach was the study's independent variable.

Possible Response Definitions

In the beginning level, application, probe, generalization, and follow-up sessions, the participants could give three types of responses: correct response, incorrect response, and no response. For a correct response, the participant provided a written, independent, and accurate solution to problems on the worksheet after receiving the skill instruction. Correct responses were reinforced with social and verbal reinforcers. For an incorrect response, the participant provided an incorrect solution to a problem on the worksheet after receiving the skill instruction. Participants' incorrect responses were verbally ignored and were only recorded on the data form as incorrect responses. To score a no response, the participant provided no solution to a problem on the worksheet within 20 sec after the instruction was given. In this case, after 20 sec, the next problem was initiated. The 20-sec waiting period was chosen because problem-solving stages include complex and time-consuming cognitive processes. During probe and follow-up sessions, the lack of participant response was ignored and only recorded as an incorrect response.

In instructional sessions, too, participants could give three types of responses: correct response, incorrect response, and no response. Since the direct instruction

method was used in instructional sessions, the opportunity to give independent responses was available to the participants only during the independent applications stage in this instructional method. For a correct response, the participant had to provide a written, independent, and accurate solution to a worksheet problem after receiving the skill instruction. Correct responses were reinforced with social and verbal reinforcers. For a response to be incorrect, the participant had to provide an incorrect solution to a problem on the worksheet after receiving the skill instruction. When the participant gave an incorrect response, error correction (i.e., modeling cue plus verbal cue) was provided. Error correction procedures were conducted after the session providing an opportunity for the participants to submit independent responses during the independent applications stage of the instructional sessions. No response was considered if the participant provided no response for one or more problems on the worksheet within 20 s after the instruction was given. No participant response was considered and recorded as an incorrect response.

General Process

Pilot scheme. To predetermine the possible problems during the tests and implement necessary adjustments, a pilot scheme was conducted with a participant who had characteristics similar to those of the study participants and met the prerequisite skills. Three instructional sessions were conducted in the pilot scheme. Each instructional session was completed in 17 min on average, and the total duration was 52 min. Additional probe sessions were not conducted in the pilot scheme, but they were conducted in the independent applications stage of the direct instruction method, which consisted of the last six questions of the instructional sessions. In this process, different from the instructional sessions in the main application, the day after both instructional sessions, the researchers decided to conduct an intermittent probe session. During the generalization posttest session, the questions were presented to the pilot participant first without the schema and then with the schema. This participant provided 50% correct responses on the problems given on the worksheet with and without a schema. After the pilot scheme, the researchers decided to provide the problems in the generalization posttest sessions with the schema.

Experimental process. The experimental process included beginning level, instruction, probe, generalization, and follow-up sessions. All sessions were conducted in a one-on-one instructional format and were recorded by a video camera. Before the application, verbal and written consent was received from the children's families for their participation and video recordings. Code names were assigned to the participants. Instructional sessions for each target behavior were conducted 4 days a week with one instructional session per day, and attention was paid not to interfere with the participants' regular education programs. Probe sessions were

conducted intermittently and immediately before the third instructional session after two consecutive instructional sessions. After the probe session, an intermission with a non-deskbound activity was implemented, and then instructional sessions were resumed. In all sessions, problems that required addition and subtraction operations were presented to the participants randomly.

Beginning level and intermittent probe sessions. Before the instruction, beginning level probe sessions were conducted for three consecutive sessions until consistent data were obtained. In beginning level sessions, the participants were asked to solve 6 given verbal mathematics problems. These verbal mathematics questions included 3 comparison-type verbal mathematics problems that required the addition of a single-digit number with another single-digit number and the result was unknown, and three comparison-type verbal mathematics problems that required the subtraction of a single-digit number from another single-digit number and the result was unknown. In these sessions, no clues were offered to the participants. In probe sessions, the participants were guided to focus on the task (e.g., "Now, we will solve mathematics problems. Are you ready to work?"). The participant's focusing behavior was verbally reinforced (e.g., "Bravo! I see that you are ready."). In these sessions, the participants were asked to solve 6 verbal mathematics problems (e.g., "Please read and solve these problems."). Intermittent probe sessions were conducted in the same way as the beginning level sessions. The percentage of correct responses given in written form by the participant to the presented verbal mathematics problems was calculated using the formula "number of correct responses/total number of questions × 100" (Erbaş, 2012, p. 117), and the results were plotted on the graph as application data.

Instructional sessions. The instructional sessions included the following three stages: (a) setting an example, (b) guided applications, and (c) independent applications. In the setting an example stage, the researcher explained how to solve the mathematics problems, of which 3 required the addition operation and 3 required the subtraction operation and were randomly presented, using the schema approach. Later, the researcher applied the problem data on the schema. Then, the researcher transformed the numbers in the schema into a family of operations. The researcher reinforced the participant for following the explanations carefully.

In the guided applications stage, the researcher made special efforts to attract the participant's attention to the study. The participant's focusing behavior was verbally reinforced. Then, the researcher told the participant that they would solve verbal mathematics problems together. In this stage, the researcher solved verbal mathematics problems, of which 3 required the addition operation and 3 required the subtraction operation and were presented randomly, together with the participant. During the guided applications stage, the researcher asked the participant to read the

verbal mathematics problem and verbally reinforced her behavior while she read. Then, the participant was asked to write down the data given in the problem on the schema and was verbally reinforced when she entered the data. At the end of the guided applications stage, the participant was asked to transform the numbers in the schema into an operation family and was again verbally reinforced while she did this.

In the independent applications stage, the researcher made an effort to attract the participant's attention to the study. The participant's focusing behavior was verbally reinforced. In this stage, the participant was asked to solve the given verbal mathematics problems independently. The length of the independent applications stage of the direct instruction method was decided after the pilot scheme. The duration was determined as 10 min. Correct participant responses during the independent applications stage were verbally reinforced. Incorrect responses were ignored during the independent applications stage and were corrected at the end of the session. The implementation flow of the instructional sessions is shown in Figure 2.

Follow-up sessions. Follow-up sessions were conducted 1, 3, and 5 weeks after the criteria for the target behavior were met for each participant. During follow-up sessions, no immediate feedback was provided for correct and incorrect responses, but participation behavior was reinforced at the end of the session. Furthermore, the reinforcers were faded on a VR3 schedule.

Generalization sessions. In the generalization phase of the study, the participants were presented with comparison-type verbal mathematics problems with unknown differences. Comparison-type verbal mathematics problems with unknown differences were designed to include the expressions *more* and *less*. The same procedures as were used in the probe and follow-up sessions were followed in the generalization sessions.

Reliability

Reliability data were collected for both the dependent and independent variables in randomly selected sessions that amounted to at least 30% of the total sessions. Dependent variable reliability data were collected with interobserver reliability, and independent variable reliability data were collected with application/procedural reliability. The formula "agreement/(agreement + disagreement) × 100" was used for the analysis of interobserver reliability data and the formula "observed practitioner behavior/planned practitioner behavior × 100" was used for the analysis of the application reliability data (Erbaş, 2012, p. 117).

Social Validity

One of the indicators of the quality of an applied research its the high social validity degree (Leko, 2014). For this purpose, social validity data for this study were
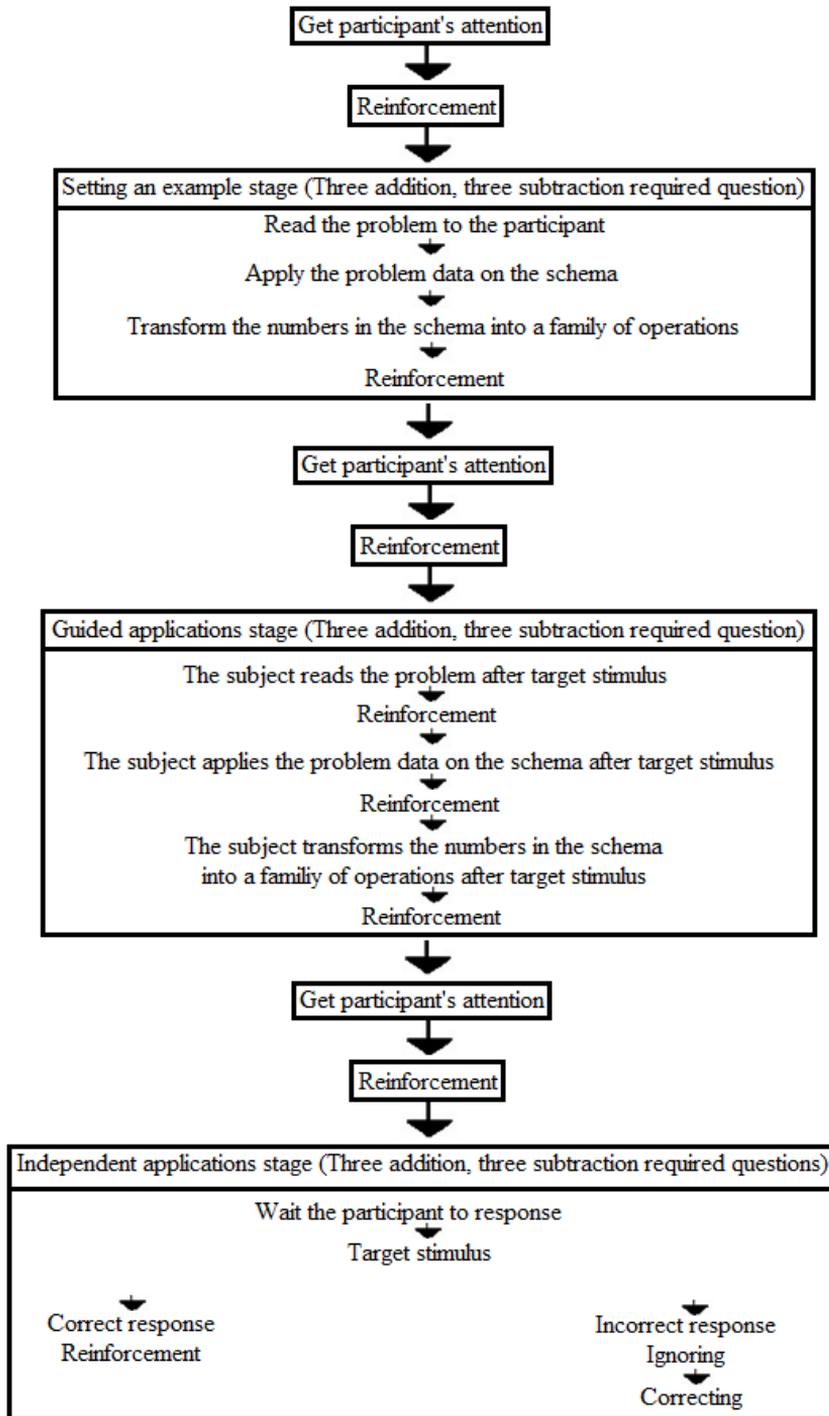
Figure 2. Implementation flow of the instructional sessions.

collected via the social validity questionnaire designed by the authors. The social validity questionnaire is shown in Table 2.

Table 2
Social Validity Questionnaire

	Questions	Respondent(s)
1	Do you think that learning verbal mathematics problem-solving skills by using the schema-based approach is important for your child/student?	M & T
2	Do you think that learning verbal mathematics problem-solving skills by using the schema-based approach contributes to your child's/student's development of daily and independent life skills?	M & T
3	Are you satisfied with your child's/student's participation in such a study related to teaching verbal mathematics problem-solving skills by using the schema-based approach?	M & T
4	Do you think that your child/student has learned verbal mathematics problem-solving skills by using the schema-based approach?	M & T
5	Have you ever observed that your child/student used the skills she learned while doing her homework or in her daily life?	M & T
6	If you answered no to the above question, have you ever given your child/student a chance to use these skills?	M & T
7	Do you want to learn to use the schema-based approach, which was used to teach verbal mathematics problem-solving skills to your child?	M
8	Do you want to implement the schema-based approach in your class, which was used to teach your student verbal mathematics problem-solving skills?	T
9	Do you want your child to participate in a similar study related with different skills that your child hasn't learned yet?	M
10	Have you noticed any changes in your child/student after this study? If the answer is yes, what are these changes?	M & T
11	What were your favorite parts of this study?	M & T
12	What were parts of this study did you dislike?	M & T
13	What do you think about the study being carried out in your house?	M
14	Do you think that the researcher abided by the Informed Consent Form you signed before the study?	M

Note. M = Mother, T = Teacher.

Findings

Effectiveness Findings

Data collected on the effectiveness of the schema approach on the performance of all participants in solving comparison-type verbal mathematics problems with unknown results are presented in Figure 3. In the graph, the horizontal axis depicts the sessions and the vertical axis depicts the participants' correct response percentages in the daily probe sessions.
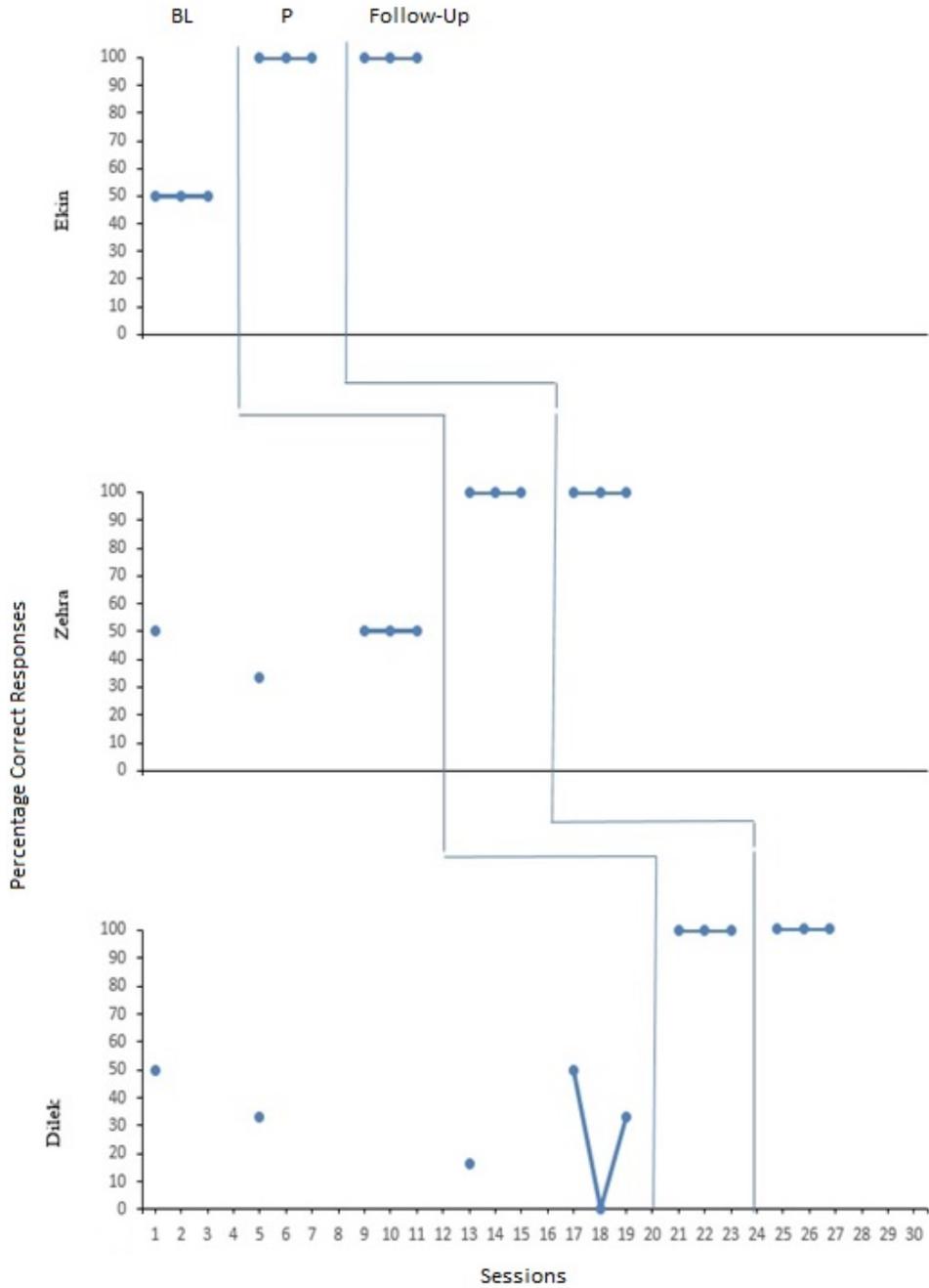
Figure 3. Correct response percentages by Ekin, Zehra, and Dilek in comparison-type problem-solving skills in the beginning level (BL), probe (P), and follow-up sessions.

Visual analysis of the data shown in Figure 3 demonstrated that there were significant differences in the correct response percentages between the probe sessions where the schema approach was applied and the beginning level sessions where it was not applied. Absolute level change analysis was conducted for the beginning level and probe stages of the application. The immediate effect was 50% for Ekin, 50% for Zehra, and 66.67% for Dilek. The absolute level change analysis conducted between the beginning level and probe stage performance scores indicated an immediate effect in the desired direction in all participant performances.

Comparison-type verbal mathematics problem-solving performance for Ekin had a mean of 50% in the three sessions conducted at the beginning level. When consistent data were obtained in the beginning level, instruction using the schema approach was initiated. In the application stage, Ekin gave 100% correct responses to comparison-type verbal mathematics problems in all the probe sessions. In the follow-up stage, Ekin gave 100% correct responses 1, 3, and 5 weeks after she met the determined performance criteria.

For Zehra, the comparison-type verbal mathematics problem-solving performance had a mean of 46.66% in the three sessions conducted at the beginning level. In the application stage, she gave 100% correct responses to comparison-type verbal mathematics problems in all the probe sessions. In the analyses conducted to determine the retention effects of the application 1, 3, and 5 weeks after Zehra met the determined performance criteria, she was observed to give 100% correct responses to the comparison-type verbal mathematics problems.

Dilek's comparison-type verbal mathematics problem-solving performance had a mean of 30.55% in the three sessions conducted at the beginning level. In the application stage, Dilek gave 100% correct responses to comparison-type verbal mathematics problems in all probe sessions. In the follow-up stage, she gave 100% correct responses to comparison-type verbal mathematics problems 1, 3, and 5 weeks after she met the determined performance criteria. Therefore, the study findings demonstrated that all three participants retained their comparison-type verbal mathematics problem-solving skills learned through the schema approach after the instruction was terminated.

Since all three participants performed at 50% accuracy in the generalization sessions, a generalization instructional session was conducted with all of them (Alberto & Troutman, 2009, p. 362). The generalization session performance percentages are presented in Figure 4.
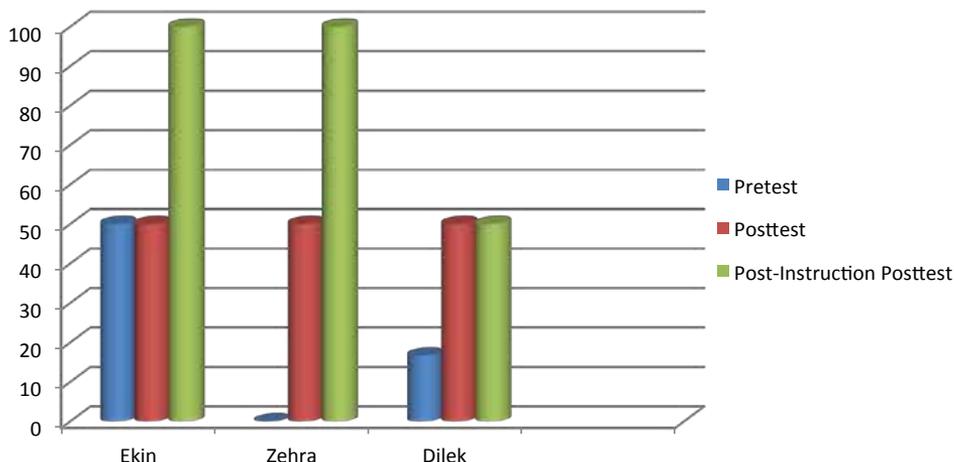
Figure 4. Generalization percentages of Ekin, Zehra, and Dilek's performances in solving comparison-type verbal mathematics problems with unknown results and verbal mathematics problems with unknown comparison and difference amounts.

In the assessments conducted with the participants prior to the instruction related to solving comparison-type verbal mathematics problems with unknown compared and difference values, Ekin responded to 50% of the problems correctly, while Zehra responded correctly to 0%, and Dilek responded correctly to 16.66% of the problems. After the instruction, each of the participants correctly solved 50% of the comparison-type verbal mathematics problems with unknown compared and difference values. Since the participants' performance did not meet the criteria for generalization of the acquired skills in generalization sessions, generalization instructional sessions were conducted. Ekin and Zehra's correct response rates increased to 100%, and Dilek's increased to 50%.

Reliability Findings

In this study, interobserver and application reliability analyses were conducted in at least 30% of all sessions. Interobserver reliability rates for all stages of the research were measured at 100%. The lowest and highest application reliability coefficients were 85.71% 100%, respectively, based on application reliability findings for all stages of the study. The most frequent error made in sessions with low application reliability occurred when the practitioner introduced reinforcers to the participants.

Social Validity Findings

A social validity questionnaire designed by the authors were administered to the mothers and teachers of the participants to assess the adequacy of the skills studied in the research, study methods, and findings. A total of 13 questions, 8 close-ended and

5 open-ended, were given to the participants' mothers, and a total of 10 questions, seven close-ended and three open-ended, were given to the participants' teachers. The collected data was analyzed descriptively, and both mothers and teachers were found to harbor positive views on the study.

Discussion

The study findings demonstrated that the schema approach was effective in teaching individuals with ASD skills of solving comparison-type verbal mathematics problems with unknown results and in fostering the retention of these skills 1, 3, and 5 weeks after the instruction was completed. Furthermore, after the generalization instructional session was conducted, all but one participant were able to generalize their skills of solving comparison-type verbal mathematics problems with unknown results to solving comparison-type verbal mathematics problems with unknown difference values. It could be argued that the study was socially valid based on the social validity data collected from the participants' mothers and classroom teachers. This study contributes to the national literature because it is the first study to demonstrate that the schema approach is applicable and usable in the instruction of solving comparison-type verbal mathematics problems with unknown results to children with ASD in Turkey. In addition, this study contributes to the international literature since it is one of a limited number of available studies in which the schema approach was used in teaching verbal mathematics problem-solving skills.

Based on the study findings, the beginning level data on the participants' skills of solving comparison-type mathematics problems requiring addition and subtraction operations were compared to end-of-instruction data. It was observed that the participants solved more verbal mathematics problems correctly after the instruction. Retention data demonstrated that the participants retained the acquired skills 1, 3, and 5 weeks after the application was over. The acquisition and retention findings of this study were consistent with other study findings (Baki, 2014; Griffin & Jitendra, 2009; Jitendra et al., 2009; Jitendra et al., 2010; Karabulut et al., 2013; Kot, 2014; Na, 2009; Rockwell, 2012; Rockwell et al., 2011; Tuncer, 2009; Xin, 2008). Since no participant was able to generalize the stimulus at the posttest generalization session based on the generalization findings, generalization instruction was conducted. One reason the participants failed to generalize could be the carrier effect. The carrier effect is the transportation of a skill acquired by the application of an independent variable to a skill related to another variable (Tekin-İftar, 2012b, pp. 113–154). In other words, the participants carried their skills of solving comparison-type verbal mathematics problems with unknown results to solving comparison-type verbal mathematics problems with unknown difference values. After the generalization instructions, Ekin and Zehra were able to generalize the skills they acquired to verbal

mathematics problems where the location of the unknown was altered, while Dilek failed to do so.

In the social validity dimension of this study, the researchers collected information from mothers and classroom teachers using a social validity questionnaire. It could be argued that the study was socially valid based on the social validity findings. The mothers stated that they appreciated that the study was conducted at home and the hours were arranged based on their family schedules, while all teachers expressed their satisfaction with the study and stated that it was effective in teaching their students and they would consider using the schema approach in their classroom activities themselves.

A review of related literature showed that studies on the instruction of verbal mathematics problem-solving skills using the schema approach have generally been conducted in school environments. The present study is unique in that it is the first study in the literature that utilized the home environment for instruction in verbal mathematics problems.

The schemas were drawn prior to the start of the experiment and presented to the participants in a ready-to-use format in this study. This was due to the fact that schemas are a means to acquiring verbal mathematics problem-solving skills, not a goal. Furthermore, it was considered that allowing the participants to draw schemas during the application would cause unproductive use of the available instructional time. Finally, since the schemas were presented that way, instructional sessions on how to draw a schema were not conducted.

The study has certain limitations as well. First, social validity data for the study were collected with a subjective assessment tool. The social comparison model was not used in social validity data collection. Second, social validity data were collected only from the participants' mothers and classroom teachers. Third, only comparison-type verbal mathematics problems that require addition and subtraction operations were used. Verbal mathematics problems that require multiplication and division operations were not used.

Based on the current study findings and application, certain recommendations can be made for future research and practice. Further studies where students are taught how to draw schemas used in the schema approach in mathematics classes could be conducted. Second, studies could be carried out in which mothers learn the schema approach so they can assist their children in doing the homework at home. Based on the study findings, the following recommendations are made for the future studies: (a) the schema approach was presented with the direct instruction method in the present study; future studies where the schema approach would be presented with one of the

near-errorless teaching methods (e.g., constant or progressive time delay procedures) could be planned; (b) the first author actually conducted the research in the present study; further studies could be planned where the schema approach is taught to students' mothers, teachers, or pre-service teachers; (c) the participants in the present study were individuals with ASD; similar studies could be conducted with different disability groups or with individuals with ASD but with different characteristics; (d) in this research, comparison-type verbal mathematics problems were studied; change and/or grouping-type verbal mathematics problems could be used in future studies; (e) in this research, comparison-type verbal mathematics problem-solving skills with the schema approach were studied; future studies where the schema approach would be used in the instruction of other mathematical skills could be planned; (f) the present study focused on instruction of verbal mathematics problems that required addition and subtraction of single-digit numbers; future studies where problem examples that require the addition and subtraction of numbers with two or more digits could be conducted; (g) in the present study, social validity data were not collected from the participants; similar studies where social validity data would be collected from the participants could be conducted; and (h) social validity data were collected via a subjective assessment method in the present study; similar studies where social validity data would be collected with the social comparison method could be conducted.

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